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Abstract

This paper introduces g3+, the new core forecasting model of the Czech National Bank (CNB), which replaced the previous g3 model in July 2019. We present the features of the new core forecasting model together with our motivation for adopting them. The new structural features and extensions were motivated by our experience with using the g3 model for more than a decade as the core forecasting tool at the CNB. The new g3+ model features a novel structural foreign economy block, oil as a production factor, heterogeneous households, and other adjustments. Also, we present a new simulation approach that allows us to emulate limited information for the simulation of conditional forecasts. Furthermore, the introduction of the g3+ model on average preserves the forecasting performance of the CNB's DSGE modeling framework.

Abstrakt

V této práci představujeme g3+, nový jádrový predikční model České národní banky (ČNB), který od července 2019 nahradil předchozí model g3. V článku popisujeme nové rysy predikčního modelu spolu s motivací pro jejich přijetí. Nové strukturální prvky a rozšíření byly motivovány našimi zkušenostmi s více než desetiletým používáním modelu g3 jako jádrového predikčního nástroje v ČNB. Mezi tyto nové rysy patří strukturální zahraniční blok, ropa jako výrobní faktor, heterogenní domácnosti a další úpravy. Představujeme také nový přístup k simulaci, který imituje omezenou znalost výhledů při tvorbě podmíněných predikcí. Zavedením modelu g3+ v průměru zachováváme predikční vlastnosti DSGE modelovacího rámce ČNB.

JEL Codes: C51, C53, E27, E37, F41.
Keywords: Conditional forecast, DSGE, g3 model, oil, small open economy, two country model.

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1. Introduction

Monetary policy conducted at the Czech National Bank (CNB) relies on the Forecasting and Policy Analysis System (FPAS), which consists of various modeling instruments. Since the release of Inflation Report III/2019 (Czech National Bank, 2019d) in the third quarter of 2019, the new g3+ model has served as the core modeling tool of our FPAS. The g3+ model is inspired by the forecasting experience with a dynamic stochastic general equilibrium (DSGE) model of a small open economy, known as g3, accumulated at the CNB over the last decade.

In this research paper we focus on the novel features of the g3+ model in a form that could be helpful for practitioners from other central banks and policy-oriented institutions. We present our motivation and the reasons behind the implementations and modeling approaches chosen. In this paper, instead of providing the full technical derivation of the DSGE model, we focus on presenting its properties and characteristics.

The new g3+ model aims at offering a more detailed structure of the Czech and foreign economy than the g3 model. It introduces a structural model of the foreign economy, a new type of households, and new production factors for final goods. Further, the new model takes into account changes in the speed of economic convergence, and also allows us to incorporate the specific role of oil prices in an oil-importing economy.

These structural upgrades provide a richer story-telling ability for forecasters and their audience. The increased complexity of the model structure allows forecasters to incorporate their expert judgments explicitly via the newly introduced elements. Beyond developing the model structure, we introduce the concept of limited rationality into the forecasting process in order to deliver an additional degree of consistency between successive forecast updates. This newly introduced concept builds on the limited information approach to exogenous outlooks when solving for the paths of conditional forecasts.

Even though the g3+ model delivers greater structural complexity than its predecessor, it is still comprehensible and easy to operate within the CNB's FPAS. Test statistics indicate that the performance of g3+ compared to the original g3 model is preserved on average. Therefore, the new core model of the CNB's forecasting and policy analysis framework is ready to support past policy analysis results and to provide long-term support for the CNB's monetary policy with a more detailed story.

The paper is organized as follows. Section 2 reminds the reader of the g3 model, which still forms the core of the new g3+ model. It briefly describes the structure of g3 and also summarizes the most important motivations behind the modeling approaches applied.[1] The section also reviews the gradual changes implemented while g3 was in operation as the CNB's core forecasting tool.

Section 3 presents the new foreign economy block of the g3+ model. As a small open economy, the Czech economy is strongly influenced by events originating abroad. The g3+ model newly includes a structural model of the foreign economy based on behavioral equations that supports the story-telling ability of the whole model. The representation of the effective euro area also offers a structural approach to the identification of foreign events via various transmission channels. Consequently, the new approach delivers intuitive and story-based impulse responses to external shocks. The new representation of the foreign economy features, among other things, a description

[1] The original, detailed description of g3 can be found in Andrle et al. (2009).

of the dollar-euro exchange rate, flexible equilibrium interest rates, and decomposition of foreign GDP into its gap and trend components. Last but not least, the new modeling approach allows us to treat the unconventional monetary policy of the European Central Bank (ECB) as an endogenous premium.

Section 3 also offers a brief description of the trade links between the effective euro area and the Czech economy. These trade links take into account the fact that the Czech Republic is a net importer of energy commodities (such as oil and natural gas) but exports mainly higher value-added goods. This asymmetry motivates us to treat energy commodities and their prices explicitly in the new model. Therefore, Section 4 presents the introduction of oil as a new production factor in the domestic economy. Furthermore, this new production factor in the model forms an explicit channel between oil prices and overall consumer price inflation.

As described in Section 5, the g3+ model newly features two types of households and internal habit formation. The first type of households has access to the financial market and hence can save or borrow, thereby smoothing its consumption over time. The second type does not have access to the financial market and spends all its income from work on consumption in the same quarter.

Sections 6 and 7 present the rest of the model upgrades and adjustments. Section 6 explains the motivation behind the implementation of a linkage between domestic investment and foreign GDP growth. Section 7 summarizes the changes made to the model parameters which reflect the structural differences between the g3+ and g3 models or observed changes in the characteristics of the Czech and foreign economies.

On top of the changes made to the model structure, applied policy work with the g3+ model is based on a novel approach to conditioning forecasts on exogenous outlooks. Full anticipation of events without reflecting the limits of agents' capacity for absorbing information from outlooks in the distant future makes it difficult to intuitively explain round-to-round forecast updates and the effects of extending the annual forecasting horizon. Section 8 therefore presents the framework for conditioning on limited information in the CNB's forecasting practice. Within this framework, the information from forecast-conditioning outlooks enters agents' expectations gradually but consistently over the whole range of the simulation. Thus, the limited information approach helps us eliminate the interpretation constraints that arise in the framework based on full information conditioning.

Lastly, Section 9 closes the description of the features of the g3+ model by discussing its properties and performance. Selected impulse responses are scrutinized, while the desired differences between the properties of the g3 and g3+ models are highlighted. A variance decomposition analysis demonstrates the share of the foreign block in explaining the variances of the key variables in g3+. A recursive forecasting and filtering exercise evaluating the model's forecasting performance is also presented. The overall results underpin the credibility of the CNB's past forecasts and provide a solid foundation for future support of the CNB's monetary policy-making.

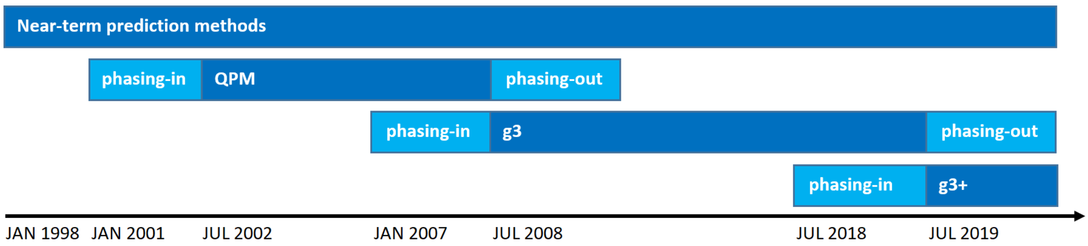
2. The g3 Model as the Forerunner of g3+

In July 2008, the CNB introduced a micro-founded model, g3, as its core tool for regular monetary policy analyses and forecasting. It replaced the Quarterly Projection Model (QPM) (see Figure 1). The QPM is a reduced-form New Keynesian gap model with several ad-hoc features.[2] With the

[2] More details on QPM can be found in the collection of contributions edited by Coats et al. (2003).

adoption of g3, the CNB became one of the few central banks relying on a dynamic stochastic general equilibrium (DSGE) model not only for policy analysis purposes, but also for regular forecasting. The g3 model was used for more than ten years without undergoing any substantial structural change. As Figure 1 shows, in July 2019 it was replaced by an upgraded version, the g3+ model.

Figure 1: Implementation of the Core Forecasting Models of the Czech National Bank



2.1 The Original g3 Model

The g3 model is a structural DSGE small open economy (SOE) type model, primarily designed for regular forecasting rounds at the CNB to support monetary policy decision-making. It follows the New Keynesian tradition and as such it features nominal and real rigidities. Through careful calibration of the model parameters, it captures the main characteristics of the Czech economy, with consistent stock-flow accounting. The introduction of the g3 model and its application in the CNB's forecasting process is thoroughly described in Andrle et al. (2009).

Designed to support the CNB's monetary policy decision-making, the g3 model is an inflation-forecast-targeting, business-cycle model. It relies on forward-looking model-consistent expectations, with endogeneity of monetary policy being its essential characteristic. The central bank's reaction function is based on a monetary policy rule which defines a nominal interest rate path consistent with reaching the inflation target within the monetary policy (MP) horizon.[3] This objective is achieved by setting the monetary policy interest rate in response to the deviation of the one-year-ahead inflation forecast (expected inflation) from the CNB's inflation target.

The g3 model-based forecasts supporting monetary policy decision-making are conditioned on the assumption of endogenous monetary policy responses as implied by the model's monetary policy rule. Further, these forecasts are conditioned on information delivered by exogenous assumptions, such as outlooks for the foreign economy, government spending, and administered prices, and expert judgment.

Since the introduction of a structural model into the core of the CNB's policy framework, the capability to process unanticipated and anticipated information has been viewed as useful for bringing the model forecasts closer to the perceptions of both forecasters and policy-makers.[4] The forecasting framework built around the g3 model has the ability to adopt information from the conditioning outlook as either unanticipated or anticipated, or as a mixture of the two, in a variable-by-variable manner, for the construction of conditional forecasts. However, even the framework with this ability faces a challenge as regards the reliability and accuracy of the conditioning outlooks. Technical

[3] The monetary policy horizon is the time horizon which monetary policy decision-making is focused on. This horizon, which takes into account the monetary policy transmission lag, is about 12–18 months, or 4–6 quarters ahead.

[4] See the explanations of the use of expert judgment and exogenous outlooks for conditioning in Coats et al. (2003).

details on using anticipated and unanticipated information for conditioning on outlooks exogenous to the model in regular forecasting exercises are presented in Appendix D.

The core of the g3 model contains the following sectors: optimizing households, firms producing intermediate and final consumption goods, monetary and fiscal authorities, and a simple foreign economy block approximating the effective euro area[5] (in terms of a two-country model "the rest of the world").

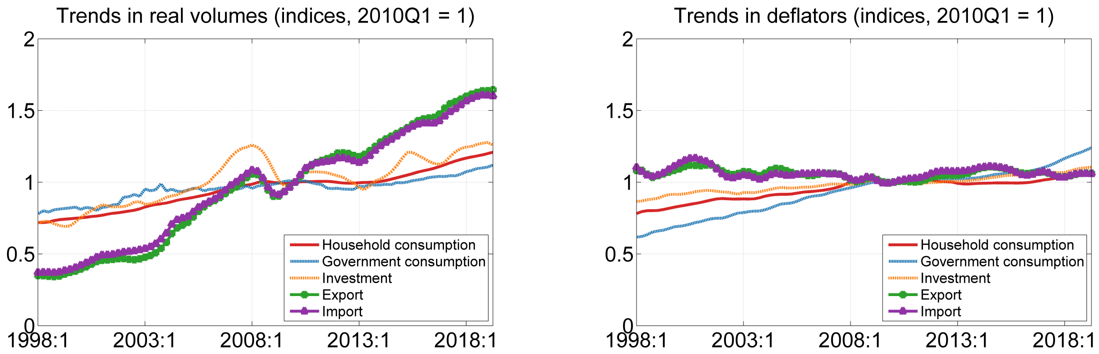
- Households consume final consumption goods under external habit formation, rent capital, and supply labor to the intermediate production sector. They receive a wage based on Calvo (1983) contracts. Households are only allowed to trade domestic nominal government bonds. Trading in international financial markets is delegated to foreign exchange (forex) dealers, whose profit-optimizing behavior implies the uncovered interest rate parity condition.[6]
- The intermediate goods production sector contains two types of firms. Domestic intermediate goods are produced by firms combining capital and labor (adjusted for labor-augmenting technology) via the Cobb-Douglas production function. Imported intermediate goods are produced by importing firms without any additional domestic component. Both types of firms use Calvo contracts to set the prices of their goods, which results in the existence of New Keynesian Phillips curves for both types of intermediate goods.
- Final goods producers manufacture consumption, export, investment, and public-spending goods. The final consumption and export sectors combine imported and domestic intermediate inputs using Leontief production technology. The final investment sector only uses imported inputs for its production, whereas producers of public goods use domestic intermediate products only. Prices are sticky à la Calvo, which results in New Keynesian Phillips curves for each good.[7] Each of the final goods sectors exerts a sector-specific degree of price stickiness.
- The monetary policy authority follows a Taylor-type rule, targeting four-quarter-ahead expected year-on-year headline inflation while paying particular attention to interest rate smoothing (monetary policy inertia). Headline inflation includes administered prices, while the effect of changes to indirect taxes is excluded.[8]
- The government follows its period budget constraint and consumes public-spending goods, which are assumed to be non-productive.
- The foreign economy, described by indicators representing the effective euro area, is modeled as a set of mutually independent AR(1) processes.

The design and calibration of the g3 model take into account the specifics and stylized facts of the Czech economy. OECD (2013) argues that participation in global value chains offered the Czech

[5] The indicators for the effective euro area are determined by weights corresponding to the average share of each country in foreign trade with the Czech Republic.
[6] This setup provides a theoretical foundation for the presence and persistence of deviations from the UIP condition. The resulting UIP form allows us to bring the model close to the data.
[7] Export prices are sticky in foreign currency, whereas the assumption of local currency pricing holds in the remaining sectors.
[8] The g3 model is built on the assumption of households consuming two consumption goods baskets: administered and market-priced goods. Headline inflation characterizes the price movement of the overall consumption basket. Administered prices thus affect both the behavior of consumers and monetary policy. Since administered prices are exogenous to the model, the monetary authority cannot influence them. As a result, changes in headline inflation expectations require that monetary policy actions feed into market prices, in line with the inflation-targeting framework.

economy a fast track to development and further industrialization. As WTO (2019) reports, the Czech Republic's forward participation rate in global value chains rose by 21% between 2007 and 2017, while backward participation followed a similar pattern. This reflects the increasing integration of domestic firms into global value chains, which results in rising trade openness. Figure 2 reveals that import and export growth outpaces that of the rest of the components of real GDP. This pattern is an essential characteristic of the Czech economy.

Figure 2: Normalized Trends in Real Volumes and Prices of GDP Components



Note: Normalized by the 2010 Q1 value of the respective variable.
Source: CNB calculation based on Czech Statistical Office, National Accounts data, Czech Statistical Office.

To account for the increasing trade openness and the common, non-stationary share of nominal imports and exports in nominal GDP, Andrle et al. (2009) introduce openness technology A_t^O and trade productivity technology A_t^X into the measurement equations of the g3 model. This approach, although not thoroughly structural, is a part of the model-consistent endogenous detrending. This approach allows trade openness A_t^O to be identified separately from the remaining elements of growth, thus helping bring the model to data.

To bring the g3 model to data, we avoid univariate methods or out-of-model prefiltering of observed data trends (see Figure 2), as such detrending may result in inconsistencies. In line with the recommendations made by Andrle (2008), the g3 model includes trends in the inner structure of the model instead. This structural approach ensures that trend-cycle interactions present in the observed data are interpreted consistently by the model. However, structural treatment of the model's balanced growth path is preferred to capture both the business cycle and the long-run trend dynamics of the economy.

In the interests of clarity of the model dynamics and application of the model's diagnostic tools, which are built on an assumption of stationarity, all the model variables are stationary around the balanced growth path. To achieve that, the g3 model is closed by following permanent technology processes: labor-augmenting, investment-specific, trade productivity, quality-view, and public goods sector-specific technologies, and a willingness-to-work preference.

2.2 Changes in the Setup of the g3 Model before the Implementation of g3+

The CNB pursues its mandate under an inflation-targeting regime to deliver its goals within the monetary policy horizon. It therefore focuses on the medium rather than the (very) long term. Since the introduction of the g3 model in 2008, its parameters describing medium-term properties have been gradually adjusted. These adjustments have reflected the ongoing structural changes in the

Czech economy and also changes in the characteristics of its more developed trade partners in the aftermath of the 2008 financial crisis.

Convergence can last for decades. It also plays a role in determining the position of the economy in the business cycle. Table 1 summarizes the adjustments made to the long-run steady-state characteristics of the g3 model over the period it served as the core forecasting tool of the CNB's Forecasting and Policy Analysis System. These adjustments reflected the results of regular forecast evaluations and experts' perceptions of the medium-term characteristics of the domestic and foreign economy in the given period. Moreover, such adjustments are supported by the observed data when considering recent data vintages.

An inflation target of 3% was announced in March 2004 and took effect in January 2006. A new value of 2%, which was announced in 2007 and became effective in 2010, additionally considered ongoing changes in the composition of administrative prices and price deregulation together with the harmonization of indirect taxes. These were perceived not to exert significant upward pressure on inflation, as explained in Czech National Bank (2007).

Table 1: Steady States of Domestic Variables

Variable (YoY, %)	**2008q3–2010q1**	**2010q2–2013q3**	**2013q4–2019q2**
Real GDP	5.0	4.0	3.0
Private Consumption	5.0	4.0	3.0
Investment	5.0	4.0	3.0
Government Consumption	5.0	4.0	3.0
Imports	9.0	8.5	7.2
Exports	9.0	8.5	7.2
CZK/EUR	−2.4	−2.4	−1.5
Variable (Level, %)			
Inflation Target	3.0	2.0	2.0

The g3 model, augmented by a measurement equations block, is non-stationary in order to allow forecasters to account explicitly for the multiple transitory and permanent technology processes present in the observed data. To make the forecasting work as straightforward as possible, the transition equations block of the model needs to be augmented with the implied aggregate rate of growth to deliver stationarity. Thus, the aggregate rate of growth ΔZ_t arises as a function of growth of labor-augmenting ΔA_t, investment-specific ΔA_t^J and willingness-to-work ΔA_t^L processes:

$$\Delta Z_t = \frac{1-\gamma}{\gamma}\Delta A_t^J + \Delta A_t - \Delta A_t^L, \tag{1}$$

where γ is the labor share in the Cobb-Douglas production function for domestic intermediate goods.[9]

Perceived adjustments in the convergence of the Czech economy and long-term growth prospects were reflected in adjustments to the growth rates of real variables (see Table 1). The accumulation of these gradual adjustments over a short period (2008–2014) motivated us to implement a common convergence process into the model structure with the aim of ensuring consistent interpretation of

[9] For more details, see the derivation of the original Equation (4.42) in Andrle et al. (2009).

the observed data and forecasts. The evolution of the convergence process is defined by use of the past and present aggregate rate of growth ΔZ_t.

In our implementation, the convergence process is common to all real quantities and is an integral part of the aggregate rate of growth. It thus has implications for identification of the stationary versions of the variables. An additional implemented trend accounts for the changing speed of convergence of the Czech economy toward its advanced euro-area trading partners over the last 20 years, as documented by Ministry of Finance of the Czech Republic (2020) in its Convergence Programme report.

The extension of the g3 model by the stationary convergence trend A_t^C considers adjustments to parameters determining the balanced growth path of real variables. Inspired by the introduction of trade openness technology A_t^O, the convergence process is implemented into the measurement equations block of the model. The convergence trend A_t^C augments the model quantities of the components of GDP when we relate the model to data. Symmetrically, a stationary foreign convergence trend A_t^{C*} was also introduced for the foreign economy. Therefore, agents in the model are assumed to view all real quantities as already deflated by convergence processes. The convergence is thus invisible to them.

With the introduction of convergence processes A_t^C and A_t^{C*}, the definition of the link between the stationary model variables and the observed data on the components of GDP is adjusted as follows:[10]

$$c_t = C_t \frac{A_t^R}{Z_t} \quad \rightarrow \quad c_t = C_t \frac{A_t^R}{Z_t A_t^C}, \tag{2}$$

$$g_t = G_t \frac{1}{Z_t A_t^G} \quad \rightarrow \quad g_t = G_t \frac{1}{Z_t A_t^G A_t^C}, \tag{3}$$

$$j_t = J_t \frac{1}{Z_t A_t^J} \quad \rightarrow \quad j_t = J_t \frac{1}{Z_t A_t^J A_t^C}, \tag{4}$$

$$x_t = X_t \frac{1}{Z_t A_t^X A_t^O} \quad \rightarrow \quad x_t = X_t \frac{1}{Z_t A_t^X A_t^O A_t^{C*}}, \tag{5}$$

$$n_t = N_t \frac{1}{Z_t A_t^X A_t^O} \quad \rightarrow \quad n_t = N_t \frac{1}{Z_t A_t^X A_t^O A_t^{C*}}. \tag{6}$$

Here, the permanent technology processes in the g3 model are: administered price goods technology in consumption A_t^R, public goods sector-specific productivity A_t^G, investment-specific technology A_t^J, trade productivity technology A_t^X, and growth in trade openness A_t^O. C_t, G_t, J_t, X_t, and N_t are the levels of growth of household consumption, government consumption, investment, exports, and imports, respectively, and c_t, g_t, j_t, x_t, and n_t are their stationary counterparts.[11]

When taking the model to data, changes in convergence processes are treated as an observed variable, while the remaining technology processes used for detrending are endogenously identified. Past deviations from the actual setting of steady-state growth are reflected as an observation of changes in the domestic ΔA_t^C and foreign ΔA_t^{C*} convergence processes. Also, it is assumed that changes in the paths of convergence processes are a smooth interpolation of these past deviations.

[10] The left-hand side presents the original definitions of the stationary variables. The right-hand side displays the definitions of the stationary variables after the model was modified by the introduction of convergence processes A_t^C and A_t^{C*}.

[11] Uppercase typeface indicates non-stationary variables, while lowercase typeface denotes the stationary version of the variable considered.

Following the arguments presented by Andrle et al. (2009)[12], the approach to the convergence process implemented is still perceived as endogenous detrending. Even though the implementation of the convergence process is not supported by microfoundations, it allows us to exploit the structure of the model in identifying stochastic trends in the economy. The stationarization of the model variables with the updated measurement block, given by Equations (2)–(6), preserves the cyclical characteristics of the model. This setup is intentional and has clear advantages when bringing the model to data.

3. New Foreign Economy Block

In the above g3 model, the foreign economy is described in simple stylized form. The g3 foreign economy is reduced to three variables – gross domestic product (GDP) and the producer price index (PPI) in the effective euro area, and the short-term interest rate – which are modeled as structurally independent autoregressive processes. This simplistic modeling does not allow for structural interpretation of developments in the foreign economy. Structural identification of foreign shocks is important, as it provides an elaborate data-based story of the foreign block forecast. It also supports smooth delivery of the forecast to policy-makers and the general public when accounting for the foreign shocks transmission into the domestic economy.

3.1 Structure of the Foreign Economy Block

The motivation for the development of a new structural foreign economy block stems from the following considerations. The CNB's conditional forecasting framework is built on the assumption that the foreign economy outlook is interpreted and included in the forecast via the foreign economy block. Therefore, our primary goal is to achieve a coherent structural interpretation of the external outlooks under the aforementioned assumption when transmitted to the domestic economy.

In our view, structural interpretation of foreign shocks results in an improvement in the robustness of the external environment analysis and in transparent interpretation of domestic developments. It thus allows the model to meet the needs of the domestic policy authority in its efforts to steer domestic inflation toward the target. Therefore, for the model update it is desirable to introduce a structural model of the foreign economy.[13]

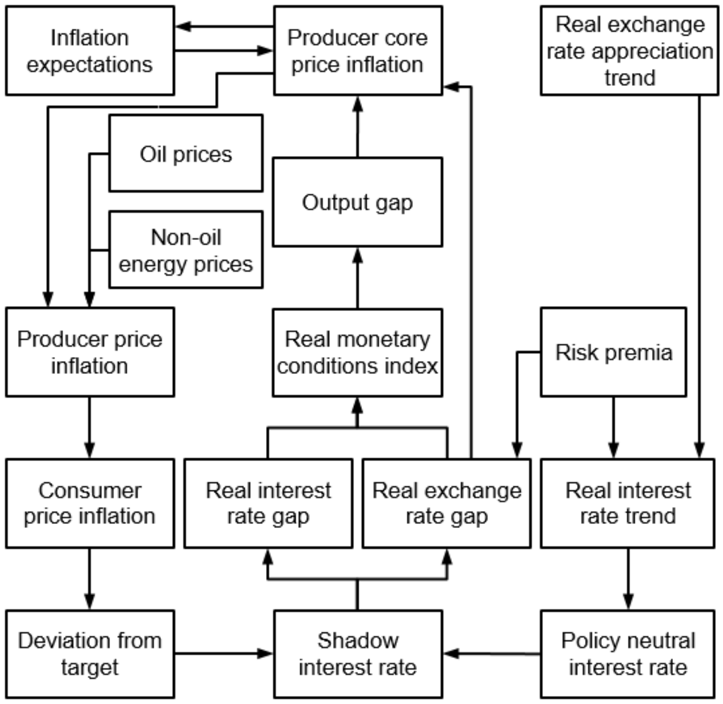
In order to meet operational needs and lay the groundwork for further development, the design of the foreign economy block should allow for its structure to be changed without altering the equations representing the domestic economy. Thus, our secondary goal is to achieve independence of the integration of the foreign economy into the model.

Furthermore, since the Czech Republic is a small open economy, such integration should also prevent spillovers from the domestic economy to the foreign one. Therefore, a two-stage approach is used to prevent such spillovers. In the first stage, foreign variables are passed to a standalone foreign model to identify unobserved variables. In the second stage, the foreign economy block variables playing a role in the integration link are treated as observed (that is, as assumptions of the forecast) from the domestic economy perspective. The remaining foreign variables define a stan-

[12] For more details see Section 4.2 in Andrle et al. (2009).

[13] In our conditional forecasting framework, the model update also includes a crucial decision on the amount of anticipated and unanticipated information when interpreting developments in the foreign economy. As conditional forecasting is the final layer of application of the model, a detailed explanation of conditional forecasting with anticipated information follows in Section 8.

Figure 3: Structural Relations within the Foreign Economy Block



dalone model block. This setup allows us to approach domestic developments as conditional on assumptions about the foreign economy but not vice versa. This foreign block integration design also allows us to analyze developments in the foreign economy independently of the domestic part of the g3+ model. In our view, this design supports operability and extends our analytical capacity.

The options for modeling the foreign block ranged from a general VAR model without a structural interpretation, through to a fully micro-founded two-country model of the euro area and the US economy with high complexity and high costs of day-to-day use. Considering the aforementioned goals, a small semi-structural gap model of the effective euro area was chosen. The preference for a semi-structural model reflects a cost-benefit choice between the theoretical soundness of the model and its complexity for regular forecasting and monetary policy analysis. Its size and structure support easy adoption by forecasters and can be smoothly communicated to policy-makers. Figure 3 presents the central relationships in the foreign economy model adopted.

Our foreign economy block, representing the effective euro area[14], in the form of a small semi-structural small-open-economy gap model, consists of a few behavioral equations and definitions. The core of the model is made of four equations: the IS curve, the Phillips curve, a monetary policy rule, and the UIP condition.

The IS curve – Equation (7) – describes the behavior of foreign aggregate demand. Its construction assumes the existence of rigidities and responsiveness to real monetary conditions.

$$\hat{y}^*_t = a_{11}\hat{y}^*_{t-1} - a_{12}\hat{r}^*_{t-1} + a_{13}\hat{z}_{t-1} + \varepsilon^{y^*}_t . \tag{7}$$

[14] The effective euro area is a fictional economy constructed as a trade-weighted aggregate of the Czech Republic's euro area partners. In this construction, Germany and Slovakia have the largest shares.

The IS curve positively relates the foreign output gap $\hat{y}_t^*$ to its lag $\hat{y}_{t-1}^*$, thus delivering persistence resembling habit formation and other rigidities. The following two terms together capture the real monetary conditions, through which monetary policy affects the real economy. The negative relation with the lagged real interest rate gap $\hat{r}_{t-1}^*$ describes the interest rate channel of euro area monetary policy.[15] In an open economy, the real effective exchange rate influences aggregate demand through net exports, as it affects exporters' competitiveness. The real exchange rate is not a policy instrument itself; however, uncovered interest rate parity links it to the monetary policy interest rate. Thus, the exchange rate channel of euro area monetary policy is described by the positive effect of the lagged real exchange rate gap $\hat{z}_{t-1}$ on aggregate demand.

The Phillips curve – Equation (8) – is adopted in the expectations-augmented open-economy version. It links foreign core inflation $\tilde{\pi}_t^{*core}$ to inflation expectations and real marginal costs proxies.

$$\tilde{\pi}_t^{*core} = a_{22}\,\tilde{\pi}_{t-1}^{*core} + (1-a_{22})\,\tilde{\pi}_{t+1}^{*core} + a_{21}\,\hat{y}_t^* + a_{23}\,\hat{z}_{t-1} + \varepsilon_t^{\tilde{\pi}^{*core}}. \tag{8}$$

Here, the lagged term $\tilde{\pi}_{t-1}^{*core}$ captures agents' backward-looking expectations based on learning, or indexation, while the lead $\tilde{\pi}_{t+1}^{*core}$ stands for expectations. The real marginal costs proxies consist of the output gap $\hat{y}_t^*$, as a representation of excess aggregate demand pressures, and the real exchange gap $\hat{z}_{t-1}$, as the pass-through from the exchange rate via imported goods prices.

Foreign core inflation $\tilde{\pi}_t^{*core}$ is defined as inflation of foreign producer prices P_t^* excluding prices of energy, as energy prices are assumed to be a factor exogenous to the foreign economy. The decomposition of the foreign producer price index P_t^* is given by:

$$P_t^* = \left(P_t^{*oil}\right)^{\rho_{\frac{oil}{ppi}}} \left(P_t^{*enerExoil}\right)^{\rho_{\frac{energy}{ppi}} - \rho_{\frac{oil}{ppi}}} \left(P_t^{*core}\right)^{1-\rho_{\frac{energy}{ppi}}}, \tag{9}$$

where P_t^{*oil} is the oil price denominated in euro, $P_t^{*enerExoil}$ represents prices of energy-related goods excluding oil, P_t^{*core} stands for the core price index, and $\rho_{\frac{oil}{ppi}}$, $(\rho_{\frac{energy}{ppi}} - \rho_{\frac{oil}{ppi}})$, and $(1-\rho_{\frac{energy}{ppi}})$ are the respective shares of the components in the producer price index (PPI). Equation (9) presents the stationary version of the PPI, while the data reveals the trend in the relative price of the core and energy components. As we assume improvements in the energy efficiency of production, we can derive the stationary version of the PPI and bring it to data.[16] Further, given that the ECB's inflation target is defined in terms of the consumer price index (CPI), a simple bridge equation links producer price inflation $\tilde{\pi}_t^*$ to consumer price inflation $\tilde{\pi}_t^{*cpi}$, as Equation (A.10) in Appendix A shows.

As the foreign economy block represents an open economy, a modified uncovered interest rate parity (UIP) condition is used to describe the nominal dollar-euro exchange rate dynamics. In this implementation, the rest of the world is approximated by the US. The foreign UIP condition takes the form:

$$\begin{aligned} 0 = {} & \rho_{usdeur}\Delta usdeur_{t+1} - (1-\rho_{usdeur})\Delta usdeur_t + \\ & + (i_t^* - i_t^{*eq} - prem_t^*) + \varepsilon_t^{\Delta usdeur}, \end{aligned} \tag{10}$$

where the expected change of the nominal dollar-euro exchange rate $\Delta usdeur_{t+1}$ and the actual change in the exchange rate $\Delta usdeur_t$ are linked to the changes in the deviations of the euro area market interest rate i_t^* from its equilibrium level i_t^{*eq} and a risk premium $prem_t^*$ represented by a

[15] As the euro area has a single monetary policy, there is no difference between the euro area interest rate and its effective version.
[16] See Equation (25) in Section 4 for more details on energy efficiency trend implementation.

simple autoregressive process. The modified UIP condition takes a very simplified form but allows us to capture some degree of persistence of capital mobility between the euro area and the rest of the world. Thus, the response of the dollar-euro exchange rate to monetary policy can mimic the properties of the data.

In recent history, unconventional monetary policy tools have been adopted by the ECB, leading to a disconnect between policy and market interest rates.[17] The nature of the unconventional policy motivated an extension of the foreign economy block through the introduction of a shadow interest rate. The unconventional component of the shadow interest rate is the CNB's estimate of the effect of the ECB's asset purchase programs introduced since 2008.[18] This estimate is based on the elasticity of the shadow interest rate with respect to the new net asset purchases of the ECB. The elasticity applied was derived using the National Institute Global Econometric Model (NiGEM).[19]

Together with the explicit treatment of foreign unconventional monetary policies in the form of the shadow interest rate, the foreign economy features a market interest rate representing the interest rate relevant for interbank and foreign exchange trades.[20] Such judgement can be introduced because the euro area policy rate – the shadow interest rate – directly affects the foreign output gap and inflation and is therefore effective mostly within the foreign economy block. However, the interbank interest rate is also important for the Czech economy, as it is relevant for financial transactions.

The interbank and policy interest rates are linked together by a premium stemming from the effects of the ECB's unconventional policies (see Equation (11)). This simple relation allows us to collapse these two rates into a single interest rate when no unconventional monetary policy tools are in effect:

$$
\begin{aligned}
i_t^* - i_t^{*shadow} &= premium_t^{*shadow}, && (11)\\
premium_t^{*shadow} &= \rho^{*shadow} * premium_{t-1}^{*shadow} + \varepsilon_t^{*shadow}. && (12)
\end{aligned}
$$

Here, $premium_t^{*shadow}$ is the premium of the interbank interest rate i_t^* over the monetary policy interest rate $i_t^{*shadow}$ and reflects the effects of unconventional monetary policy tools. Equation (12) states an AR(1) process for this premium with persistence given by an AR parameter $\rho^{*shadow}$ and subject to disturbances given by a shock $\varepsilon_t^{*shadow}$.[21] A presentation of the micro-foundations of the adopted relationship between the foreign shadow rate and the interbank rate is out of scope of this paper, as both rates are exogenous inputs to the CNB's forecasts.

[17] The ECB started purchasing bonds from commercial banks in March 2015. In the first round of quantitative easing (QE), which lasted until the end of 2018, the ECB spent EUR 2.6tn on purchases. The ECB launched a second round of QE in September 2019 and by November 2020 had expanded its balance sheet by an additional EUR 1tn.

[18] The refinancing operations of the ECB are excluded from the definition of the shadow interest rate.

[19] More details on the NiGEM model can be found in Hantzsche et al. (2018).

[20] In many applications the 3-month EURIBOR is used as the representation of the interbank interest rate. However, practical forecasting may require adoption of expert judgment into interest rate path if one believes that there are other financial market forces (such as internal exposure limits in banks) that could influence the UIP condition.

[21] This setup implies that under the assumption of no change in the unconventional monetary policy, the shadow and interbank interest rates co-move. However, should the unconventional monetary policy change, the paths of the shadow and interbank interest rates will differ. This difference will result in presence of non-zero realizations of a premium shock $\varepsilon_t^{*shadow}$. Our model structure allows for modeling an unconventional monetary policy expansion as a negative (expansionary) monetary policy shock $\varepsilon^{i^{*shadow}}$ in Equation 13 leading to a decrease of the shadow interest rate. Model structure allows also for flexibility, when under assumption that this expansion is not passed further to the interbank interest rate, such expansion will be rendered by a positive (restrictive) premium shock $\varepsilon^{*shadow}$.

The introduction of a monetary policy reaction function, represented by a Taylor-type reaction function consistent with achieving the medium-term inflation target (Equation (13)), complements the main behavioral equations of the foreign economy block. The ECB sets its policy interest rate in response to the expected deviations of the consumer inflation forecast from the inflation target and the deviations of aggregate demand from its trend value as follows:

$$\begin{aligned} i_t^{*shadow} &= a_{33} i_{t-1}^{*shadow} + (1-a_{33}) \Bigg(i_t^{*shadow,eq} + a_{31} \hat{y}_t^* + \\ &\quad + a_{32} \left(\frac{\tilde{\pi}_{t+4}^{*cpi4}}{4} - \tilde{\pi}^{*cpi4} \right) \Bigg) + \varepsilon_t^{i^{*shadow}} . \end{aligned} \tag{13}$$

Here, nominal interest rate $i_t^{*shadow}$, representing foreign monetary policy tool, follows expected deviation of harmonized consumer price inflation (four quarters ahead) $\tilde{\pi}_{t+4}^{*cpi4}$ from the foreign inflation target $\tilde{\pi}^{*cpi4}$ and the contemporary foreign output gap $\hat{y}_t^*$. The monetary policy rule also accounts for presence of the smoothing of the foreign shadow interest rate, which represents the persistence, parameterized by a_{33}, characteristic for the behavior of a conservative central bank.

The model of foreign economy is closed by definitions (Equations (A.6)–(A.12)) and autoregressive processes (Equations (A.13)–(A.18)). The respective equations and parameterizations can be found in Appendices A and B.

3.2 Taking the Foreign Economy Block to Data

As mentioned earlier, the foreign economy block of the g3 model is built on three observed variables. Effective euro area GDP and PPI inflation are employed to represent foreign demand and price developments. Foreign monetary policy is characterized by the shadow interest rate, which also accounts for non-standard (unconventional) monetary policy measures. The effective GDP and PPI indicators are the result of CNB calculations combining individual country data from the Eurostat database with export structure data from the Czech Statistical Office to create export-weighted indicators. The *shadow* interest rate is a result of a CNB calculation based on market interest rates (such as 3M EURIBOR) obtained from Bloomberg and the European Central Bank (ECB) and data on various asset purchase programs from the ECB.

The introduction of structural representation of the foreign economy into the g3+ model expands the set of observed variables. Alongside effective euro area GDP y_t^*, a decomposition into its trend y_t^{*trend} and gap $\hat{y}_t^*$ components is added. Identification of these components is a result of a set of approaches ranging from the simple Hodrick and Prescott (1997) filter, supported by identification in the NiGEM model, to expert interpretation within the independent foreign block.

Next, effective euro area PPI inflation $\tilde{\pi}_t^*$ is supplemented by a decomposition into its energy $\tilde{\pi}_t^{*enerExoil}$ and core non-energy $\tilde{\pi}_t^{*core}$ components. Also, effective euro area CPI inflation $\tilde{\pi}_t^{*cpi}$ is added. The effective euro area PPI and CPI indicators are a result of CNB calculations based on country data from the Eurostat database. The decomposition into their energy and core components is based on CNB computations drawing on the Statistical Classification of Economic Activities in the European Community (NACE).[22]

[22] Details can be found in Czech National Bank (2019a).

Further, the Brent crude oil price denominated in USD $\tilde{\pi}_t^{oil,usd}$ is added together with the nominal dollar-euro exchange rate $usdeur_t$. These data come from Bloomberg. Monetary policy in the euro area is described by the shadow interest rate $i_t^{*shadow}$ and its equilibrium level $i_t^{*shadow,eq}$. These data stem from CNB calculations using Bloomberg data together with the estimation of the effects of the ECB's unconventional monetary policies.

The observed data cover all key macroeconomic variables of the foreign block and the data set starts from 1998 Q1.[23] Variables such as the real interest rate gap $\hat{r}_t^*$, the real exchange rate gap $\hat{z}_t$, and the risk premium $premium_t^{*shadow}$ are not observed. These variables are identified by the structural mechanisms of the model via Kalman smoothing.

3.3 Properties of the Foreign Economy Block

In this section, we present simulated impulse responses of the main macroeconomic variables to selected shocks to illustrate the basic dynamic properties of the foreign economy model. The illustration of the model properties is supplemented by the identification of structural shocks.

There are four fundamental shocks in the foreign economy block: a demand shock represented by a shock in the IS curve $\varepsilon_t^{y^*}$, a supply shock represented by a shock in the Phillips curve $\varepsilon_t^{\tilde{\pi}^{*core}}$, a monetary policy shock represented by a shock in the monetary policy rule $\varepsilon_t^{i^{*shadow}}$, and an exchange rate shock $\varepsilon_t^{\Delta usdeur}$. The rest of model's shocks are considered as residuals, for example, a shock to the decomposition of the foreign producer price index ($\varepsilon_t^{\tilde{\pi}^*}$) and a shock to the equation which implies the dynamics of CPI inflation $\varepsilon_t^{\tilde{\pi}^{*cpi}}$, or shocks leading variables defined by AR processes. These AR drivers are: a shock to the equilibrium shadow interest rate $\varepsilon_t^{i^{*shadow,eq}}$, a shock to the risk premium $\varepsilon_t^{prem^*}$, a shock to the PPI energy equation $\varepsilon^{\tilde{\pi}^{*enerExoil}}$, a shock to the Brent crude oil price $\varepsilon_t^{\tilde{\pi}^{oil,usd}}$, a shock to the oil price trend $\varepsilon_t^{\tilde{\pi}^{oil,trend}}$, and a shock to foreign GDP trend growth $\varepsilon_t^{\Delta y^{*trend}}$.

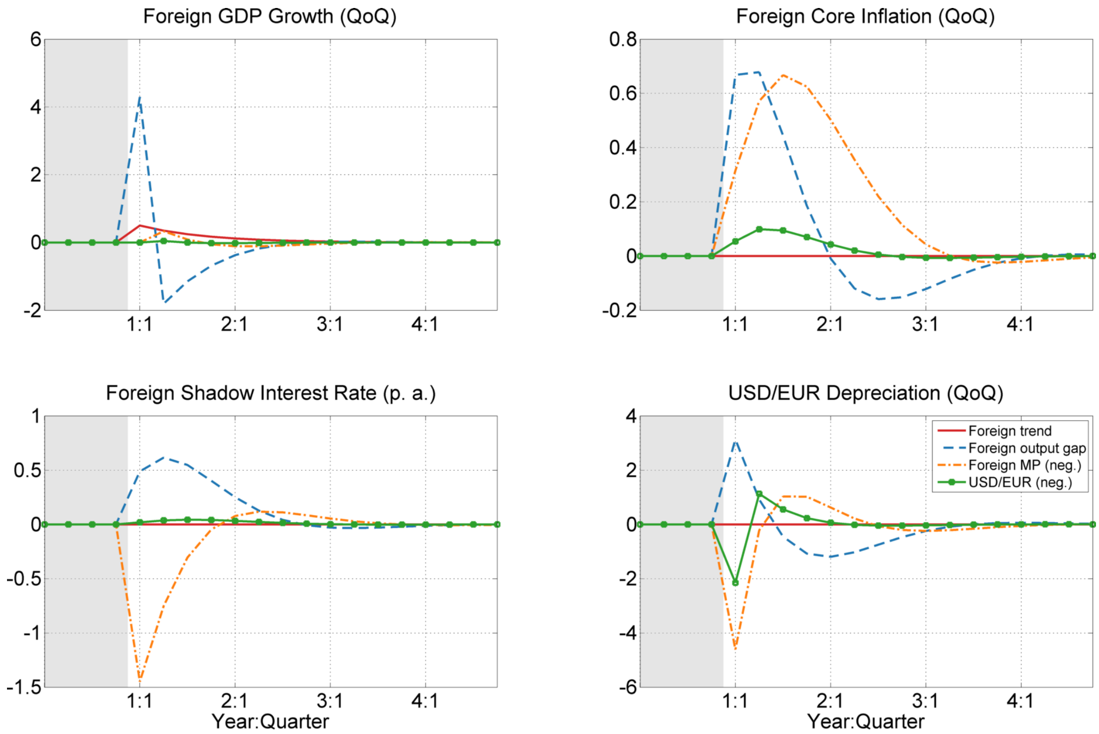
Figures 4 and 5 summarize the responses of the selected variables to a set of one standard deviation shocks. In these figures, the vertical axis shows the deviation in percentage points from the steady state, while the horizontal axis shows time with years as the base unit.

There are four demand-type shocks (see Figure 4) in the foreign economy block of the g3+ model.[24] A positive shock to foreign trend demand $\varepsilon_t^{\Delta y^{*trend}}$ increases overall foreign demand itself without having any effects on the rest of the foreign variables. With the exception of this trend shock, the realizations of shocks $\varepsilon_t^{y^*}$, $\varepsilon_t^{i^{*shadow}}$, and $\varepsilon_t^{\Delta usdeur}$ support a positive demand response and also intuitively increase inflation. The foreign output gap and USD/EUR shocks increase the shadow interest rate, while the negative foreign monetary policy shock increases demand by reducing the foreign shadow interest rate. As the response to the foreign output gap shock shows, inflation reacts strongly and persistently to aggregate demand, thus forcing a tightening of euro area monetary policy to curb future inflation, which leads, in turn, to a short-lived appreciation of the euro.

[23] In the process of development of the foreign economy block, the parameters of the foreign economy block equations were initially estimated. Nevertheless, as the foreign economy block is connected to the rest of the g3+ model, the parameters were adjusted to ensure plausible model properties. While the literature and partial estimation were used as a general guide, expert judgment was needed to achieve intuitive model behavior and to capture the most important features in the data.

[24] The definition of a positive demand-type shock is a positive response of both the output gap and inflation. The definition of a positive supply-type shock is based on positive and negative responses of the output gap and inflation, respectively. Such classification serves for ease of grouping of the impulse responses in the presented figures.

Figure 4: Demand Shocks in the New Foreign Economy Block (deviation, %)



Note: The figure shows the impulse responses of the variables to a one standard deviation surprise shock. The responses are measured in percentage point deviations from the steady state.

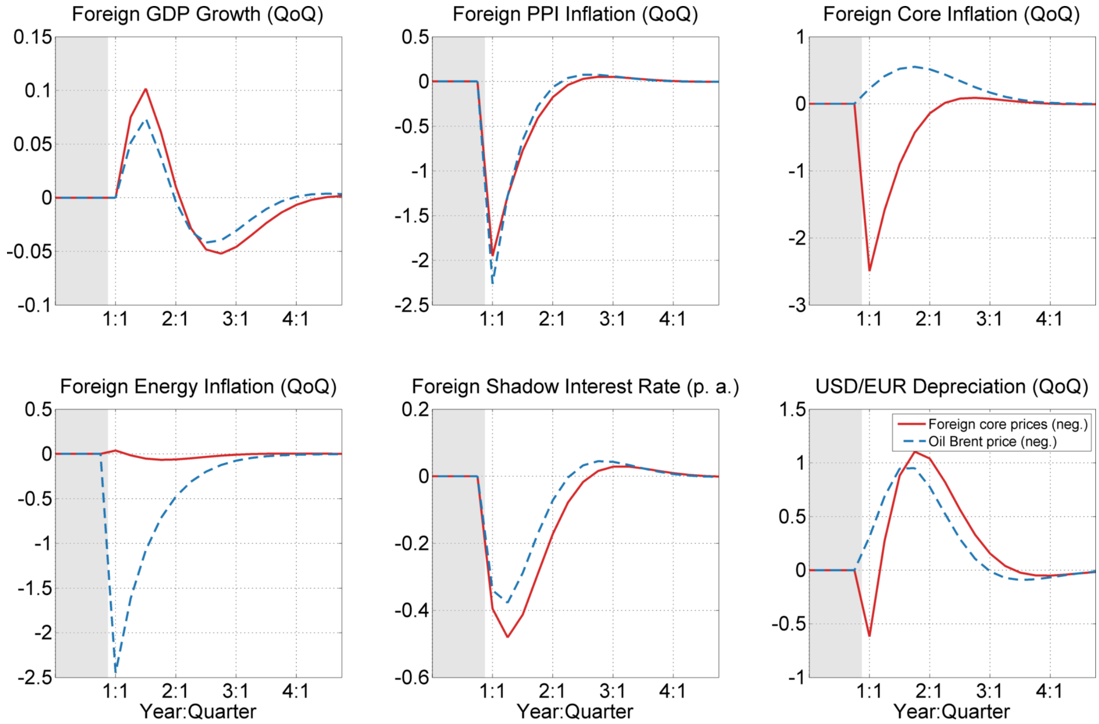
The negative dollar-euro exchange rate shock $\varepsilon_t^{\Delta usdeur}$ sparks euro depreciation, which supports growth in aggregate demand. The euro area monetary policy authority thus responds by increasing the shadow interest rate.

There are two shocks in the g3+ foreign economy block which can be considered supply-type shocks (see Figure 5). First, the negative foreign core inflation shock $\varepsilon_t^{\tilde{\pi}^{*core}}$ leads to a decrease in core inflation. The monetary authority responds by easing its stance to shift inflation back to the target by reducing the shadow nominal interest rate. The expansionary policy and lower prices stimulate growth in foreign aggregate demand. A decrease in the shadow interest rate results in a short-term depreciation of the euro. However, as this response leads to an improvement in competitiveness and supports the euro area trade balance, the euro appreciates in the medium term.

Second, the negative oil price shock $\varepsilon_t^{\tilde{\pi}^{oil,trend}}$ results in lower headline CPI and PPI inflation, so monetary policy is eased. A decrease in the shadow interest rate positively supports aggregate demand growth and results in depreciation of the euro against the dollar. After that, core inflation speeds up on the back of policy easing and higher demand supported by the low price of energy. The depreciation of the euro against the dollar starts to decelerate after a while, reflecting an improvement in the net foreign assets position. However, it depreciates in real terms, since the lower PPI inflation outweighs this effect.

To present the new features of the foreign economy block, we document its ability to interpret historical developments by identifying selected structural shocks and assessing their impact on inflation and monetary policy. In contrast to g3, the g3+ model offers a detailed foreign economy

Figure 5: Supply Shocks in the New Foreign Economy Block (deviation, %)



Note: The figure shows the impulse responses of the variables to a one standard deviation surprise shock. The responses are measured in percentage point deviations from the steady state.

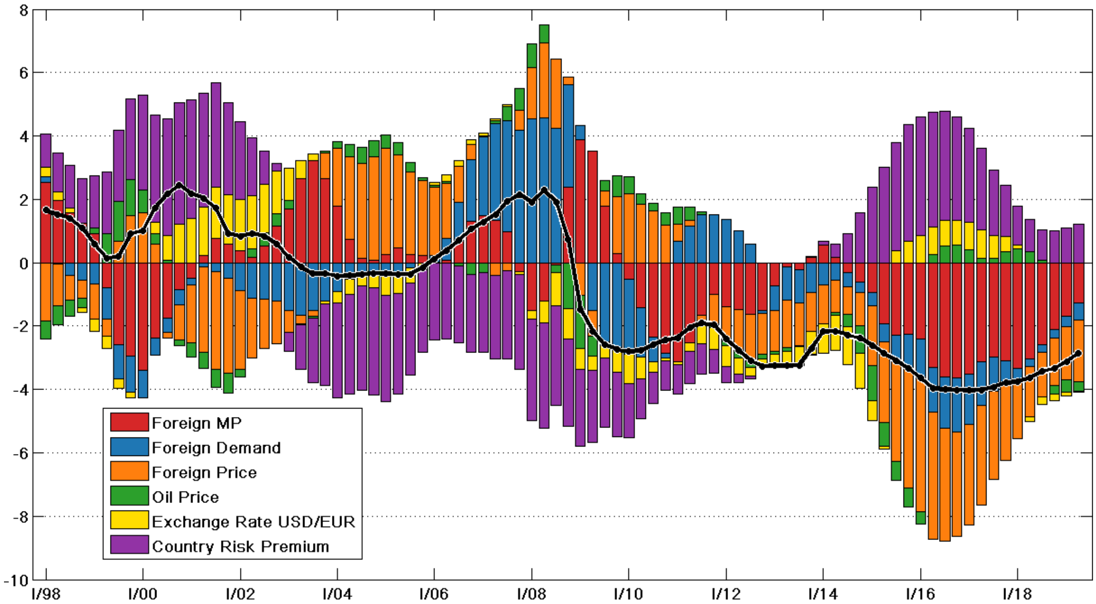
structure and therefore provides a structural interpretation of developments in the foreign economy while being able to consider conventional and unconventional policy responses. The identification of selected structural shocks allows forecasters to understand the story provided by the outlooks for the foreign economy.

Figure 6 presents the decomposition of the deviations of the shadow policy interest rate $i_t^{*shadow}$ from its steady state into structural shocks. It documents a significant contribution of foreign demand prior to the financial crisis in 2008 (blue bars indicate a positive contribution of foreign demand) to interest rate increases, while monetary policy contributed with additional easing (negative red bars) with respect to the endogenous response as prescribed by the foreign policy rule. The sudden drop in foreign demand during the financial crisis was not matched by monetary policy easing as implied by the model policy rule (positive red bars).

From 2011 on, strong anti-inflationary pressures are present, accompanied by appreciation of the euro against the dollar. Further, monetary policy easing (negative red bars over 2010–2012) contributed to a low shadow policy interest rate. The anti-inflationary pressures were exacerbated by the decline in oil prices after 2014. Since then, the ECB has tried to mitigate the strong anti-inflationary effects through a low shadow interest rate, which deviates considerably from the level implied by the endogenous response of the foreign monetary policy rule.

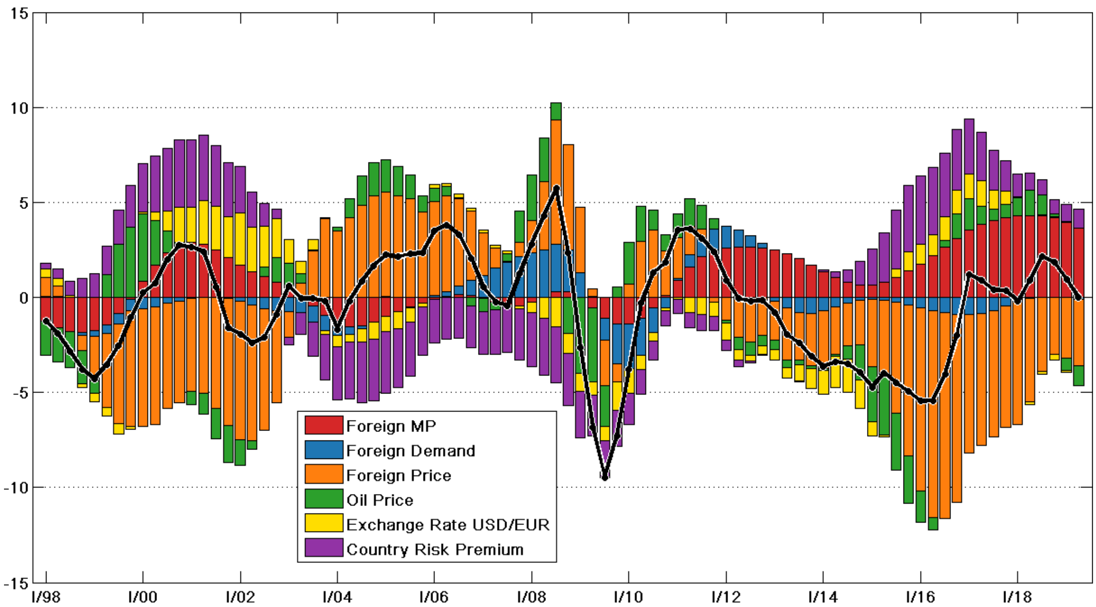
Similarly, the decomposition of the deviation of foreign PPI inflation into structural shocks (Figure 7) reveals a significant presence of positive price and strong foreign demand shocks as the source of foreign price growth prior to the 2008 financial crisis. Our decomposition identifies subdued foreign demand along with strong anti-inflationary effects of core and oil prices, supported by

Figure 6: Foreign Shadow Interest Rate Deviations: Structural Shocks Decomposition (p.a., %)



Note: The figure shows the decomposition as the deviation from the steady state to facilitate interpretation of the contributions. The solid black line is the sum of these contributions.

Figure 7: Foreign PPI Inflation Deviations: Structural Shocks Decomposition (YoY, %)



Note: The figure shows the decomposition as the deviation from the steady state to facilitate interpretation of the contributions. The solid black line is the sum of these contributions.

appreciation of the dollar against the euro, as the main contributors to the low inflation rate. However, the easing of monetary conditions beyond the levels suggested by the model policy rule in the aftermath of the financial crisis (positive red bars) was not able to offset these anti-inflationary effects and return inflation to its target value until 2017.

3.4 Linking the Small Open Domestic Economy

The foreign economy block of the g3+ model delivers a higher degree of complexity than the foreign economy representation in g3. Therefore, a more elaborate approach to linking foreign variables to the domestic economy needs to be applied. Although the variables from the original foreign economy block of the g3 model are still present in g3+, they no longer play an essential role in the evolution of the foreign economy, as their components have mostly taken over the role.

In g3+, the foreign economy's output gap $\hat{y}_t^*$ is the primary determinant of foreign demand and its cyclical position for the domestic economy. This is in contrast to the g3 model, where foreign economic activity was not explicitly decomposed into its trend and gap components. Similarly, inflation of core (non-energy) foreign producer prices $\tilde{\pi}_t^{*core}$ is the main driver of foreign prices, whereas g3 did not feature a decomposition of foreign producer prices. So, for foreign demand and the foreign price index, the foreign-domestic economy link has been modified.

Further, according to the uncovered interest rate parity (UIP) condition for assets in Czech koruna and euro, changes in the foreign interest rate result in movements of the CZK/EUR exchange rate, which in turn lead to changes in the competitiveness of domestic exporters and therefore in domestic demand for imports. However, based on our discussions with the foreign exchange traders at the CNB, the dominance of the foreign market interest rate i_t^* over the foreign monetary policy interest rate $i_t^{*shadow}$ prevails for the relevant contracts.

The UIP condition therefore features the foreign market interest rate i_t^* as the relevant interest rate for defining exchange rate changes.[25] In g3+, the domestic-foreign economy UIP condition takes the same log-linearized form as in the g3 model:

$$i_t = i_t^* + [(1-\rho)E_t\Delta s_{t+1} + \rho(2\overline{\Delta s} - \Delta s_t)] + prem_t + \varepsilon_t^{uip}, \tag{14}$$

where i_t is the domestic monetary policy interest rate, Δs_t denotes the change in the CZK/EUR exchange rate (with its steady-state appreciation rate $\overline{\Delta s}$), $prem_t$ is the Czech economy risk premium, and ε_t^{uip} is a transitory shock to the UIP condition. Thus, the foreign shadow interest rate $i_t^{*shadow}$ does not directly enter the exchange rate channel of transmission of foreign monetary policy to the domestic economy. Nevertheless, the foreign shadow interest rate i_t^* indirectly influences domestic variables via the international trade channel.

The introduction of energy prices into the structure of the foreign economy has resulted in an additional layer in foreign prices. The g3 model features only the aggregate foreign producer price index, whereas g3+ provides a decomposition of the aggregate price index into three components (see Equation (9), Figure 10, and Equation (25) for a detailed description).

The detailed price index structure results in a modification of the domestic-foreign links, which, in turn, leads to changes in their properties. The price competitiveness of exporters is now defined vis-à-vis the core component of foreign producer prices (see Equation (16)). The motivation for this modification stems from the foreign trade structure of the Czech Republic, in which energy exports play only a limited role. Generally, in most of the equations originating in the g3 model, the core component of foreign producer price inflation replaces the overall foreign price index in g3+. The linkage of the energy and oil components of the producer price index is described in detail in Section 4.

[25] For more details, see the explanation presented in footnote 20.

Further, when linking domestic export dynamics to foreign demand, the g3 model relies on an endogenously detrended measure of foreign demand for domestic goods. In this process, foreign demand is represented by effective euro area GDP multiplied by a factor of four. The detrended measure is defined as the ratio of foreign demand to its steady-state growth rate augmented by the growth of the remaining technologies (for example trade openness technology, see Andrle et al., 2009). Hence, it is mainly the cyclical component of foreign demand (given the price of exports relative to the foreign price level) that drives the demand for domestic exports. The new g3+ model adopts the decomposition of foreign GDP into its gap and trend components and transforms it into the measure of foreign demand.

To preserve consistency with the domestic block of the model, the stationarity of demand for Czech exports X_t needs to be preserved. To achieve this, domestic-economy-related technologies accounting for the openness trend, the aggregate trend, and exports are employed (see Equation (5)). Also, we need to account for the trends in the foreign demand proxy $(Y_t^*)^4$ and in the price of exports relative to the foreign core price level $\frac{P_t^X}{P_t^{*core}}$.

The derivation of the trade link definition starts with the following modified equation for the demand for domestic exports implied by CES aggregation:

$$X_t = \left(\frac{P_t^X}{P_t^{*core}} \right)^{-\theta_x} (Y_t^*)^4, \tag{15}$$

where θ_x is the price elasticity of exports given the foreign core price level and the price of exports. The modification comes from the use of core producer prices P_t^{*core} instead of the overall producer price index P_t^* as employed in Equation (3.29) by Andrle et al. (2009).

When adopting a detailed foreign demand structure, it is necessary to account explicitly for differences in the trend movements of the domestic and foreign economies. Therefore, the steady-state growth rates of the individual variables together with growth in openness are used to derive the stationary version of Equation (15) as follows:

$$x_t = \left(\frac{p_t^X}{p_t^{*core}} \right)^{-\theta_x} (\hat{y}_t^*)^4 \frac{(Y_t^{*trend})^4}{tech_t^x}, \tag{16}$$

where $tech_t^x$ is the export-stationarizing technology and $\hat{y}_t^*$ and Y_t^{*trend} are the foreign output gap and trend, respectively.

The derivation of the domestic-foreign economy link is closed by defining a new variable $wedge_t^{trends}$ that aggregates the difference between the foreign and domestic trends as follows:

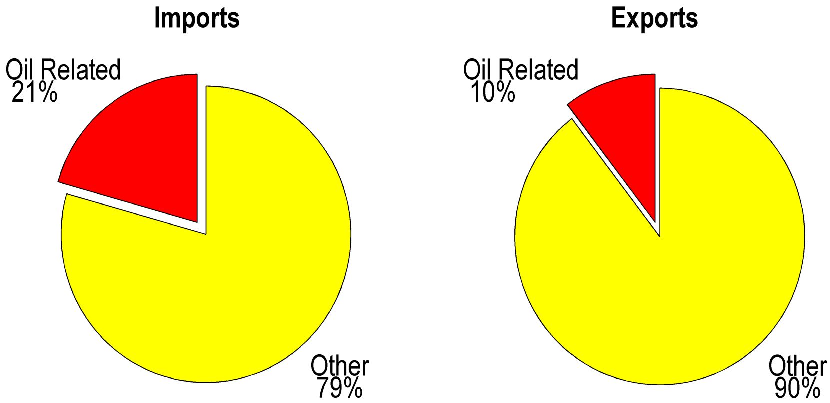
$$wedge_t^{trends} \equiv \frac{(Y_t^{*trend})^4}{tech_t^x}. \tag{17}$$

Additional details on the derivation and identification of the level of the domestic-foreign growth wedge are presented in Appendix C.

4. Energy in Production

The use of the g3 model in the regular forecasting rounds helped us identify the analytical limits and missing behavioral relationships related to energy prices. Our motivation for introducing detailed structural relations is supported by an analysis of the shares of oil-related commodities (SITC groups chemicals, mineral fuels, and oils) in overall Czech imports and exports.[26] The results of this analysis, summarized in Figure 8, reveal the presence of asymmetry between exports and imports. This reflects the Czech economy's focus on exports of goods with high value-added while importing raw materials used for production.[27] Due to the presence of structural asymmetries, the use of a single foreign price index generates limitations in capturing the effects of price fluctuations. As a result, the effects of foreign prices on the domestic economy may be over or underestimated, depending on their transmission channel, as argued by Kilian (2009).

Figure 8: Commodity Structure of Foreign Trade



Note: The numbers indicate the shares in the total value.
Source: Czech Statistical Office, CNB calculation, aggregation of chemicals, fuels and oils.

First, when considering the international markets competitiveness channel, abstracting from the difference between foreign prices relevant for exporters and importers limits analysis of sectoral profit margins. Second, as Kilian (2009) and Peersman and Robays (2009) document, energy price fluctuations have a direct impact on the price level of the economy. They affect consumption decisions and influence the cost structure of firms at the same time through the import price channel. This motivates us to consider energy, represented by oil, as a production factor. Also, our analysis of input-output tables reveals that the import content varies across the final goods representing the components of GDP. The importance of using a detailed structure for the cost transmission channel is stressed by the fact that the costs of oil and non-oil imported goods affect domestic prices, as wage and price indexation further propagates the effects of imported price shocks. Therefore, we extend firms' cost transmission channel by introducing energy and other intermediate imported goods into the production function for all final goods.

The g3+ model newly distinguishes importers of oil and other imported goods.[28] Inspired by An and Kang (2011), imports are used as an intermediate good to produce final goods in the sector of household and government consumption, investment, and export goods.

The continuum of individual importers differentiates a single foreign bundle available on the international goods trade market into a variety of goods. They face exchange rate movements and a

[26] Prices of these commodities are characterized by high correlation with oil prices. This arises from the nature of their production processes, where oil is the main ingredient.
[27] The prevalence of non-energy goods in exports supports our choice of core producer prices to represent the competitiveness price for linking domestic production and foreign demand in the g3+ model – see Equation (16).
[28] The previous g3 model relied on an aggregate import bundler only.

downward-sloping demand curve for their production by the imported goods bundler, and follow Calvo pricing. The local currency pricing assumption implies that imported goods are sticky in the domestic currency. In addition, importers of oil and other import goods are identical in terms of their production technology.

The newly introduced structure of intermediate goods importers implies the following laws of motion for real marginal costs in the oil import sector rmc_t^{oil} and in the other import goods sector rmc_t^{other}:

$$rmc_t^{oil} = \frac{p_t^{oil}}{p_t^{N,oil}}, \quad rmc_t^{other} = \frac{p_t^{other}}{p_t^{N,other}}, \tag{18}$$

where p_t^{oil} and p_t^{other} are the per unit prices of imported goods in domestic currency. The import prices at which goods are sold to domestic agents are $p_t^{N,oil}$ and $p_t^{N,other}$.

Just like in other production sectors, the price-setting behavior of importers follows Calvo (1983) pricing, where the signal to change their prices comes with probability $(1-\beta\xi^{oil})$ or $(1-\beta\xi^{other})$, which introduces nominal stickiness into their output prices. This setup implies the following laws of motion for prices of imports of oil and non-oil goods:

$$log\left(\frac{\Pi_t^m}{\Pi_{t-1}^m}\right) = \beta log\left(\frac{\Pi_{t+1}^m}{\Pi_t^m}\right) + \frac{(1-\rho^m)(1-\beta\xi^m)}{\xi_m} log(rmc_t^m \Theta^m) + \varepsilon_t^m, \tag{19}$$

where index $m \in \{oil, other\}$ stands for either the oil or non-oil imports sector, Θ^m is the markup in the given sector, $\Pi_{t-1}^m = \frac{p_t^{N,m}}{p_{t-1}^{N,m}}$ is import price inflation, and ε_t^m is a cost-push shock.

As the scheme of goods production shows (see Figure 9), final goods producers bundle imported oil goods, other imported goods, and domestically produced intermediate goods. Generally, for final good $m_t \in \{C, G, X, J\}$ (that is, the GDP components household and government consumption, exports, and investment), $n_t^{oil,m}$ is the imported oil content, $n_t^{other,m}$ is the non-oil import content, and y_t^m is the domestic content used in its production. Each of the final goods sectors applies a Leontief production function (a fixed proportions production function).[29] This implies that the factors of production are used in fixed (technologically pre-determined) proportions without the possibility to substitute between the factors:

$$m_t = min\left\{\frac{n_t^{oil,m}}{oilN^m\omega^m}, \frac{n_t^{other,m}}{(1-oilN^m)\omega^m}, \frac{y_t^m}{1-\omega^m}\right\}. \tag{20}$$

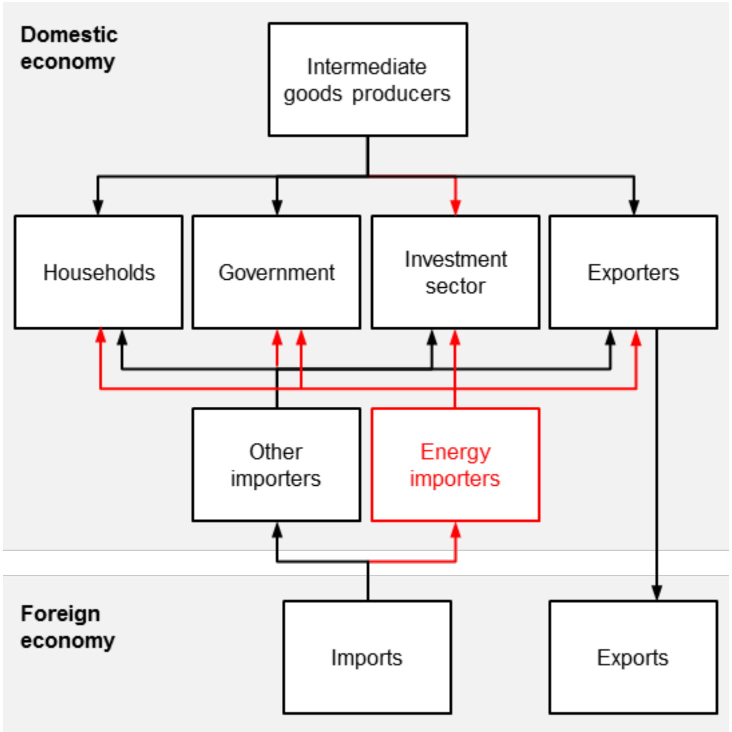
The shares $oilN^m$ and ω^m are calibrated based on input-output tables and supplementary analyses (such as impulse response functions and recursive filtering and forecasting).

Given the demand for final goods, the first-order conditions for the Leontief production function imply demands for factors:

$$\begin{aligned} n_t^{oil,m} &= oilN^i\omega^m m_t, \\ n_t^{other,m} &= (1-oilN)^m\omega^i m_t, \\ y_t^m &= (1-\omega^m)m_t. \end{aligned} \tag{21}$$

[29] The Leontief production function is a limit case of the general constant elasticity of substitution production function.

Figure 9: Production of Final Goods in the g3+ Model



Note: The newly added production sector and links are highlighted in red.

Last but not least, the market clearing conditions imply that total imports n_t are given by aggregating demand for oil and non-oil imports:

$$n_t = n_t^{oil} + n_t^{other}, \tag{22}$$

where the demands for imports of oil and other import goods are given as an aggregation of the individual demands from final goods producers:

$$n_t^{oil} = n_t^{oil,c} + n_t^{oil,g} + n_t^{oil,j} + n_t^{oil,x}, \tag{23}$$

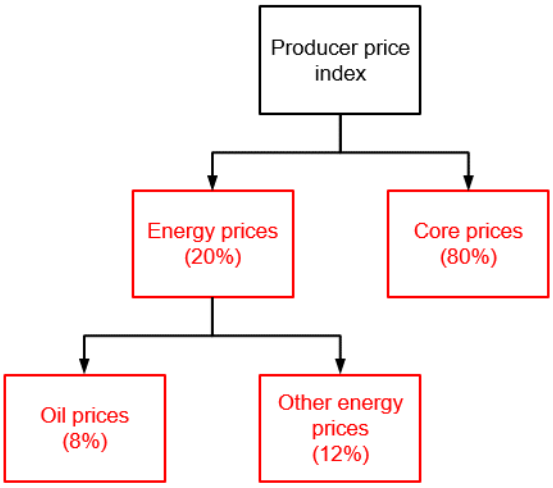
$$n_t^{other} = n_t^{other,c} + n_t^{other,g} + n_t^{other,j} + n_t^{other,x}, \tag{24}$$

where n_t^{oil} is the amount of oil imported and n_t^{other} the total amount of other goods imported.

The application of the same form of production function, given by Equation (20), for all final goods sectors is an additional extension introduced within the g3+ model. An integral part of this modification, however, is the choice of import intensities of the individual real expenditure components of the national accounts. Newly, the investment sector J uses a non-zero share of domestic intermediate goods (in the g3+ model, the share $\omega^J < 1$, keeping in mind that the g3 model features only a single type of imported good). Also, the government sector G newly consumes a portion of imported goods as its final goods production, that is, $\omega^G > 0$.

The features above related to goods production and the construction of the exports-foreign demand link in the g3+ model result in a rich theoretical structure of foreign producer prices, as shown in Figure 10. Such a structure makes forecasting challenging when the model needs to be brought to data. As it originates from domestic production and the foreign economy block structure at the same time, accurate identification of the contribution of core imported goods prices is of high interest. However, due to limited availability of data on producer prices, we are not able to directly isolate the contribution of the oil price to the overall effective euro area PPI, so we cannot identify the core producer prices component as a residual.

Figure 10: Components of the Foreign Producer Price Index



Note: The numbers indicate the weight in the total index. The newly added components are highlighted in red.

Therefore, rather than identifying the contribution of oil prices, we estimate the contribution of the energy industry to the overall PPI. Our inspiration for the desired decomposition originates in the granular data collected by Eurostat in accordance with the Statistical Classification of Economic Activities in the European Community, known as NACE. The Main Industrial Groupings (MIGs) provide a useful alternative, as they lie at an intermediate level between the NACE sections on the one hand and the NACE divisions and groups on the other.

Using this statistical breakdown, we can decompose the PPI along the industry type dimension. Thus, by separating out the contribution of energy-producing industries to the overall PPI, we can identify the contribution of competitiveness-relevant industries defining core goods prices (see the diagram in Figure 10). Further, by employing the theoretical structure and the oil price, we can decompose the energy prices under consideration into the price of oil and other energy prices.

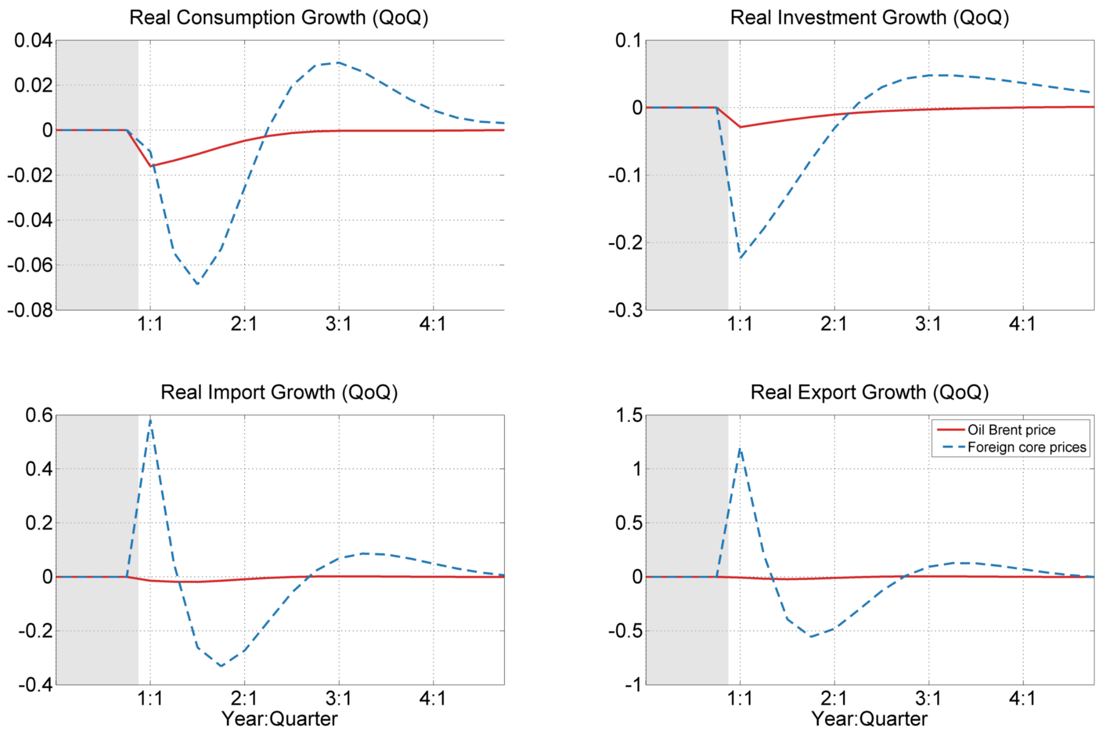
Our data exploration shows that the price of oil grows faster than the other components of the PPI. Quarter-on-quarter growth of oil prices has averaged just over 7% in the last 25 years and about 4% in the last 40 years. Under the assumption of 2% growth of overall producer prices, ceteris paribus, this property of the oil price would lead to an increase in the share of the oil price index in the PPI over the period considered and render the relative oil core price non-stationary. However, we do not observe such a characteristic in the data. The faster growth of the oil price together with rising environmental concerns is motivating producers to use oil in production with increasing efficiency. After exploring the data on the changes in the relative prices of energy and core goods, we assume that oil price growth and efficiency growth balance each other out.

Therefore, in the presence of technology delivering improvements in oil usage $tech^{oilTrendWedge}$, we can abstract from the time-varying weights in the construction of the producer price index given by Equation (9) and assume constant weights of the oil price $\rho_{\frac{oil}{ppi}}$ and the energy price $\rho_{\frac{energy}{ppi}}$. Using the oil efficiency improvement technology $tech^{oilTrendWedge}$, the stationarity of the relative oil core price is preserved by removing the contribution of the efficiency improvement technology from the growth of the oil price. Foreign PPI inflation is defined as:

$$\pi_t^* = \left(\frac{\pi_t^{oil}}{\Delta tech^{oilTrendWedge}}\right)^{\rho_{\frac{oil}{ppi}}} \left(\pi_t^{enerExoil}\right)^{\left(\rho_{\frac{energy}{ppi}} - \rho_{\frac{oil}{ppi}}\right)} \left(\pi_t^{*core}\right)^{\left(1-\rho_{\frac{energy}{ppi}}\right)} + \varepsilon_t^{ppi}, \tag{25}$$

where π_t^* is the overall PPI inflation rate and π_t^{oil}, $\pi_t^{enerExoil}$, and π_t^{*core} are the growth rates of oil prices, non-oil energy prices, and core prices, respectively (all denominated in euro). Here, $\rho_{\frac{oil}{ppi}}$, $(\rho_{\frac{energy}{ppi}} - \rho_{\frac{oil}{ppi}})$, and $(1 - \rho_{\frac{energy}{ppi}})$ are the shares of the respective components in the PPI and ε_t^{ppi} is a cost-push shock. Further, the steady-state level of oil efficiency improvement technology growth is given by the difference between the steady-state levels of oil price growth and foreign core producer price growth.

Figure 11: Increase in Oil and non-Oil Prices: Domestic GDP Components (deviation, %)



Note: The figure shows the impulse responses of the variables to a 1% shock. The responses are measured in percentage deviations from the steady state.

We present impulse responses to illustrate the basic dynamic properties of the g3+ model resulting from the extension of the production sector structure. Figure 11 presents the transmission of a one percent increase in oil and foreign core prices to the components of domestic GDP. The presented responses document that Czech foreign trade, and in particular exports, show relatively attenuated deviations in response to a positive oil price shock compared to the effects of a foreign core price shock. Such dynamics are in line with our forecasting experience, as changes in world oil prices affect Czech exporters and foreign producers in a similar fashion. Oil price shocks do not significantly alter world market shares. The household consumption and investment growth rates reveal a negative response, reflecting the increase in production costs.

By contrast, higher foreign *core prices* markedly improve Czech exporters' price competitiveness. Exports increase and, due to the high import intensity of exports, imports rise as well. The reactions of consumption and investment are also more pronounced when compared to the responses to the oil price shock. A more detailed description of the IRFs for these shocks is included in Section 9.

5. Heterogeneity of Households

As mentioned in Section 2, all households in the g3 model are made up of Ricardian-type consumers with access to financial markets.[30] Given that all households are infinitely lived, their consumption is a function of their permanent, not disposable, income. This modeling approach implies that the permanent income hypothesis, formulated by Friedman (1957), holds.

In the g3 model, the representative Ricardian-type household maximizes its utility given by the logarithmic utility function which is standard in modern structural models:[31]

$$\mathcal{U}(h) \equiv \log\left(\frac{C_t^{ric}(h) - \chi H_t}{1-\chi}\right), \tag{26}$$

$$H_t = \bar{C}_{t-1}^{ric} \exp(\varepsilon_t^H), \tag{27}$$

where C_t^{ric} is consumption of households, χ is the habit formation parameter, and H_t represents the external habit, specified as the economy-wide level of previous period consumption $\bar{C}_{t-1}^{ric}$ augmented for an i.i.d. habit shock ε_t^H.[32] Therefore, for the stationary consumption level of Ricardian households:

$$\lambda_t(h) p_t^C = \frac{1-\chi}{c_t^{ric}(h) - \chi \bar{c}_{t-1}^{ric} \exp(\varepsilon_t^H)}, \tag{28}$$

where λ_t is the shadow price of nominal household wealth.

However, many empirical studies emphasize the presence of systematic deviations of household consumption from the permanent income hypothesis. Campbell and Mankiw (1989), Mankiw (2000), andGalí et al. (2007) argue that the consumption decisions of economic agents are affected by their current level of income, myopia, lack of access to capital markets, fear of saving, and ignorance of intertemporal trading opportunities. Therefore, consumption smoothing is far from being as perfect as implied by the permanent income hypothesis.

Moreover, based on estimates of a modified Euler equation, Campbell and Mankiw (1989) provide evidence of the presence of "rule-of-thumb" (alternatively "hand-to-mouth" or "non-Ricardian")

[30] Ricardian-type consumers optimize their lifetime utility given that they have access to capital markets. They can trade a full set of contingent securities and can therefore borrow to consume in bad times or, conversely, deposit a portion of their income in good times. As a result, they can smooth their consumption over their lifetime.
[31] Andrle et al. (2009) use the logarithmic functional form to deliver consistency with a balanced growth path when deriving the g3 model.
[32] Generally, as discussed by Fuhrer (2000), the introduction of habit formation allows models to be brought closer to data. Additionally, it generates hump-shaped and persistent responses to various shocks.

consumers in the US and other industrialized economies. Those consumers do not have access to financial markets, do not own any assets, and do not possess any liabilities. They only consume their current labor income. Rule-of-thumb households cannot smooth their consumption over their lifetime. Therefore, they deviate from the optimizing behavior implied by the permanent income hypothesis.[33]

In addition to the international evidence, Czech data reveals that the proportion of financially constrained households characterized by financial distress is not negligible. These households' disposable income is just enough to cover their subsistence expenditures only. They are not able to accumulate buffers to be used for consumption smoothing.[34] As well as low income, these households often have limited access to financial services.

We adopt the recipe of Mankiw (2000) for deviating from perfect consumption smoothing. He allows for deviations from the permanent income hypothesis resulting from optimizing behavior of households with access to financial markets. Following this micro-founded recipe, non-Ricardian households are introduced into the g3+ model. In g3+, the economy is populated by a continuum of households, of which a fraction $(1-\omega_{liq})$ is represented by Ricardian households, while the remaining fraction ω_{liq} is represented by non-Ricardian households consuming their entire labor income in each period. Since the economy is populated by two types of households, aggregate stationary consumption c_t is specified as the sum of Ricardian and non-Ricardian consumption:

$$c_t = c_t^{liq} + c_t^{ric}, \tag{29}$$

where c_t^{liq} and c_t^{ric} represent the consumption of non-Ricardian and Ricardian households, respectively.

The fraction of non-Ricardian households ω_{liq} is set at 0.3. This calibration originates from our internal analysis of microeconomic data on households' wealth.[35] Our calibration is supported by Palas (2016), who delivers a mean estimate of the share of non-Ricardian households in the Czech Republic of 27.5%.

Non-Ricardian households supply labor and receive the same wage as Ricardians, based on wage contracts negotiated by an employment agency. At the same time, they do not have access to financial markets (and thus use their entire income for consumption in each period). Given that, a simple budget constraint describes the aggregate consumption of non-Ricardian households:

$$c_t^{liq} = \omega_{liq} \frac{w_t l_t}{p_t^C}, \tag{30}$$

where w_t is the (average) wage prevailing in the economy, l_t represents labor, and p_t^C is the consumption price level.

Besides the introduction of non-Ricardian households, the g3+ model presents a modification of the g3 utility function for optimizing households given by Equation (26). While the newly adopted utility function is still logarithmic, internal habit formation ("time nonseparable") replaces external

[33] As mentioned by Campbell and Mankiw (1989), such behavior is a result of consumers' deviation from behavior consistent with rational expectations, or the existence of binding borrowing constraints.
[34] For additional details and statistics see the survey report available at https://www.stem.cz/hodnoceni-prijmu-vlastni-domacnosti-je-vyrazne-lepsi-nez-v-minulosti/. Available in Czech only.
[35] The analysis is based on the ratio of the value of households' bank holdings to households' income.

habit formation ("catching up with the Joneses"). Further, together with the modification of the preference shock, the following specification of Ricardian households' utility function is introduced into the g3+ model:

$$\mathcal{U}(h) \equiv \log\left(\frac{C_t^{ric}(h) - \chi C_{t-1}^{ric}(h)}{1-\chi}\right)\kappa_t^H. \tag{31}$$

In here, C_t^{ric} is the consumption of Ricardian households and $\kappa_t^H = (\kappa_{t-1}^H)^{\rho^H}\exp(\varepsilon_t^H)$ is a stochastic process that introduces a persistent deviation into households' preferences for consumption relative to time spent at work. This is in contrast to the role of the external habit formation shock ε_t^H, given by Equation (27), which drives a Ricardian household's deviation in the perceived level of other households' consumption.[36]

The introduction of internal habit formation is motivated by its success in explaining the highly correlated savings and growth (see Carroll et al., 2000) and the relatively low variability in the consumption growth rate (as documented by Constantinides, 1990). We believe that this modification of the utility function and the introduction of a persistent preference shock κ_t^H brings the model closer to the data by making it more realistic than in the case of external habit formation. We also think that omitting the assumption that individual consumers usually do not track aggregate consumption choices beyond several quarters and almost certainly do not do so over the long run as external habit formation may suggest, improves the model's credibility. The adoption of internal instead of external habit formation is also supported by empirical evidence presented by Ferson and Constantinides (1991), Heaton (1995), and Chen and Ludvigson (2009).

After the derivations, the stationary consumption of Ricardian households in the g3+ model is given by:

$$\lambda_t(h)p_t^C = (1-\chi)\left(\frac{1}{c_t^{ric}(h) - \chi c_{t-1}^{ric}(h)} - \beta\frac{\chi}{c_{t+1}^{ric}(h) - \chi c_t^{ric}(h)}\right)\kappa_t^H, \tag{32}$$

where β denotes the household discount factor of future utility.

The presented changes in the household sector bring the g3+ model closer to the data, which reveal substantial deviations from the permanent income hypothesis. The modifications (primarily the addition of non-Ricardians) support a stronger relationship between current income and consumption, as justified by the data. Also, the introduction of non-Ricardian households is consistent with the observation that a significant fraction of households have near-zero net worth. These modifications thus partially improve the forecasting performance in terms of consumption.

6. Investment Cycle

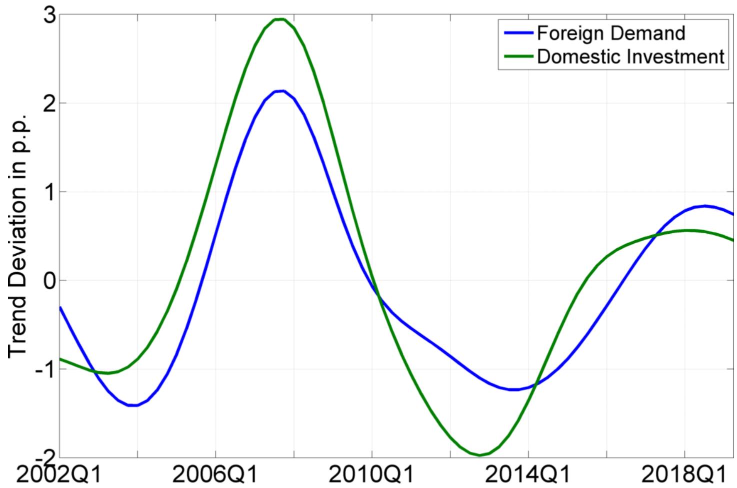
Empirical evidence presented by Jansen and Stokman (2014) supports the view that business cycles have tended to be more synchronized across countries since the mid-1990s. Apart from the trade channel, such behavior is caused by foreign direct investment, which constitutes a separate channel through which economies may affect each other. The authors also conclude that foreign

[36] The persistence parameter of the AR(1) process for κ_t^H is set at a modest value of 0.3. For a comparison of the impulse responses of the g3 and g3+ models to the preference shock, see Figure E6 in Appendix E.

disturbances influence domestic economies for a longer time span when transmitted through the investment channel rather than the trade channel.

This evidence motivates us to explore the properties of the Czech investment cycle and its long-term component. The results of our simple analysis, displayed in Figure 12, show that strong co-movement of the external demand cycle and the investment cycle can also be observed for the Czech economy. Our data analysis reveals a strong correlation (of just above 0.7) between domestic investment and external demand growth approximated by effective euro area GDP. As discussed by Gürtler (2018), the strong relation of investment to foreign demand is also due to the intensive involvement of the Czech Republic in global value chains, since investment is often associated with the creation of production capacities. Further, our analysis shows that domestic investment growth is more elastic than the other components of GDP to changes in external demand.

Figure 12: Co-movement of the Long-Term Components of Domestic Investment and External Demand



Note: The Hodrick and Prescott (1997) filter (with $\lambda = 1,600$) is applied to the levels of both time series to identify the trend components. The deviation of the trend component growth from its steady-state growth is also computed.

These findings are in line with the theoretical predictions of Cavallari (2010). The author uses a small DSGE model with nominal rigidity and endogenous choice whether to invest in the domestic and foreign markets to explain that changes in world demand are found to stimulate trend investments in all sectors of the domestic economy. Such international spillovers can be magnified for countries that are characterized by higher productivity, as the foreign investment channel can transmit even global monetary policy actions through changes in world demand and effects on the terms of trade.

Our forecasting experience, as gained from quarterly forecast evaluation exercises, revealed that the g3 model was often unable to deliver a robust connection between foreign demand and domestic investment. Our evaluation of the g3 model forecasts points to exogeneity of the investment-specific technology A_t^J. This technology augments investment growth and is modeled as a simple autoregressive process. In our view, this exogeneity weakens the investment-foreign demand link. Therefore, enhancement of data properties, theoretical studies, and past forecasting experience motivate us to introduce a link between external demand and domestic investment in the g3+ model.

We introduce an additional transient technology to support long-term deviations in the investment-external demand growth relationship. As we focus on long-term effects, we believe that the deviation of effective external demand trend growth from its long-term value provides a robust proxy for long-lived deviations of foreign demand. In the foreign block of the g3+ model, the foreign trend growth Δy_t^{*trend} is relevant for domestic firms' medium- to long-term investment decisions. Therefore, it is used to define the foreign demand-domestic investment link.

Since we require this link not to alter the balanced growth path of the model, it is introduced as a domestic investment growth add-on $\Delta aJnp_t$ as follows:

$$\Delta j_t = \frac{j_t}{j_{t-1}} \Delta tech_t^j \Delta aJnp_t, \tag{33}$$

$$\Delta aJnp_t = \left(\frac{\Delta y_t^{*trend}}{\Delta \bar{y}^{*trend}} \right)^{4\rho_{aJnp}} \varepsilon_t^{aJnp}, \tag{34}$$

where j_t denotes the stationary level of gross capital formation and Δj_t is its growth rate. All domestic technologies ΔZ_t, ΔA_t^J, and ΔA_t^C used to augment stationary investment, as Equation (4) shows, are aggregated into a single technology $tech_t^j$ for ease of interpretation. The trend component of foreign aggregate demand growth Δy_t^{*trend} and its steady-state value $\Delta \bar{y}^{*trend}$ are used to identify long-lived deviations in foreign demand growth. Finally, the elasticity of the foreign demand-domestic investment link is ruled by parameter ρ_{aJnp}, while a non-productive investment shock ε_t^{aJnp} allows for short-term deviations.

7. Recalibration of Parameters

Given that the g3+ model offers a structural upgrade, the values of some parameters of the original g3 model have been updated to reflect the new structural features. The parameter adjustments aim to preserve properties of the model – such as impulse responses – that are important for policy analysis. We are also motivated to keep the model characteristics in line with current views on medium-term growth so that the model preserves its role as the main analytical tool of the CNB's inflation forecast-targeting framework.

The most visible parameter adjustments involve changes to the import intensities of production in the final goods sectors reflecting the introduction of new production factors (for more details see Section 4). Furthermore, some deep parameters describing the model agents' behavior have also been adjusted to reflect novel features such as the new type of households.

Table 2 reviews the changes to the long-term characteristics of the Czech and foreign economies that were introduced along with the g3+ model. The most extensive adjustment concerns the parameters describing the characteristics of international trade along both the domestic and international dimensions. This reflects the observed slowdown in the growth of the openness of the Czech economy. These changes in the characteristics of international trade are in line with data observations, as such a slowdown is observed as a declining slope of imports and exports over time (see Figure 2 for a reminder).

As the data reveal, effective euro area GDP growth has declined in recent years. This is reflected in a slight decrease in its steady-state growth. The growth of the foreign economy, as a proxy for foreign demand, has to match the characteristics implied by the international goods market clearing

Table 2: Steady State Assumptions

Variable	g3	g3+
Domestic Economy (YoY, %)		
Real GDP	3.0	3.0
Private Consumption	3.0	3.0
Investment	3.0	3.0
Government Consumption	3.0	3.0
Imports	7.2	6.4
Exports	7.2	6.4
CZK/EUR	−1.5	−1.5
Domestic Economy (Level, %)		
Inflation Target	2.0	2.0
Foreign Economy (YoY, %)		
Foreign GDP	1.8	1.6
Foreign Economy (Level, %)		
Foreign Interest Rate	3.5	2.5
Inflation Target	2.0	2.0

condition for export demand given by Equation (15). As Equations (5) and (6) show, import growth and export growth are defined as the augmentation of the aggregate rate of growth ΔZ_t and growth in trade openness A_t^O, which is a common component of imports and exports. While we do not find enough empirical evidence for an adjustment to domestic aggregate growth, the decrease in the growth of the foreign economy implicitly results in a decrease in the growth of Czech trade openness.

Together with the observation of decreasing global economic growth and declining global real interest rates, the steady-state level of the foreign interest rate is revised downward. This change is in line with current empirical studies exploring the decline in real interest rates in developed countries over the last half-century (see Rachel and Summers, 2019, for example).

Table 3 also presents the changes to the parameters responsible for the dynamic properties of the model. The introduction of non-Ricardian households is accompanied by a switch from external to internal habit formation. The habit formation parameter χ increases. This, in turn, increases the persistence of Ricardian households' consumption and preserves the hump shape of the impulse responses.

Parameter ρ_s, controlling the degree of forward-lookingness of forex dealers in the UIP equation, has been reduced slightly. This adjustment improves the model's predictive ability, especially for the exchange rate.

The structure of the foreign economy block of the g3+ model includes various sources of foreign demand and price persistence in comparison with the g3 model approach based on three mutually independent AR processes. Therefore, the persistence in the international goods market is rebalanced to preserve the dynamic properties of the domestic export sector. To achieve this goal, we have reduced the price elasticity of exports θ_x and the autoregressive parameter ρ_{aX} (controlling the persistence of export-specific technology) in the export demand Equation (16).

Table 3: Comparison of Model Parameters

Name	**Symbol**	**g3**	**g3+**
Deep Parameters			
Habit Formation	χ	0.75	0.80
Forward-lookingness of UIP	ρ_s	0.70	0.65
Price Elasticity of Exports	θ_x	1.20	1.00
Calvo Wage	ξ_L	0.80	0.85
Calvo Exports	ξ_X	0.60	0.70
Calvo Imports Other	$\xi_{N^{other}}$	0.60	0.40
Calvo Imports Oil	$\xi_{N^{oil}}$		0.15
Government Consumption Rule	ρ_g	0.95	0.85
AR			
Export-Specific Technology	ρ_{aX}	0.60	0.40

We have increased the Calvo parameter ξ_L for the Phillips wage curve to match the high observed rigidity of the Czech labor market. Furthermore, the Calvo parameters for exports ξ_X and imports $\xi_{N^{other}}$, which measure the price re-optimization probabilities $(1-\xi_X)$ and $(1-\xi_N)$, are adjusted to better reflect the price rigidities in these production sectors. We have reduced the government consumption rule parameter ρ_g linking government and private consumption. This reduction improves the model's predictive ability for household consumption, as government consumption is a conditioning variable in the forecast construction process.

8. Limited Information in Expectations

As mentioned in Section 2, the Forecasting and Policy Analysis System at the CNB has been supported by conditional forecasts since the early years of the QPM framework, as explained in Czech National Bank (2002). Given that the labels "conditional" and "unconditional" may be potentially misleading, we start with a description of the CNB's version of forecast conditioning.

The Czech Republic is a small open economy that does not influence developments in the large foreign economy. Our modeling framework takes this assumption into account. Thus, the foreign economy outlook is treated as an exogenous assumption of the conditional forecast. Further, the forecast is also conditioned on model-exogenous domestic assumptions that include outlooks for administered prices (such as electricity prices for households) and government spending. These domestic variables are under the control of regulatory and fiscal authorities. Their link to the business cycle is relatively weak due to the discretion in these authorities' decision-making processes. However, we still include them in the forecast conditioning, as they are important drivers of domestic inflation and developments in the real economy.

An important feature of the CNB forecast is that it conditions on the endogenous response of the model policy rule rather than on the assumption that the central bank does not change its interest rate throughout the forecast period. As explained in Box 1 in Coats et al. (2003), such a forecast is often labeled as unconditional despite the fact that it is also conditional on a whole range of assumptions, such as the outlooks for foreign and domestic variables.[37]

[37] To make the use of "conditional" and "unconditional" even more complicated, in the general approach an "unconditional" forecast is just one special type of conditionality.

When solving for the conditioning forecast paths, the information included in the conditioning assumptions is important for the formation of economic agents' expectations and hence for their decision-making. In our view, a key factor is how the conditioning information is treated in the process of solving for the forecast paths.

Therefore, besides structural improvements, the g3+ forecasting infrastructure applies a novel conditional forecast solution concept. This concept is able to consistently limit the amount of anticipated information available period by period. The novelty of our limited information rational expectations (henceforth LIRE) concept consists in a further refinement of the current LIRE simulation technique of the g3 framework. The current LIRE concept has superseded the original full information rational expectations (henceforth FIRE) as presented in Andrle et al. (2009).

In the first part of this section, we explain our motivation driving the introduction of the LIRE concept and briefly describe the approach used in forecasting rounds organized around the g3 model (henceforth LIRE:g3). Additional technical details relating to this simulation approach can be found in Appendix D. In the second part of this section, we introduce the main principles of the novel LIRE approach used in the g3+ model (henceforth LIRE:g3+). In addition, we discuss the implications of the new simulation technique for conditional forecasting.

8.1 Full Information Rational Expectations

The method used to solve for models with anticipated structural variations, as applied by Andrle (2008), builds on the solution method proposed by Sims (1982) and Sims (2002). The theoretical solution of the model with an infinite horizon of shock visibility ε_{t+k} for $k = 1,2,3,\ldots$, that is, full information rational expectations, is given by:

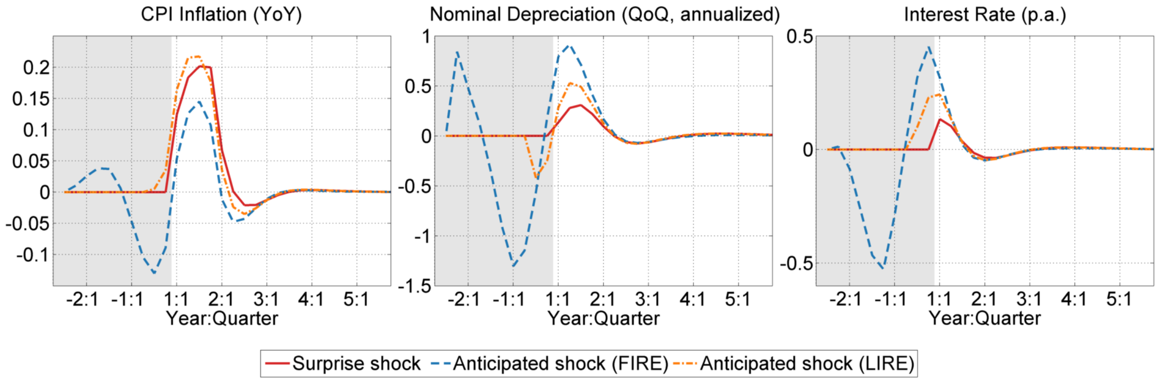
$$\xi_t = F\xi_{t-1} + R^{[0]}\varepsilon_t + R^{[1]}\varepsilon_{t+1} + R^{[2]}\varepsilon_{t+2} + R^{[3]}\varepsilon_{t+3} + \ldots. \tag{35}$$

The transition matrix F describes the transition from the past state given by the vector of all the model variables ξ_{t-1} to the current state ξ_t. As Equation (35) defines, the response of the model economy to anticipated shocks ε_{t+k} is given by the response matrices $R^{[k]}$ that originate from the model solution.

The FIRE concept, though theoretically sound, may deliver results that are questioned in forecasting practice on the grounds that future information discounting is ignored. Comparing the impulse responses for selected variables to anticipated and surprise consumption cost-push shocks under FIRE provides an example of where such questioning originates. Figure 13 displays the impulse responses to a distant anticipated shock under FIRE (blue dashed line) and a surprise shock (yellow dashed line). Figure 13 also presents the responses delivered by the novel LIRE:g3+ approach able to gradually deliver anticipated information. The resulting dynamics of the anticipated shock are challenging to communicate even though they are entirely consistent with the structure of the underlying model.

A typical challenge that arises under the FIRE approach is the grueling explanation of an ambiguous impulse response – in the presented example, resembling a sine curve – to an anticipated shock in periods before the shock hits the economy. Another challenging result under FIRE is the increasing amplitude of the immediate response to an anticipated shock when the shock hits the economy further in the future. That is, the economy reacts more profoundly to shocks with anticipated realization in the more distant future. These anticipated shock responses are challenging to communicate even though they are entirely consistent with the structure of the underlying model.

Figure 13: Surprise and Anticipation: Consumption Cost-push Shock (deviation, %)



Note: The responses are measured in percentage deviations from the steady state. The shocks occur in period 1:1. The anticipated shocks are known since period -3:3, that is, ten periods ahead.

As Romer (2016) and Blanchard (2018) state, the FIRE concept is built on several strong assumptions that most likely do not hold in reality. An evident problem is the infinite horizon, which for computational practice has to be capped by some finite period. Even then, the reliability and availability of reasonable exogenous outlooks beyond several quarters ahead (or a couple of years at most) are highly dubious. Therefore, the existence of a future period after which the following information is discounted seems to be a plausible assumption. De Grauwe (2012) acknowledges these cognitive limitations of economic agents in reality as well. At the same time, following Sims (2003), we also acknowledge that human information-processing capacity is limited.

Hence, it is plausible to assume that economic agents process near-term information more intensely than information located in the distant future. This assumption arises from the design whereby the ratio of economic agents responding to information (or even learning about it) gradually declines (in line with the time distance of the shock) with increasing distance of information. Even if all economic agents followed the same information set, it would be reasonable to assume that information discounting is an increasing function of time as uncertainty increases toward the distant future. Following this reasoning, the immediate response to distant future anticipated shocks should be less pronounced than that to near-term ones.

Similarly to studies on learning in DSGE models such as Slobodyan and Wouters (2012) we adopt considerations about the efficient use of available information. This leads us to introduce an additional layer into the solved linearized model given by Equation (35). This additional layer is designed to adjust the anticipated information set entering the solution for the paths conditioned on that information. However, unlike Slobodyan and Wouters (2012), we assume that economic agents know the full structure of the model economy and form their expectations based on this knowledge. In our implementation, the added layer adjusts the information set so that agents treat information delivered in the distant future as a surprise. Economic agents, however, remain oblivious to this adjustment.

In April 2014, we adopted the LIRE:g3 approach in the g3 framework to mitigate the issues related to the dynamics implied by the FIRE impulse responses, which were implausible for forward guidance policy. The LIRE:g3 approach builds on the adjustment of the efficiency in the adoption of anticipated information available over the simulation horizon (that is, the switch from full anticipation to surprise shocks).

When solving for the forecast paths under LIRE:g3 at time t, the model operator could decide that all shocks beyond period $t+k$ should be unanticipated. In other words, the model operator could cut off the information visibility at horizon $t+k$. The shocks defining the paths of the exogenous outlooks until period $t+k$ would remain fully anticipated. This decision would result in the model dynamics as captured by Equations (D.1) through (D.3). The corresponding expectation scheme is presented in Table D1 in Appendix D.

While the LIRE:g3 approach mitigates the issues with intricate impulse responses to anticipated shocks delivered under FIRE, it also introduces an inconsistency. Since forecasting is a periodical activity, the forecasting horizons, as well as the information visibility horizons of the expectation schemes, shift between individual forecasting rounds. In the case of the fixed horizon expectation scheme (see Table D1), an inconsistency arises whenever the visibility horizon is shifted forward. While the forecast path assumed anticipated shocks only until period $t+k$ in the original projection, the new forecast (that is, the next one) assumes anticipated shocks until period $t+k+1$.

Generally, such inconsistency arises between any two projections with different time horizons $t+k$ and $t+k+\Delta$ for anticipated information, where $\Delta > 0$. Similarly, such inconsistency arises even in the case of the fixed horizon expectation scheme with diminishing information, as presented in Table D2. However, the adverse effects would likely be smaller due to a gradual fading out of the anticipated information in this case. These shortcomings of the shock adjustment approaches for limiting anticipated information motivate us to introduce an alternative approach together with the g3+ model.

8.2 Limited Information Rational Expectations in the g3+ Model

The novel approach to processing conditioning information introduced together with the g3+ model is based on the adoption of a moving-horizon expectation scheme by altering the response matrices R. This modification delivers time-consistent treatment of future shocks in subsequent forecasts. Under the LIRE:g3+ approach, we still assume a horizon beyond which all shocks are taken as unanticipated, and we also allow for a gradual fading out of anticipated information. However, the expectation scheme forms a moving window that shifts forward as a whole along the forecast path even within a single forecasting round.

The adoption of the LIRE:g3+ approach effectively mitigates the communication issues arising from forecast changes between individual forecasting rounds induced by shifting the forecast horizon (or the end of the history), even when the conditioning outlooks themselves remain unchanged.[38] Moreover, the information from exogenous outlooks eventually fully enters the forecast. It does so only gradually (as the anticipation window moves forward).

The dynamics of the model economy under the LIRE:g3+ concept are captured by Equations (36) through (39). The indices i of the weight matrices $W^{[i]}$ no longer correspond to vectors of shocks ε_{t+i} (as in Equations (D.5) through (D.8)) but to response matrices $R^{[i]}$. Also, the response matrices $R^{[i]}$ are still obtained as a model solution under the assumption of fully rational expectations as given

[38] The issue is eliminated for distant anticipated shocks, which do not enter the forecast at the shorter end. However, as shocks become "visible" under the given expectation scheme, the anticipated response are function of their time to materialize distance. Thus, the impulse response to the same shock in the same period will be slightly different when the forecast horizon shifts forward. The response to an unanticipated shock under FIRE is not time to materialize dependent. Due to this time to materialize difference between responses to unanticipated shock and to an anticipated shock, the problem with changing forecast under the same outlook can only be entirely eliminated with anticipated shocks switched off all the way through, that is, only if $R^{[i]} = 0, \forall i > 0$.

by Equation 35.[39]

$$
\begin{aligned}
\xi_t &= F\xi_{t-1} + R^{[0]}W^{[0]}\varepsilon_t \quad + R^{[1]}W^{[1]}\varepsilon_{t+1} + R^{[2]}W^{[2]}\varepsilon_{t+2} + \cdots + R^{[k]}W^{[k]}\varepsilon_{t+k}, && (36)\\
\xi_{t+1} &= F\xi_t \quad + R^{[0]}W^{[0]}\varepsilon_{t+1} + R^{[1]}W^{[1]}\varepsilon_{t+2} + R^{[2]}W^{[2]}\varepsilon_{t+3} + \cdots + R^{[k]}W^{[k]}\varepsilon_{t+k+1}, && (37)\\
\xi_{t+2} &= F\xi_{t+1} + R^{[0]}W^{[0]}\varepsilon_{t+2} + R^{[1]}W^{[1]}\varepsilon_{t+3} + R^{[2]}W^{[2]}\varepsilon_{t+4} + \cdots + R^{[k]}W^{[k]}\varepsilon_{t+k+2}, && (38)\\
&\vdots\\
\xi_{t+s} &= F\xi_{t+s-1} + R^{[0]}W^{[0]}\varepsilon_{t+s} + R^{[1]}W^{[1]}\varepsilon_{t+s+1} + R^{[2]}W^{[2]}\varepsilon_{t+s+2} + \cdots + R^{[k]}W^{[k]}\varepsilon_{t+k+s},\\
&\vdots && (39)
\end{aligned}
$$

Using the definition $R^{[i]}W^{[i]} = R^{*[i]}$, we can derive the following general form of the forecast path under the LIRE:g3+ concept.

$$
\begin{aligned}
\xi_t &= F\xi_{t-1} + R^{*[0]}\varepsilon_t \quad + R^{*[1]}\varepsilon_{t+1} + R^{*[2]}\varepsilon_{t+2} + \cdots + R^{*[k]}\varepsilon_{t+k}, && (40)\\
\xi_{t+1} &= F\xi_t \quad + R^{*[0]}\varepsilon_{t+1} + R^{*[1]}\varepsilon_{t+2} + R^{*[2]}\varepsilon_{t+3} + \cdots + R^{*[k]}\varepsilon_{t+k+1}, && (41)\\
\xi_{t+2} &= F\xi_{t+1} + R^{*[0]}\varepsilon_{t+2} + R^{*[1]}\varepsilon_{t+3} + R^{*[2]}\varepsilon_{t+4} + \cdots + R^{*[k]}\varepsilon_{t+k+2}, && (42)\\
&\vdots\\
\xi_{t+s} &= F\xi_{t+s-1} + R^{*[0]}\varepsilon_{t+s} + R^{*[1]}\varepsilon_{t+s+1} + R^{*[2]}\varepsilon_{t+s+2} + \cdots + R^{*[k]}\varepsilon_{t+k+s},\\
&\vdots && (43)
\end{aligned}
$$

In general, we assume that discounting of future information (that is, the set of proposed LIRE:g3+ weight matrices $W^{[i]}$) follows these assumptions:

1. *Diagonality*. The weight matrices should be diagonal because the structural shocks are uncorrelated:

$$
W^{[i]} = \begin{bmatrix} w^i_{11} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & w^i_{nn} \end{bmatrix}. \tag{44}
$$

2. *Identity matrix of surprise shocks.* The weight matrix corresponding to surprise shocks, $W^{[0]}$, is an identity matrix:

$$
W^{[0]} = \mathbb{I}. \tag{45}
$$

3. *Boundedness of expectations*. There is a horizon h beyond which all shocks are taken as unanticipated:

$$
\exists h, 0 < h < \infty : \forall i, i > h, w^i_{kk} = 0, \forall k \in \{1, \ldots, n\}. \tag{46}
$$

[39] The g3+ model is solved under the assumption of fully rational expectations. We implement the LIRE concept via ex-post modification of the anticipated information amount rendered as shocks over the forecast horizon via the response matrices.

4. *Normalization*. The elements of the weight matrices can only take values between zero and one:

$$\forall i \geq 0, \forall k \in \{1, \ldots, n\} \quad : \quad w^{i}_{kk} \in [0,1]. \tag{47}$$

5. *Monotonicity*. The elements of the weight matrices at given coordinates cannot increase with increasing horizon:

$$\forall i, j, i < j, \forall k \in \{1, \ldots, n\} \quad : \quad w^{i}_{kk} \geq w^{j}_{kk}. \tag{48}$$

The proposed moving-horizon LIRE:g3+ expectation scheme with decreasing weights applied to adopt the information from the foreign variables outlook in the g3+ model forecast is presented in Table 4. Under the LIRE:g3+ approach, the expectation abilities do not vary across model agents, but they can vary across model variables.[40] Therefore, the expectation scheme concerns all agents in the economy indiscriminately, including the central bank.[41] For forecasting purposes under the inflation-targeting regime, forecasters need to set the full information horizon far enough to cover the monetary policy horizon. In the case of the Czech economy, this is especially crucial for foreign variables, since they are essential drivers of the domestic business cycle. This restriction imposes another constraint on the information scheme.

Table 4: Moving-Horizon Expectation Scheme with Decay: Foreign Variables Outlook

Horizon / Period	year $y+1$				year $y+2$				year $y+3$				year $y+4$			
	Q1	Q2	Q3	Q4	Q1	Q2	Q3	Q4	Q1	Q2	Q3	Q4	Q1	Q2	Q3	Q4
current period	1	1	1	1	1	1	0.8	0.6	0.4	0.2	0	0	0	0	0	0
quarter $q+1$	P	1	1	1	1	1	1	0.8	0.6	0.4	0.2	0	0	0	0	0
quarter $q+2$	P	P	1	1	1	1	1	1	0.8	0.6	0.4	0.2	0	0	0	0
quarter $q+3$	P	P	P	1	1	1	1	1	1	0.8	0.6	0.4	0.2	0	0	0

Note: We assume that historical data is available until Q4 of year y and the forecast in the current forecasting round starts in year $y+1$, that is, in Q1. The columns represent the periods of the exogenous outlook in the forecast range, while the rows stand for the forecasted periods in the current forecasting round. The values show the proportion of the shock that is taken as anticipated. P stands for the predicted value from the previous period.

The monetary policy authority should take into account all available information when producing its macroeconomic forecast, if not for other reasons, then to give a reliable policy interest rate path over the forecast horizon.[42] The LIRE:g3+ approach delivers this in a smooth, controlled manner. As the LIRE:g3+ expectation scheme moves along the forecast path within the simulation, the full information set is eventually incorporated into the forecast paths. In other words, the LIRE:g3+ approach ensures that all agents in the economy eventually learn about the complete path of the exogenous outlook. Nevertheless, economic agents will react only to a subset of the whole outlook at any given time. Therefore, to preserve the consistency of the monetary policy response, the monetary policy authority takes into account the same information set, that is, the information relevant to the whole economy at the time.

[40] In practice, this is implemented via one to one relation of variable to structural shock.

[41] The LIRE approach does not allow for different expectation schemes for different economic agents of the DSGE model.

[42] In fact, due to the delay in monetary policy transmission, this path also has to take into account information related to periods further beyond the end of the forecast horizon itself.

Table 5: Moving-Horizon Expectation Scheme with Decay: Domestic Variables Outlook

Horizon / Period	year $y+1$				year $y+2$				year $y+3$				year $y+4$			
	Q1	Q2	Q3	Q4	Q1	Q2	Q3	Q4	Q1	Q2	Q3	Q4	Q1	Q2	Q3	Q4
current period	1	0.75	0.5	0	0	0	0	0	0	0	0	0	0	0	0	0
quarter $q+1$	P	1	0.75	0.5	0	0	0	0	0	0	0	0	0	0	0	0
quarter $q+2$	P	P	1	0.75	0.5	0	0	0	0	0	0	0	0	0	0	0
quarter $q+3$	P	P	P	1	0.75	0.5	0	0	0	0	0	0	0	0	0	0

Note: The same explanation as presented in the note under Table 4 applies here.

Table 5 presents an expectation scheme that implements faster expectation decay, thus delivering a shorter anticipated information horizon. This expectation scheme is used for conditioning on the outlooks for domestic variables, that is, administered prices and government consumption. As mentioned in the introduction to conditional forecasting, these variables are characterized by a large amount of discretion in the authorities' decision-making process, which weakens their link to the business cycle. This discretion is also reflected in outlooks that can be substantially revised in response to new information contained in the authorities' communications. The information included in these outlooks is therefore taken with lower degree of anticipation in comparison to variables more closely related to the business cycle.

9. Model Analysis and Forecasting Performance

In developing the g3+ model, we apply several approaches to investigate its stochastic properties and to test its forecasting performance. In the following subsections, we present the results of some of these analyses: (i) *impulse-response analysis*, (ii) *forecast error variance decomposition*, (iii) *model-consistent identification of structural shocks* in the observed data, and (iv) *recursive filtering and forecasting*. We focus on presenting analyses (i) through (iv), as we believe they offer the most valuable insights into the model's channels of transmission and highlight the main implications of the changes implemented with respect to the g3 model.

9.1 Impulse Response Analysis

In this subsection, we illustrate the properties of the new model in terms of the *impulse responses* of selected domestic variables to one percent surprise shocks. The shocks presented here are selected to demonstrate the most significant differences in the responses between the g3 and g3+ models. The shocks are classified into two groups – foreign and domestic. The foreign shocks include a foreign demand shock, a foreign monetary policy shock, a shock to foreign core prices, and an oil price shock. The domestic shocks under consideration are a wage-push shock, a consumption preference shock, a monetary policy shock, and a white-noise UIP shock. The impulse responses are summarized in Figures E1 through E8 in Appendix E.

9.1.1 Foreign Shocks

In the g3+ model, *the foreign demand shock* (see Figure E1) drives an increase in foreign inflation and the foreign shadow interest rate. The increase in foreign aggregate demand raises demand for domestic export goods. Therefore, the increase in domestic exports implies an increase in domestic imports, reflecting the import intensity of production. The higher exports stimulate the domestic investment and labor demand, causing wages to increase. The increase in wages also pushes domestic consumption up. Altogether, the impact on domestic GDP is positive. The domestic interest rate rises in response to higher inflation pressures arising predominantly from the labor market. The nominal exchange rate appreciates due to the improved net foreign asset position, which outweighs the effect of the decrease in the domestic-foreign interest rate differential.[43] Compared to the g3 model, the impulse responses of most domestic variables are more pronounced due to a simultaneous increase in foreign inflation and foreign interest rates that accompanies the positive foreign demand shock. In the g3 model, the appreciation of the Czech koruna nearly compensates for the increased domestic inflation pressures, so a significant monetary policy response is not realized.

The foreign monetary policy shock (see Figure E2) has two important transmission channels in the g3+ model. First, an increase in the foreign shadow interest rate lowers foreign demand and also dampens foreign inflation. Second, a tightening of monetary policy leads – via the financial market rate and exchange rate channels – to depreciation of the Czech koruna, partly offsetting the drop in foreign prices.[44] The decrease in foreign core prices worsens Czech exporters' price competitiveness, which is mirrored in lower exports. Consumption and investment decrease in response to the lower foreign demand. Due to the import intensity of the individual components of domestic demand, imports decrease as well, but this only partly offsets the negative effect on net exports. Overall, GDP declines. The koruna depreciation fuels an increase in imported goods inflation, causing the domestic interest rate to rise. The reaction of domestic prices and monetary policy is more subdued in the g3+ model, as the hike in the foreign interest rate leads to a decline in foreign demand and imported inflation, which is not present in the g3 model.

The shock to foreign core prices (see Figure E3) increases foreign core inflation. The foreign monetary authority responds by increasing the shadow interest rate to return inflation to the target. Foreign GDP growth deteriorates in response to the restrictive real monetary conditions. The increase in foreign core prices improves Czech exporters' price competitiveness, as Czech exporters compete on foreign non-energy goods and services markets. Thus, exports and imports both increase (the latter due to the high import intensity of exports). The koruna appreciates via the net foreign assets (NFA) channel, which dominates the interest rate differential effect. Growth in import prices together with higher demand for Czech exports pushes overall domestic inflation up. In response, the domestic interest rate increases, inducing lower consumption by Ricardian households. Conversely, higher labor demand increases the wage income and consumption of non-Ricardians. However, the overall effect on domestic consumption growth is slightly negative. The impulse responses of the domestic variables in the g3+ model are amplified in comparison to the g3 model, where the increase in foreign inflation is largely compensated by appreciation of the Czech koruna.[45]

The oil price shock (see Figure E4) results in foreign HICP inflation increasing, to which the central bank responds by increasing the shadow interest rate. Foreign core inflation goes down in response to the monetary policy tightening. Simultaneously, the oil price shock increases prices in the foreign production sector, leading to a decrease in foreign aggregate demand growth. The higher import prices of oil-related goods and services are quickly reflected in domestic producers' costs, fueling

[43] As explained by Andrle et al. (2009), quadratic foreign exchange trading costs related to serving foreign debt imply a foreign debt-elastic UIP premium, which is a concept similar to the debt-elastic risk premium proposed by Schmitt-Grohé and Uribe (2003). An increase in net exports improves the net foreign asset position, thus driving this premium down and inducing domestic currency appreciation. Domestic monetary conditions therefore become more restrictive, even in a situation where domestic policy does not match the increase of the foreign policy rate one to one.

[44] In the absence of a shadow rate premium shock, the two foreign interest rates – the shadow rate and the interbank rate – move in tandem.

[45] Note that the g3 model does not distinguish between foreign core prices and foreign energy prices. We therefore provide the impulse response of the g3 model to a general foreign price shock for comparison.

positive domestic inflation pressures. As a change in energy prices does not directly affect the price competitiveness of Czech exporters, the decrease in foreign core prices considerably worsens the price competitiveness of Czech exporters. The lower foreign core prices together with the subdued foreign demand lead to a decrease in Czech exports followed by a decrease in imports as implied by import intensities. The nominal exchange rate depreciates via the NFA channel. The dampened domestic production restrains labor demand and wage growth. However, the increase in foreign inflation together with the depreciated currency results in domestic price growth, so the domestic central bank also responds by raising its interest rate. By comparison with the g3 model, the impulse responses are generally less pronounced.[46] This result occurs because the change in oil prices affects the domestic and foreign economies simultaneously in the same direction. Mutual trade and the exchange rate are thus affected only slightly, and the reaction of domestic inflation and monetary policy is less pronounced than in the case of the g3 model.

9.1.2 Domestic Shocks

The wage push shock (Figure E5) delivers an initial increase in wages. Although firms suppress labor demand due to higher costs, the consumption of households increases due to their higher income. As a result of the higher wage growth, the inflation pressures from the labor market increase, leading to higher overall inflation. The exchange rate appreciates as a result of the central bank raising the interest rate in response to the higher inflation. As the competitiveness of exporters deteriorates, a decrease in exports is followed by a slight decrease in imports. Even though investment and consumption growth increase, the lower export growth prevails. This translates into lower GDP growth. In comparison to the g3 model, the impulse responses are qualitatively similar, but more gradual and persistent overall. Also, the initial positive response of private consumption is more intuitive than the relatively strong negative response in the g3 model.

The consumption preference shock (Figure E6) temporarily increases the growth of private consumption of households. The increased domestic demand leads to an increase in domestic inflation pressures and subsequently in inflation. The central bank reacts by increasing its interest rate, which leads to a short-lived appreciation of the Czech koruna and lower labor supply from Ricardian households, as the higher interest rate improves income from savings. Simultaneously, wage growth slows in response to the restrictive monetary policy, as a decrease in demand growth is expected. The higher inflation propagates via inflation indexation to faster wage growth. The initial increase in consumption results in overall GDP growth; however, as the trade balance is negatively affected by lower demand for consumption of foreign goods and a decrease in demand due to the restrictive monetary policy, GDP growth changes negligibly in response to this shock. In comparison to the g3 model, the impulse responses of the g3+ model are qualitatively very similar but distinctly less pronounced.

The monetary policy shock (Figure E7) captures temporary deviations of the nominal interest rate from the central bank's policy rule. A positive deviation in interest rates is reflected in exchange rate appreciation, as it widens the interest rate differential with respect to the foreign economy. The higher interest rate in the domestic economy also dampens current consumption due to intertemporal substitution by households. The lower consumption and consequently domestic production lead to lower wages, which translates into lower domestic inflation pressures. Inflation declines due to the decrease in both domestic and imported inflation pressures. The appreciation of the koruna reduces the competitiveness of domestic exporters and results in a decline in exports, which negatively affects GDP growth. Overall, the impulse responses of the g3+ model are very similar to those of

[46] Note again that the impulse response to a general foreign price shock in the g3 model is offered for comparison.

the g3 model. One of the most notable differences resides in the more subdued reaction of private consumption and GDP growth in the g3+ model.

The UIP shock (see Figure E8) captures temporary deviations of the exchange rate from the uncovered interest parity condition in the model. A positive UIP shock leads to koruna depreciation, which sparks inflation pressures via an increase in import prices. The central bank reacts by increasing the nominal interest rate. The koruna depreciation also boosts exports, which, in turn, increases imports. The higher demand for domestically produced goods leads to higher demand for labor, which pushes wages upward. The increased import prices lead to lower investment activity due to the high import intensity of investment. Despite the increase in wages, consumption falls due to the higher costs of imported goods and tighter monetary policy. The overall impact on domestic GDP is positive, since the contribution of export growth outweighs the negative contribution of the remaining components of domestic demand. Compared to the g3 model, the responses of wage growth and inflation and the monetary policy reaction are slightly more pronounced. However, the decline in private consumption growth is not as significant as in the g3 model.

9.2 Variance Decomposition

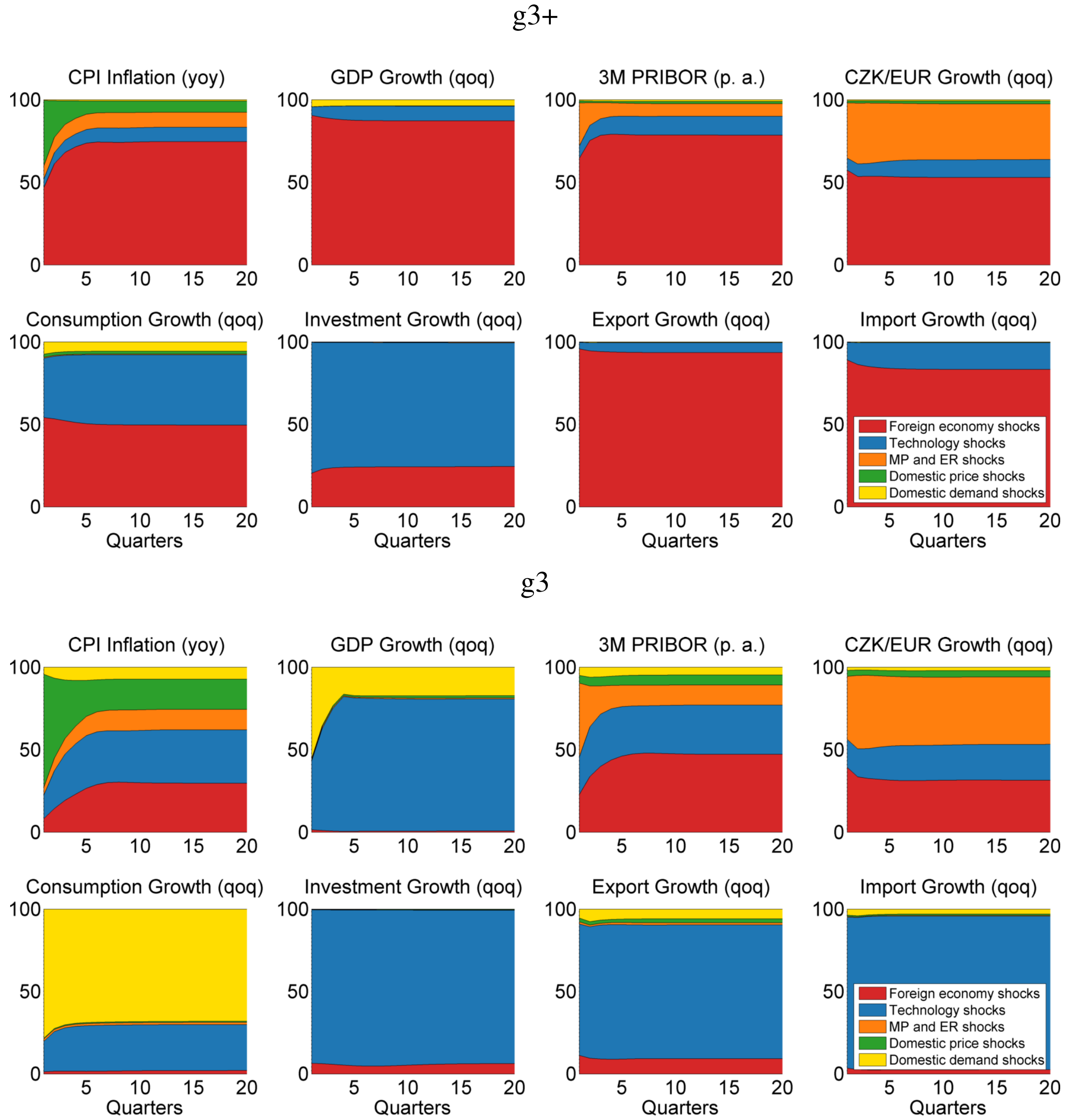
We use *forecast error variance decomposition* (or, alternatively, variance decomposition) to explore the stochastic properties of the g3+ model. Variance decomposition analysis allows us to explore the contributions of shocks to the volatility of the variable under scrutiny. This analysis illustrates the implications of the structure of the new model (especially the foreign economy block) and its calibration for the spread of structural shocks across model variables. It also allows us to highlight differences between the g3+ and g3 models in terms of shock propagation via the transmission channels of the model.

Figure 14 presents a comparison (between the g3+ and g3 models) of the variance decomposition of the main variables, namely, CPI inflation, real GDP growth (and its components), the nominal interest rate, and nominal exchange rate depreciation. The most notable change in the variance structure is a shift to higher explanatory power of foreign economy shocks (at the expense of technology shocks for most variables) and the demand shock for consumption.

In the g3 model, the variation in export and import growth is mostly driven by technology shocks, while the variation in consumption arises from domestic demand shocks. In the g3+ model, almost all the variation in CPI inflation, the nominal interest rate, and real export and real import growth originates in foreign economy shocks. The contribution of foreign shocks is higher for the rest of the variables presented. This change stems from the introduction of the new foreign economy block, which replaced the original representation consisting of three mutually independent AR processes.

The new structure of the foreign economy block provides feedback loops that crowd out the contribution of exchange rate shocks in the explanation of export and import dynamics. Similarly, the direct link between investment and foreign demand also increases the importance of foreign shocks for the variance in real investment growth. Further, the implementation of new rule-of-thumb consumers introduces an additional channel through which domestic technology shocks can generate variation in consumption. Therefore, the g3+ model does not rely on domestic demand shocks as a source of GDP fluctuations. Overall, the results highlight the importance of foreign shocks for developments in a small open economy and underline the differences in the story-telling ability of the new g3+ model.

Figure 14: Relative Variance Decomposition of Main Variables: g3+ vs g3



9.3 Model-consistent Identification of Structural Shocks

The CNB's Forecasting and Policy Analysis System makes intensive use of structural shock identification to communicate the story behind the observed data. Structural shock analysis plays a regular part in explaining the initial conditions of the economy at the start of each forecasting round.

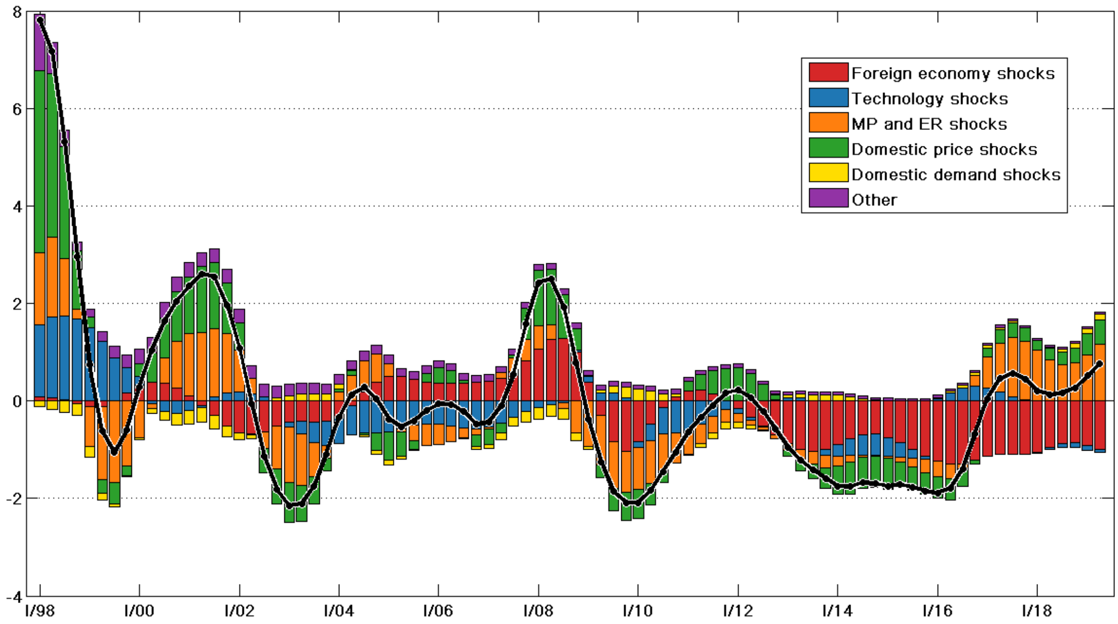
The CNB as an inflation-targeting central bank focuses on monetary policy-relevant inflation (MP inflation).[47] In the following analysis, we therefore focus on the drivers of MP inflation over the period of inflation targeting.

Figure 15 presents a comparison between the g3 and g3+ models of the historical deviation of the decomposition of MP inflation from its target.

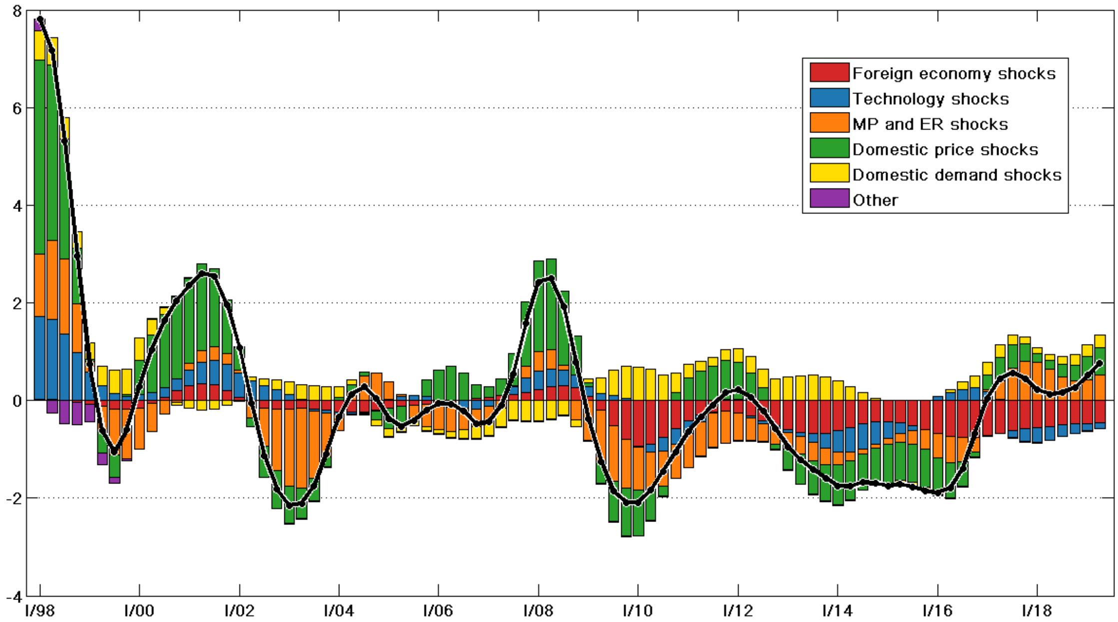
[47] Monetary policy-relevant inflation is inflation to which monetary policy reacts in the model framework. It is defined as headline inflation adjusted for the first-round effects of changes to indirect taxes, since the CNB's mandate allows it to apply escape clauses for changes in the level and structure of indirect tax rates.

Figure 15: MP Inflation Deviations: Decomposition into Structural Shocks (YoY, p.p.)

Model g3+



Model g3



For MP inflation, our decomposition into structural shocks reveals that prices were driven by the positive contribution of foreign shocks in the early stages of the financial crisis in 2008. Negative demand shocks were present in the later stages of this crisis. In our interpretation of history, these resulted from the high uncertainty affecting consumption behavior at that time. Over the 2012–2016 period, negative domestic and foreign price shocks were the most important drivers of MP inflation. After this period, domestic monetary policy and exchange rate shocks were driving domestic inflation upward.

When focusing on the contributions of foreign shocks, the comparison reveals that foreign shocks play a more vital role in the g3+ model than in the g3 model. The presented decomposition offers an intuitive interpretation of movements in the business cycle for a small open economy with close trade relations with its large counterpart. This supports our belief that the setup of the dynamic properties of the g3+ model is plausible. The results of the deviation decompositions also provide a solid foundation for accepting a higher share of foreign shocks in the variance decompositions.

The comparison of the decompositions for the g3+ and g3 models, as presented in Figure 15, reveals qualitative similarities. Presence of the same groups of shocks over the analyzed history can be observed. This allows us to conclude that the introduction of the g3+ model does not challenge the interpretation of historical developments as identified by the g3 model. However, some quantitative differences occur as a result of the newly introduced transmission channels and the changes to the model parameters.

9.4 Recursive Filtering and Forecasting

Before g3+ was introduced as the core model, its properties within the CNB's Forecasting and Policy Analysis System were evaluated using the regular full-fledged forecasts. The results of the final one of these forecasting exercises were presented as an alternative scenario in April 2019 (Czech National Bank, 2019b). For a thorough statistical evaluation of the model's forecasting properties, we would need a large set of g3+ forecasts. To reduce the complexity of constructing a large set of historical forecast replications, we resort to the *recursive filtering and forecasting* approach for evaluating the "fit" of the model forecasts to the data.[48] Our evaluation exercise is based on the regular set of conditioning assumptions used for outlooks (for foreign variables, among others), which are applied as actual realizations of the exogenous outlooks. No forecast-round-specific expert judgments are applied, as this may lead to bias.

The recursive filtering and forecasting mimic the regular forecasting process. Starting from the first period of the evaluation (2000 Q2), the initial conditions of the forecast are identified by filtering the data covering the history period of the corresponding forecast round. Subsequently, building on the initial conditions identified, an eight quarters ahead forecast is constructed by conditioning on the regular set of outlooks. In this exercise, the outlooks are constructed as the actual observations of the variables used for forecast conditioning. In the next round, the data sample used for filtering is extended by one period and a new forecast is made and so on, until the last period of the evaluation (2018 Q2) is reached.[49]

To evaluate the forecasting performance of the model, we calculate the mean absolute errors (MAEs) and root mean square errors (RMSEs) as follows:

$$MAE^m = \sum_{j=1}^{N}\sum_{i=1}^{H} |x^m_{j,i} - \bar{x}_{j,i}|, \tag{49}$$

$$RMSE^m = \sqrt{\sum_{j=1}^{N}\sum_{i=1}^{H} (x^m_{j,i} - \bar{x}_{j,i})^2}, \tag{50}$$

where m stands for the model used, $j = 1,\ldots,N$ represents the rounds of recursive filtering and forecasts, $i = 1,\ldots,H$ stands for the number of data points included in the computation of the statistics

[48] Here we follow our traditional terminology. We therefore use the term filtering for the Kalman smoother, since in principle it just involves the application of a filter.
[49] For a more detailed description of the recursive filtering and forecasting exercise see Andrle et al. (2009).

based on the horizon considered (full forecast vs MP horizon),[50] $x^m_{j,i}$ is the forecasted value of a variable, and $\bar{x}_{j,i}$ is the true observed value of the respective variable. g3+ to g3 ratios are computed in order to facilitate analysis of the forecasting performance statistics.

Table 6 summarizes these relative statistics for the selected variables over the full horizon (1–8 quarters) and the MP horizon (4–6 quarters) of the forecast. Due to the complexity of the exercise, we do not evaluate the contribution of each new feature in the g3+ model separately. Therefore, the reasoning for the mechanisms behind improvements or deteriorations in performance is based on our expert judgment.

Table 6: Forecasting Performance: Recursive Forecasts Statistics

	Full Forecast Horizon		**MP Horizon**	
Variable	**MAE**	**RMSE**	**MAE**	**RMSE**
Exchange Rate CZK/EUR	**0.87**	**0.79**	**0.85**	**0.76**
MP Inflation (YoY, ann., %)	1.01	**1.00**	1.01	**1.00**
Wage Growth (YoY, ann., %)	1.24	1.25	1.27	1.27
Real GDP Growth (YoY, ann., %)	1.04	1.00	**0.94**	**0.96**
Real Consumption Growth (YoY, ann., %)	**0.81**	**0.77**	**0.85**	**0.82**
Real Investment Growth (YoY, ann., %)	1.14	1.13	1.22	1.18
Real Import Growth (YoY, ann., %)	1.07	1.06	1.05	1.05
Real Export Growth (YoY, ann., %)	1.10	1.08	1.05	1.08
Consumption Price Deflator (YoY, ann., %)	1.04	1.02	1.07	1.04
Investment Price Deflator (YoY, ann., %)	1.01	1.01	**0.93**	**0.95**
Import Price Deflator (YoY, ann., %)	**0.86**	**0.87**	**0.75**	**0.78**
Export Price Deflator (YoY, ann., %)	1.09	1.06	1.11	1.09

Note: The table reports the relative values of the statistics as given by Equation (49) and (50) for g3+ vs g3. Numbers less (greater) than one indicate an improvement (worsening) of the in-sample performance of g3+ relative to g3.

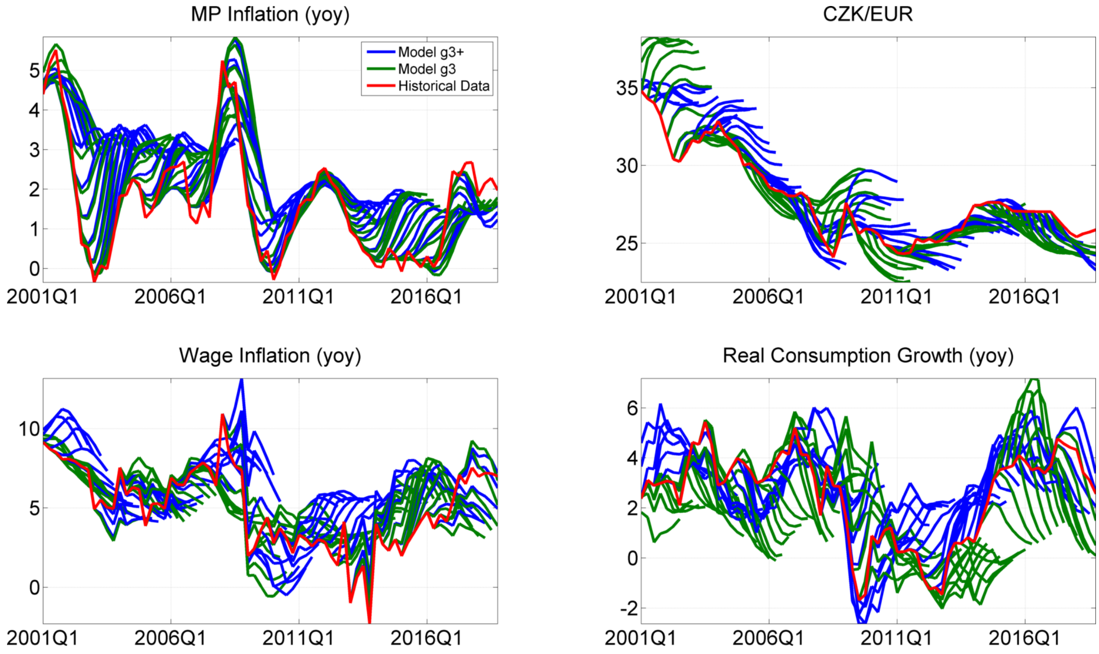
First, the relative MAE and RMSE measures reveal the same results qualitatively across all the variables. Second, there is some variation across horizon selection. The g3+ model offers improvements for the MP horizon, while the opposite holds for the full forecast horizon. As Table 6 indicates, the g3+ model outperforms the original g3 for the exchange rate and real consumption growth, where the differences are the most significant. In this case we attribute the improvement in performance for the exchange rate to the introduction of the new structural foreign economy block and the recalibration of the forward-lookingness parameter of the UIP condition. The improvement in the household consumption forecast can be attributed directly to the introduction of heterogeneous households.

There are also several variables for which the forecasting ability of g3+ with respect to g3 remains almost unchanged (MP inflation, deflators, exports and imports). Further, the results of our analysis also show that the improvements come at a cost, as there are several variables for which the forecasting performance deteriorates. The most notable difference arises in the case of wage growth, where the observed wages seem to be more persistent than those forecasted.

To illustrate the outcome of recursive forecasting, Figure 16 presents recursive forecasts of the level of MP inflation, the nominal CZK/EUR exchange rate, wage inflation, and real consump-

[50] The full forecast horizon covers eight quarters, while the MP horizon is the 4^{th}–6^{th} quarter of the forecast.

Figure 16: Recursive Forecasts – Simulations



tion growth.[51] As Figure 16 shows, the g3+ model delivers a better fit in the case of the nominal exchange rate and real consumption growth. However, the forecasting accuracy for wages is substantially lower, while the accuracy of the MP inflation forecast is almost unchanged.

Overall, we conclude that the recursive filtering and forecasting exercise indicates that the g3+ model keeps the same predictive ability as g3 on average. We should note that even though the forecasting performance of the g3+ model remains nearly the same, the new implementations described in the previous sections provide us with a richer model structure that delivers a more intuitive interpretation of key macroeconomic variables. Furthermore, the presence of performance tradeoffs between the g3 and g3+ models signals that the CNB's forecasting framework consistently provides stable performance.

10. Conclusion

This paper summarizes the development of g3+, a new DSGE model used for forecasting and policy analysis at the Czech National Bank (CNB). The implementation of the g3+ model is motivated by our need for more structural details capturing the essential characteristics and drivers of the Czech economy. The new model also reacts to the practical forecasting challenges that the CNB has been facing over the last decade using the g3 model. We also document the setup of the medium-term characteristics of the Czech economy representing the current view of the CNB.

The core model of the CNB's forecasting framework needs to incorporate relevant transmission mechanisms and sectors of the economy to support successful implementation of the inflation-targeting regime. The CNB's forecasters have constantly expanded their expertise in modeling issues and areas that were not endogenous to the previous core model, g3, since its introduction in

[51] Given the mechanical structure and the absence of expert judgments, the presented forecasts are different from the CNB's official ones.

2008. Expert judgment has become an integral part of our analysis and has been extensively used as an extra input in the regular forecasting rounds.

Expert judgment has been applied to deal, for example, with the prolonged period of extraordinarily low interest rates in the euro area coupled with the use of unconventional monetary policy tools, the time-varying speed of economic convergence of the Czech economy to the euro area, the use of energy in production, the presence of financially constrained households, and the limited capacity of model agents to process the conditioning information used as an external input to the forecast. As these issues are often structural, they can be addressed with a micro-founded model to facilitate their communication and support the transparency of the CNB's forecasts.

After extensive testing, the g3+ model was smoothly adopted as the core forecasting tool of the CNB in July 2019, as Czech National Bank (2019b) and Czech National Bank (2019c) document. Despite its (so far) short life, its new features have proved to be well-chosen. It provided crucial insights in the analysis of the impact of oil price growth in the aftermath of the oil-infrastructure attacks in Saudi Arabia in September 2019. The new foreign economy block within the g3+ model offered a rich story-telling ability for analyzing the effects of a possible disorderly Brexit in autumn 2019. Further, the possibility to incorporate the unconventional monetary policy of the ECB has facilitated communication with policy-makers and the general public. Last but not least, the most recent experience includes forecasting during the COVID-19 pandemic, when the g3+ model framework has allowed experts' insights and the effects of lockdown and government measures and support programs to be integrated with a high degree of transparency and reliability. The Forecasting and Policy Analysis System (FPAS) built on the g3+ model has proved its flexibility in supporting the decision-making process by enabling the CNB to draw up a number of sensitivity scenarios describing forecast risks in an environment of unprecedented uncertainty.

As our tests and most recent experience demonstrate, the g3+ model helps us identify sources of fluctuations, answer questions about structural changes, forecast and predict the effect of policy changes, and perform counterfactual experiments with a higher degree of detail than in the previous g3 model. The g3+ model is, in our view, a powerful tool that provides a coherent framework for policy discussion and analysis and, most importantly, supports the credibility of the FPAS. As explained by Christiano et al. (2018), a model with such features is an essential policy tool, as it provides policy-makers with the ability to assess policy tradeoffs by conducting quantitative experiments that are unavailable when considering real-time developments.

We believe that the expertise gained with developing and implementing the g3+ model will open up avenues for further research and model development. In this regard, the recent crisis – with interest rates near the zero lower bound – and the emergence of various unconventional monetary policy measures have already supplied new challenges for DSGE model-based forecasting, challenges that need to be tackled in the future.

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Appendix A: Model Equations

The following equations form the semi-structural model of the foreign economy (the effective euro area) in the g3+ model.[52] The basic behavioral equations are:

$$
\begin{aligned}
\hat{y}^*_t &= a_{11}\,\hat{y}^*_{t-1} - a_{12}\,\hat{r}^*_{t-1} + a_{13}\,\hat{z}_{t-1} + \varepsilon^{y^*}_t, && \text{(A.1)}\\
\tilde{\pi}^{*core}_t &= a_{22}\,\tilde{\pi}^{*core}_{t-1} + (1-a_{22})\,\tilde{\pi}^{*core}_{t+1} + a_{21}\,\hat{y}^*_t + a_{23}\,\hat{z}_{t-1} + \varepsilon^{\tilde{\pi}^{*core}}_t, && \text{(A.2)}\\
i^{*shadow}_t &= a_{33}\,i^{*shadow}_{t-1} + (1-a_{33})\left(i^{*shadow,eq}_t + a_{31}\,\hat{y}^*_t + \right. && \text{(A.3)}\\
&\left. + a_{32}\left(\frac{\tilde{\pi}^{*cpi4}_{t+4}}{4} - \tilde{\pi}^{*cpi4}\right)\right) + \varepsilon^{i^{*shadow}}_t, && \text{(A.4)}\\
0 &= \rho_{usdeur}\Delta usdeur_{t+1} - (1-\rho_{usdeur})\Delta usdeur_t + \\
&+ (i^*_t - i^{*eq}_t - prem^*_t) + \varepsilon^{\Delta usdeur}_t, && \text{(A.5)}
\end{aligned}
$$

where the definitions of the variables are:

$$
\begin{aligned}
\hat{r}^*_t &= i^{*shadow}_t - \tilde{\pi}^*_{t+1} - (i^{*shadow,eq}_t - \tilde{\pi}^*), && \text{(A.6)}\\
\hat{z}_t - \hat{z}_{t-1} &= -prem^*_t - \Delta usdeur_t - \tilde{\pi}^*_t + \tilde{\pi}^*, && \text{(A.7)}\\
\tilde{\pi}^*_t &= \rho_{\frac{oil}{ppi}}(\tilde{\pi}^{oil,usd}_t - \tilde{\pi}^{oil,trend}_t - \Delta usdeur_t) + \left(1 - \rho_{\frac{energy}{ppi}}\right)\tilde{\pi}^{*core}_t + \\
&+ \left(\rho_{\frac{energy}{ppi}} - \rho_{\frac{oil}{ppi}}\right)\tilde{\pi}^{*enerExoil}_t + \varepsilon^{\tilde{\pi}^*}_t, && \text{(A.8)}\\
\tilde{\pi}^{ener}_t &= \rho_{\frac{oil}{ppi}}(\tilde{\pi}^{oil,usd}_t - \tilde{\pi}^{oil,trend}_t - \Delta usdeur_t) + \left(\rho_{\frac{energy}{ppi}} - \rho_{\frac{oil}{ppi}}\right)\tilde{\pi}^{*enerExoil}_t, && \text{(A.9)}\\
\tilde{\pi}^{*cpi}_t &= \tilde{\pi}^{*cpi} + \frac{1}{4}(\tilde{\pi}^*_t - \tilde{\pi}^*) + \varepsilon^{\tilde{\pi}^{*cpi}}_t, && \text{(A.10)}\\
\tilde{\pi}^{*cpi4}_t &= \tilde{\pi}^{*cpi}_t + \tilde{\pi}^{*cpi}_{t-1} + \tilde{\pi}^{*cpi}_{t-2} + \tilde{\pi}^{*cpi}_{t-3}, && \text{(A.11)}\\
\Delta y^*_t &= (\hat{y}^*_t - \hat{y}^*_{t-1}) + \Delta y^{*trend}_t, && \text{(A.12)}
\end{aligned}
$$

and the rest of the variables follow autoregressive processes as a law of motion:

$$
\begin{aligned}
i^{*shadow,eq}_t &= \rho_{i^{*shadow,eq}}\,i^{*shadow,eq}_{t-1} + (1-\rho_{i^{*shadow,eq}})\,i^{*shadow,eq} + \varepsilon^{i^{*shadow,eq}}_t, && \text{(A.13)}\\
prem^*_t &= \rho_{prem^*}\,prem^*_{t-1} + \varepsilon^{prem^*}_t, && \text{(A.14)}\\
\tilde{\pi}^{*enerExoil}_t &= \rho_{\tilde{\pi}^{*enerExoil}}\,\tilde{\pi}^{*enerExoil}_{t-1} + (1-\rho_{\tilde{\pi}^{*enerExoil}})\tilde{\pi}^{*enerExoil} + \varepsilon^{\tilde{\pi}^{*enerExoil}}, && \text{(A.15)}\\
\tilde{\pi}^{oil,usd}_t &= \rho_{\tilde{\pi}^{oil,usd}}\,\tilde{\pi}^{oil,usd}_{t-1} + (1-\rho_{\tilde{\pi}^{oil,usd}})\tilde{\pi}^{oil,usd} + \varepsilon^{\tilde{\pi}^{oil,usd}}_t, && \text{(A.16)}\\
\tilde{\pi}^{oil,trend}_t &= \rho_{\tilde{\pi}^{oil,trend}}\,\tilde{\pi}^{oil,trend}_{t-1} + (1-\rho_{\tilde{\pi}^{oil,trend}})\tilde{\pi}^{oil,trend} + \varepsilon^{\tilde{\pi}^{oil,trend}}_t, && \text{(A.17)}\\
\Delta y^{*trend}_t &= \rho_{\Delta y^{*trend}}\,\Delta y^{*trend}_{t-1} + (1-\rho_{\Delta y^{*trend}})\Delta y^{*trend} + \varepsilon^{\Delta y^{*trend}}_t. && \text{(A.18)}
\end{aligned}
$$

[52] Here, all variables are in logarithms. For notation and parametrization, see Appendix B.

Appendix B: Model Parameters

This table describes the notation and parametrization of the foreign economy block in both the g3 and g3+ models.

Table B1: Foreign Economy: Steady State Parameters

Name	Symbol	g3	g3+
Steady States			
Foreign GDP (QoQ, ann., %)	Δy^*	1.8	1.6
Foreign Trend GDP (QoQ, ann., %)	Δy^{*trend}	1.8	1.6
Foreign CPI (QoQ, ann., %)	$\tilde{\pi}^{*cpi}$	2	2
Foreign Forecasted CPI (QoQ, ann., %)	$\tilde{\pi}^{*cpi4}$	2	2
Foreign PPI (QoQ, ann., %)	$\tilde{\pi}^*$	2	2
Foreign Core PPI (QoQ, ann., %)	$\tilde{\pi}^{*core}$		2
Foreign Energy excl. Oil PPI (QoQ, ann., %)	$\tilde{\pi}^{*enerExoil}$		2
Oil Price (QoQ, ann., %)	$\tilde{\pi}^{brent,usd}$		4
Oil Price Trend (QoQ, ann., %)	$\tilde{\pi}^{brent,trend}$		2
Foreign Interest Rate (p.a., %)	i^*	3.5	2.5
Foreign Equilibrium Interest Rate (p.a., %)	i^{*eq}		2.5
Foreign Premium (p.a., %)	$prem^*$		0
USD/EUR (QoQ, ann., %)	$\Delta usdeur$		0
Deep Parameters			
IS: Lag	a_{11}	0.75	0.6
IS: Real Interest Rate Gap	a_{12}		0.15
IS: Real Exchange Rate Gap	a_{13}		0.025
PC: Output Gap	a_{21}		0.07
PC: Lag	a_{22}	0.3	0.45
PC: Exchange Rate Gap	a_{23}		0.01
MPR: Output Gap	a_{31}		0.25
MPR: Elasticity to Target	a_{32}		2
MPR: Lag	a_{33}	0.8	0.7
UIP: Lead	ρ_{usdeur}		0.65
Oil Share in PPI	$\rho_{\frac{oil}{ppi}}$		0.06
Energy Share in PPI	$\rho_{\frac{energy}{ppi}}$		0.2
AR Processes Parameters			
Foreign Equilibrium Interest Rate	$\rho_{i^{*eq}}$		0.8
Foreign Premium	ρ_{prem^*}		0.7
Foreign Energy excl. Oil Inflation PPI	$\rho_{\tilde{\pi}^{*enerExoil}}$		0.5
Oil Price Growth	$\rho_{\tilde{\pi}^{brent,usd}}$		0.65
Oil Price Trend Growth	$\rho_{\tilde{\pi}^{brent,trend}}$		0.75
Foreign Trend GDP Growth	$\rho_{\Delta y^{*trend}}$		0.7

Appendix C: Identification of Domestic-Foreign Growth Wedge

The steady-state growth of the model economy is defined with help of exogenous growth processes known as technologies. As these technologies are only applied for stationary variables growth augmentation, given by Equations (2)–(6), they are defined in terms of quarter-on-quarter growth rates. Abstracting from definition of technology levels allow to preserve the stationarity of model. Such augmentation of stationary variable growth with the stochastic growth of the respective technologies is used to link the model variables to observed non-stationary data.

To account for the difference for the domestic and foreign economy growth rates, the domestic-foreign growth wedge $wedge_t^{trends}$ can be derived from the growth of the respective growth augmenting technology processes. The export-stationarizing technologies have, by definition, the same steady-state growth rate to a factor of four as the foreign GDP trend. In order to close the model, we pin down the level of the domestic-foreign growth wedge $wedge_t^{trends}$ as follows:

$$\Delta\left(Y_t^{*trend}\right)^4 = \frac{wedge_t^{trends}}{wedge_{t-1}^{trends}}\Delta tech_t^x. \tag{C.1}$$

Here, to allow for more flexibility, an AR(1) process is introduced:

$$wedge_t^{trends} = \left(wedge_{t-1}^{trends}\right)^{\rho_{wedgetrends}}\left(wedge_{ss}^{trends}\right)^{1-\rho_{wedgetrends}}\exp\left(\varepsilon_t^{wedgetrends}\right), \tag{C.2}$$

where parameter $\rho_{wedgetrends}$ defines the speed of return of domestic-foreign growth wedge $wedge_t^{trends}$ to its steady state $wedge_{ss}^{trends}$ while being a subject to an i.i.d. shock $\varepsilon_t^{wedgetrends}$.

Appendix D: Limited Information Rational Expectations in the g3 Model

One possible approach to mitigating the problem with intricate FIRE impulse responses is via shock size modification. This is, in fact, how this issue was addressed in the g3 model. When calculating the forecast under LIRE:g3, the model operator could decide that all shocks beyond period $t+k$ should be unanticipated. In other words, the model operator would set the shock visibility horizon at $t+k$. The shocks explaining the exogenous outlooks until period $t+k$ would then remain fully anticipated. This decision would result in the model dynamics as captured by Equations (D.1)–(D.4). The corresponding expectation scheme is depicted in Table D1.

$$
\begin{aligned}
\xi_t &= F\xi_{t-1} + R^{[0]}\varepsilon_t \quad + R^{[1]}\varepsilon_{t+1} + R^{[2]}\varepsilon_{t+2} + \cdots + R^{[k]}\varepsilon_{t+k}, && \text{(D.1)}\\
\xi_{t+1} &= F\xi_t \quad + R^{[0]}\varepsilon_{t+1} + R^{[1]}\varepsilon_{t+2} + R^{[2]}\varepsilon_{t+3} + \cdots + R^{[k-1]}\varepsilon_{t+k}, && \text{(D.2)}\\
\xi_{t+2} &= F\xi_{t+1} + R^{[0]}\varepsilon_{t+2} + R^{[1]}\varepsilon_{t+3} + R^{[2]}\varepsilon_{t+4} + \cdots + R^{[k-2]}\varepsilon_{t+k}, && \text{(D.3)}\\
&\vdots\\
\xi_{t+k} &= F\xi_{t+k-1} + R^{[0]}\varepsilon_{t+k}. && \text{(D.4)}
\end{aligned}
$$

Table D1: Fixed Horizon Expectation Scheme

Horizon / Period	year $y+1$				year $y+2$				year $y+3$				year $y+4$			
	Q1	Q2	Q3	Q4	Q1		Q3	Q4	Q1	Q2	Q3	Q4	Q1	Q2	Q3	Q4
current period	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0
quarter $q+1$	P	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0
quarter $q+2$	P	P	1	1	1	1	0	0	0	0	0	0	0	0	0	0
quarter $q+3$	P	P	P	1	1	1	0	0	0	0	0	0	0	0	0	0

Note: We assume that historical data is available until Q4 of year y and the forecast in the current forecasting round starts in year $y+1$, that is, Q1. The columns represent the periods of the exogenous outlook on the forecast range. The rows stand for the forecasted periods in the current forecasting round. The values show the proportion of the shock that is taken as anticipated. P stands for the predicted value from the previous period.

Additionally, the model operator could assume some discounting scheme or decay of expectations in the periods beyond the full visibility horizon. With increasing distance into the future, the proportion of shocks taken as anticipated would be reduced in line with some pre-determined scheme.[53] The model dynamics would then be described by Equations (D.5)–(D.8), where $W^{[i]}$ stand for the weight matrices that describe the gradual fading out of expectations. The corresponding expectation scheme is depicted in Table D2.

[53] The rest of the exogenous outlook path would be explained by unanticipated shocks. Unanticipated (i.e., surprise) shocks would take effect only in the future and would not influence the values of the model variables and agents' expectations before they hit the economy. Therefore, unanticipated shocks are not included in the system of equations in this section.

$$\xi_t = F\xi_{t-1} + R^{[0]}W^{[0]}\varepsilon_t + R^{[1]}W^{[1]}\varepsilon_{t+1} + R^{[2]}W^{[2]}\varepsilon_{t+2} + \cdots + R^{[k]}W^{[k]}\varepsilon_{t+k}, \tag{D.5}$$

$$\xi_{t+1} = F\xi_t + R^{[0]}W^{[1]}\varepsilon_{t+1} + R^{[1]}W^{[2]}\varepsilon_{t+2} + R^{[2]}W^{[3]}\varepsilon_{t+3} + \cdots + R^{[k-1]}W^{[k]}\varepsilon_{t+k}, \tag{D.6}$$

$$\xi_{t+2} = F\xi_{t+1} + R^{[0]}W^{[2]}\varepsilon_{t+2} + R^{[1]}W^{[3]}\varepsilon_{t+3} + R^{[2]}W^{[4]}\varepsilon_{t+4} + \cdots + R^{[k-2]}W^{[k]}\varepsilon_{t+k}, \tag{D.7}$$

$$\vdots$$

$$\xi_{t+k} = F\xi_{t+k-1} + R^{[0]}W^{[k]}\varepsilon_{t+k}. \tag{D.8}$$

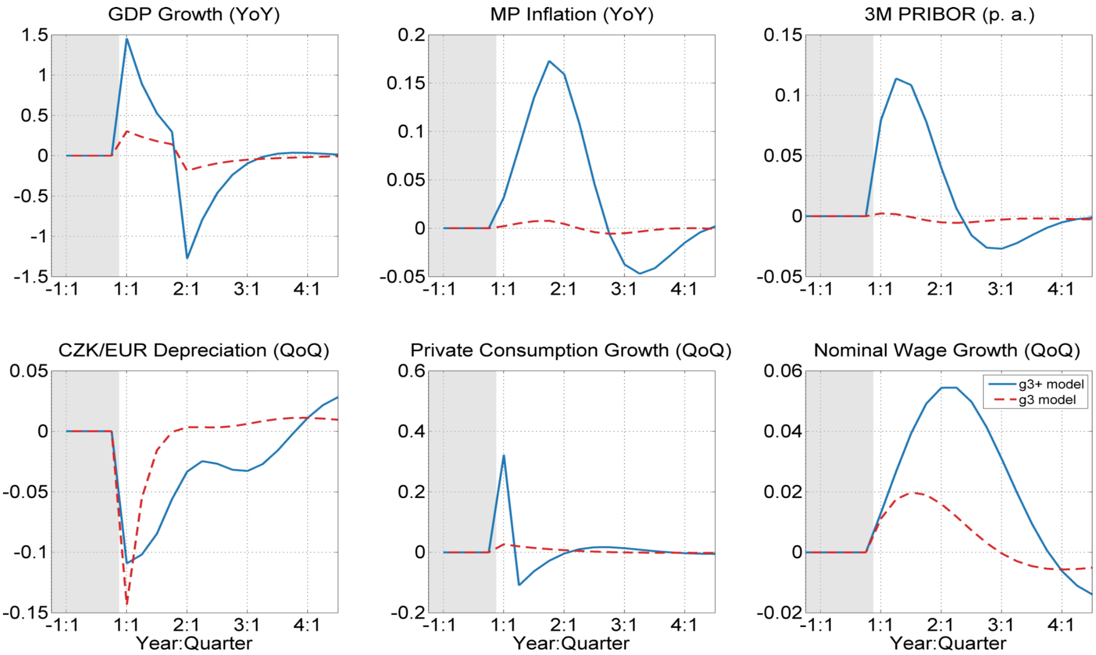
Table D2: Fixed Horizon Expectation Scheme with Decay

Horizon / Period	year $y+1$				year $y+2$				year $y+3$				year $y+4$			
	Q1	Q2	Q3	Q4	Q1	Q2	Q3	Q4	Q1	Q2	Q3	Q4	Q1	Q2	Q3	Q4
current period	1	1	1	1	1	1	0.8	0.6	0.4	0.2	0	0	0	0	0	0
quarter $q+1$	P	1	1	1	1	1	0.8	0.6	0.4	0.2	0	0	0	0	0	0
quarter $q+2$	P	P	1	1	1	1	0.8	0.6	0.4	0.2	0	0	0	0	0	0
quarter $q+3$	P	P	P	1	1	1	0.8	0.6	0.4	0.2	0	0	0	0	0	0

Note: The same explanation as presented in the note under Table D1 applies here.

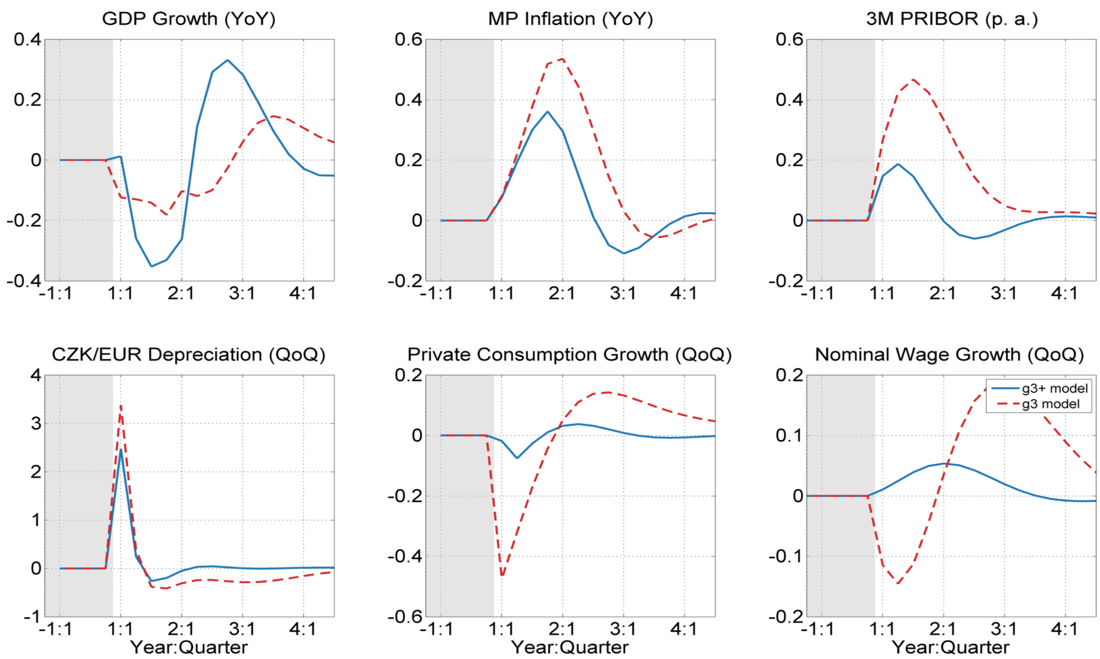
Appendix E: Impulse Responses

Figure E1: Foreign Demand Shock: Domestic Economy Variables (deviation, %)



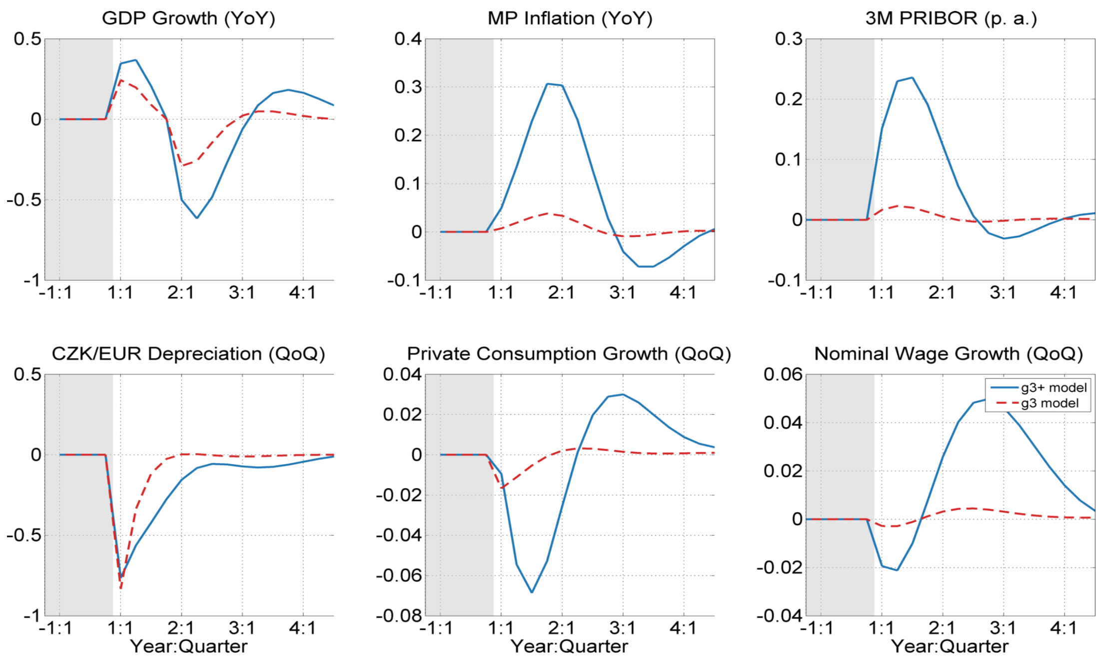
Note: The figure shows the impulse responses of the variables to a one percent deviation surprise shock. The responses are measured in percentage deviations from the steady state.

Figure E2: Foreign Monetary Policy Shock: Domestic Economy Variables (deviation, %)



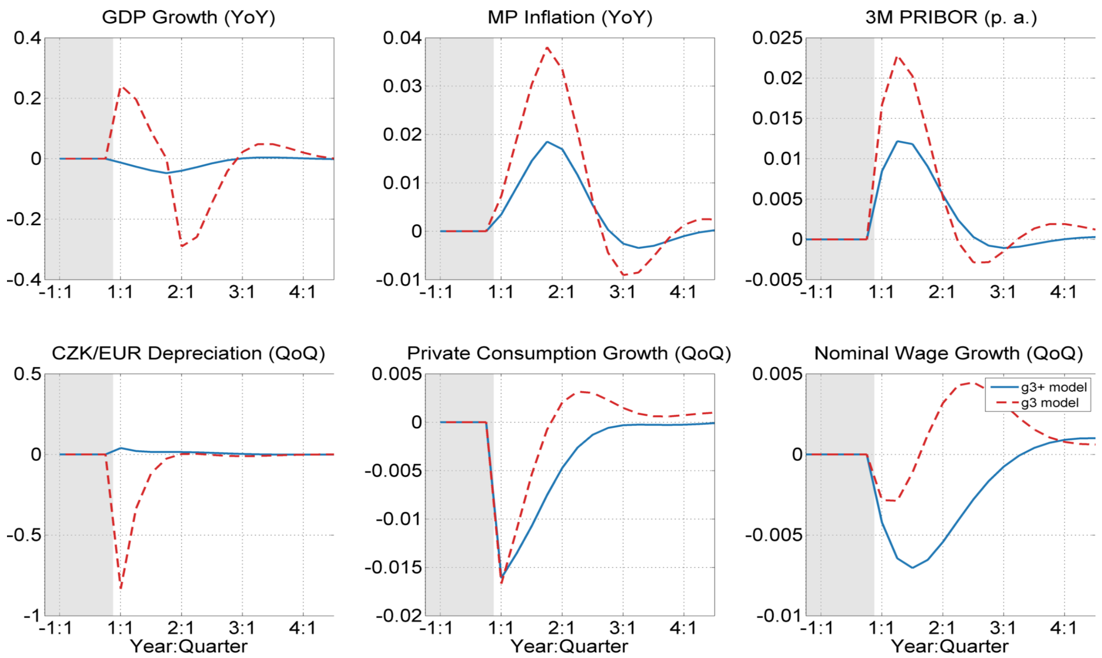
Note: The figure shows the impulse responses of the variables to a one percent deviation surprise shock. The responses are measured in percentage deviations from the steady state.

Figure E3: Foreign Core Prices Shock: Domestic Economy Variables (deviation, %)



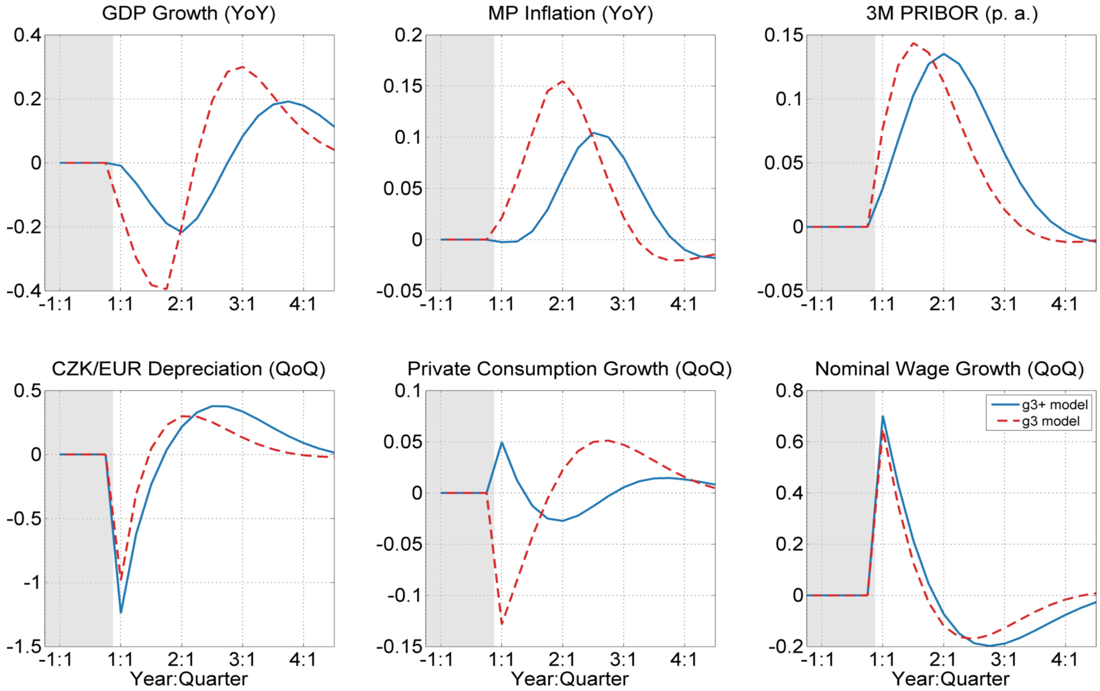
Note: The figure shows the impulse responses of the variables to a one percent deviation surprise shock. The responses are measured in percentage deviations from the steady state. Impulse responses to the overall foreign prices shock are depicted in the case of the g3 model.

Figure E4: Oil Price Shock (USD): Domestic Economy Variables (deviation, %)



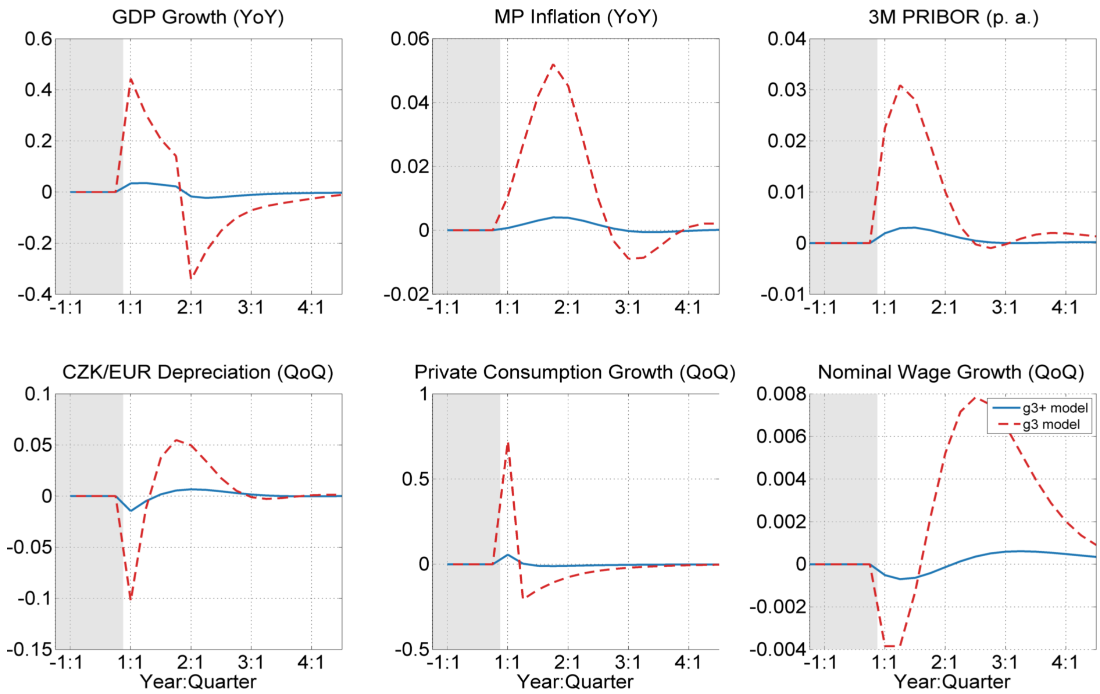
Note: The figure shows the impulse responses of the variables to a one percent deviation surprise shock. The responses are measured in percentage deviations from the steady state. Impulse responses to the overall foreign prices shock are depicted in the case of the g3 model.

Figure E5: Domestic Wage Push Shock: Domestic Economy Variables (deviation, %)



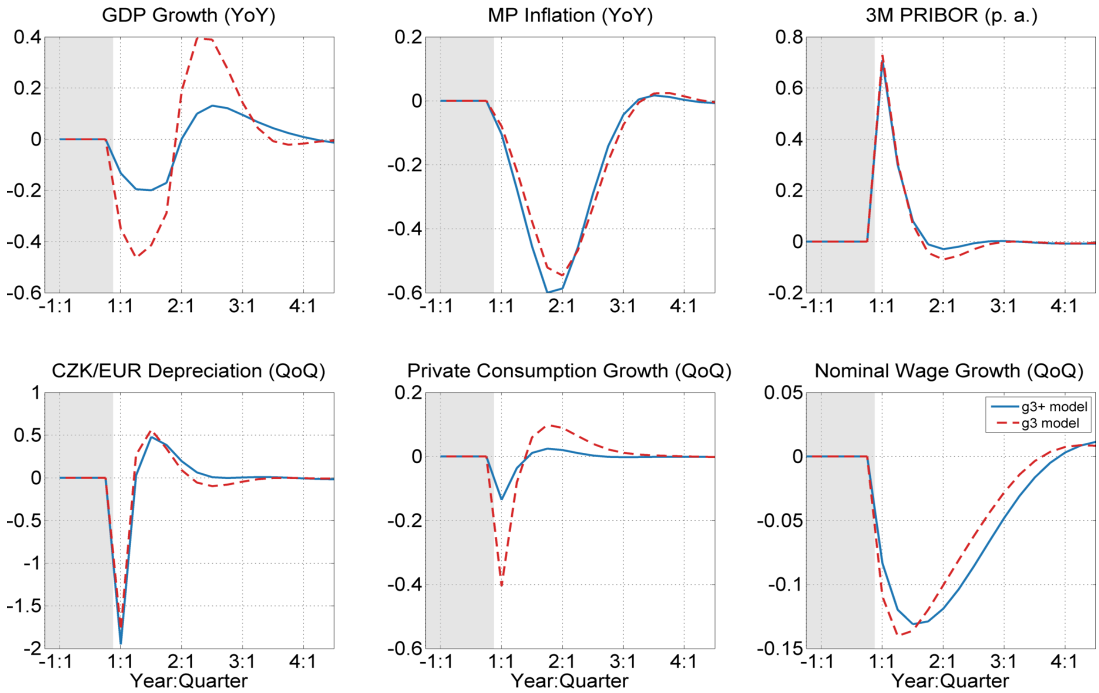
Note: The figure shows the impulse responses of the variables to a one percent deviation surprise shock. The responses are measured in percentage deviations from the steady state.

Figure E6: Domestic Consumption Preference Shock: Domestic Economy Variables (deviation, %)



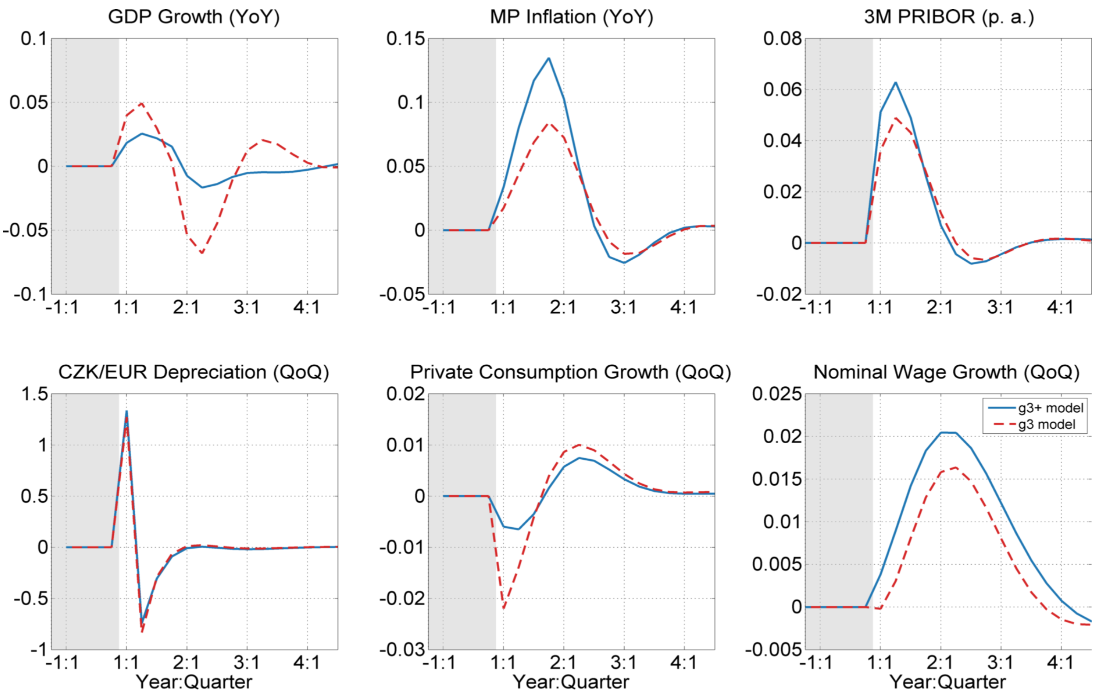
Note: The figure shows the impulse responses of the variables to a one percent deviation surprise shock. The responses are measured in percentage deviations from the steady state.

Figure E7: Domestic Monetary Policy Shock: Domestic Economy Variables (deviation, %)



Note: The figure shows the impulse responses of the variables to a one percent deviation surprise shock. The responses are measured in percentage deviations from the steady state.

Figure E8: Domestic UIP Shock: Domestic Economy Variables (deviation, %)



Note: The figure shows the impulse responses of the variables to a one percent deviation surprise shock. The responses are measured in percentage deviations from the steady state.

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