Martin Cincibuch, Pavel Bouc

INTERPRETATION OF CZECH FX OPTIONS

WP No. 36
Praha 2001

Abstract

We describe the Czech koruna option market and explore the behaviour of option prices during three eventful periods of the history of the Czech koruna. We point out their forward-looking nature and also show how implied risk neutral distribution might be used for monitoring and interpretation of market sentiment. Finally, we analyse the predictive power of the at-the-money-forward implied volatility. Our results for koruna options are consistent with results published for other currencies and show the efficiency of the market.

Martin Cincibuch is affiliated with CNB and Centre for Economic Research and Graduate Education in Prague. Part of the article was written when he participated on the Academic Project of the Centre for Central Banking Studies (CCBS) of the Bank of England. He is grateful to CCBS for strong support. Pavel Bouc is affiliated with Československá obchodní banka. We express only our views and not views of the institutions. We also thank Gordon Gemmill, Tony Gravelle, Richhild Moessner, Jan Strakoš, Martin Špaňár and David Vávra for valuable suggestions and worthwhile discussions. All errors and omissions are ours.

Contents

1 Introduction and summary

Currency options are interesting for market and central bank economists, because option prices contain information, which is independent from that provided by spot and interest rate markets. Option prices reflect the market perception of uncertainty incidental to the future development of the exchange rate, and therefore, they could be used for monitoring the current market sentiment. It is then natural to ask how rational this sentiment is, i.e. how much about the actual future development options can predict.

These issues have been studied for main currency pairs, but our article is one of the first[1] that addresses them in the Czech koruna context. Indeed, the OTC (over-the-counter) koruna option market has been developing and expanding now for several years, and thus data time series are becoming rich enough for analytical purposes.

In the first section of the article, we briefly describe the structure and history of the Czech FX option market. We characterise four main categories of contracts traded on the market and suggest how to interpret the prices of these structures. In addition, we show how probabilistic distributions implied

[1] Probably the first research that employed Czech currency options was pursued by Campa and Chang [6] who studied the behaviour of the option market before the currency crisis in 1997.

by option prices may be used for monitoring market sentiment.

Next, we explore the behaviour of option prices during three eventful periods of the history of the Czech koruna between January 1997 and August 2000. First, we observe that at-the-money-forward[2] (ATMF) implied volatility-signalled market worries weeks before the crisis unfolded in May 1997. Moreover, other option contracts - risk reversals - demonstrated even a greater information lead than ATMF straddles. This finding is consistent with the results of Campa and Chang [6], who formally showed that options warned against the looming crisis much earlier than other markets. Second, we briefly discuss the sharp drop in risk reversal prices that occurred during the first months of 1999. It coincided with the outflow of short-term capital, which had speculated on the interest rate differential between Czech koruna, German mark and dollar instruments. And finally, we analyse the behaviour of the option market in the context of forex interventions of the Czech National Bank in autumn 1999. We also point out how shifts in market sentiment during this period are reflected in risk neutral distributions (RND) implied by option prices.

The behaviour of option prices before the currency crisis in 1997 is an anecdote that supports a hypothesis of the rational market expectations. In the final part of our study, we investigate this question more formally and test whether the ATMF implied volatility of koruna options predicts actual future volatility. We found that the koruna option market is quite efficient in this sense. This result is consistent with the results for other currencies.

[2] At-the-money forward moneyness refers to the option whose exercise price is equal to the current forward price. Similarly, out-of-the-money (OTM) moneyness refers to call (put) options with the exercise price higher (lower) than the current spot price of the underlying asset.

2 Koruna FX option market

The Czech koruna option market is relatively young. Its origin may be dated back to approximately 1996. It is also comparatively small due to the small size of the local economy. Therefore, it still has some emerging market features. Nevertheless, it has already overcome the early stages of its development, and now it may serve as a source of valuable market information.

In the beginning, the market was dominated by a couple of local banks. Later, the market expanded, and foreign, namely London banks, took the lead. The market is structured as a typical OTC (over-the-counter) market where the bulk of transactions takes place on the interbank market. Banks sometimes trade directly with each other, but more often they make deals via brokers[3].

In currency composition and average turnover, the koruna option market shadows the spot market. While in 1997 both mark and dollar options were traded in parallel, since the year 2000, euro options have dominated the market and dollar options have become marginal. Average daily turnover may be roughly estimated to be about 80 - 100 mil. euro[4]. In terms of notional

[3] Approximately fifteen to twenty banks actively trade koruna options, but only about five of them form the market core. About five broker institutions serve the market as well.

[4] Unlike exchange markets the OTC markets are traditionally opaque, and one often has to rely on more or less precise estimates. The turnover

Table 1

Approximate daily averages in billions of US dollars

	Spot, Outright Forwards and Forex Swaps	Options
All currency pairs	1500	87
USD - DEM (1998)	250	17
USD - JPY (1998)	260	32
CZK - EUR	0.75	0.09

BIS [3], CNB [4] and own estimates

amounts, which, as the Table 1 shows, is very little in comparison with the trading volume of global FX options. Having turnover and liquidity lower than may be found for main currency pairs, the koruna option market shows bigger bid-ask spreads.[5] In addition, a smaller number of contract types and strategies is actively traded, and contrary to some more developed markets, continuous trading does not exist.

However, while the koruna option market is smaller than option markets for main currencies, it is not small in comparison to the size of the koruna spot market. The ratio of turnovers in options and the cumulation of spot, forwards and swaps is not too different from other markets. It points to market satiation and its structural maturity.

As is usual on the OTC market, option prices are quoted in terms of the Black-Scholes implied volatility. The Black-Scholes formula serves as a 1-1 mapping between the price of the option and the implied volatility parameter. This convention does not require that the Black-Scholes model hold in reality. Further, the Black-Scholes delta ("hedge ratio") is used instead of the exercise price for quotes as a measure of moneyness. Delta is a marginal change in the Black-Scholes price of an option with respect to a change in the spot rate, e.g. the more a call is in the money, the higher its delta, even though

number is a consensual estimate of several market participants, and it is also consistent with the indication from the Czech National Bank survey of foreign exchange market turnover realised through Czech banks.

[5]While a typical bid-ask spread on the Czech market is about 0.5-1 percentage point in terms of implied volatility, on the developed markets it might be as low as 0.2% Nevertheless, Czech spreads are lower than in other European emerging markets.

the relationship is non-linear. Most often, ATMF and 0.25-delta options are quoted. To a great extent, this market convention enables the separation of volatility as the measure of underlying uncertainty from other observable market variables that affect option prices. A more detailed description of the market quoting conventions is given in the Appendix 5.1.

2.1 Traded contracts and their interpretation

Contracts traded on the koruna market might be classified into four main categories. The first one consists of the ATMF straddle, which is the main contract of the market and represents about 30%-40% of the option turnover. Its quoted volatility run (its term structure) ranges from one week to one year, but a one-month horizon seems to be most representative. A long ATMF straddle is a combination of long positions in the ATMF call and ATMF put options. ATMF implied volatility reflects the level of uncertainty perceived by the market and it relates closely to the variance of the distribution of expected returns.

Plain vanilla single call or put options fall into the second most important group which accounts for about 20%-40% of the option turnover. Singles are traded as OTM options for various strikes between approximately 20 to 50 delta. Nevertheless, ATMF and 25-delta contracts are most usual.

The third category of contracts is formed by risk reversals, which account for about 20% of the total option turnover. A long risk reversal is also a combination of two OTM (most often 25 delta) options: a long position in the OTM call and a short position in the OTM put. The risk reversal price is quoted as the difference between the volatilities of its two components.

Risk reversals are also quite interesting from the analytical point of view. A decrease in the risk reversal price means that the call option becomes less expensive than the put option. It might be interpreted in a way that the market attributes a smaller likelihood to the euro appreciating significantly (koruna depreciating) than the likelihood of the euro depreciating significantly (koruna appreciating)[6]. Therefore, risk reversals could be seen as an indicator of the asymmetry of market expectations. As such, risk reversals relate to the skewness[7] of the distribution of expected returns.

This phenomenon, however, might be seen from a different 'dynamic' perspective, which is shared by many market practitioners (Gemmill [12]). Assume that the koruna is expected to become more volatile in the wake of a remarkable depreciation than after appreciation of a similar magnitude. Then, an option which would become e.g. at-the-money after the depreciation scenario would have higher time value than the option that would be at-the-money in the appreciation scenario. Consequently, the first option should have a higher price (implied volatility) than the latter one and the risk reversal would be positive. Indeed, volatility asymmetry between the two scenarios would transform into the asymmetry of the terminal distribution of returns, and in the risk neutral world the two interpretations are completely equivalent.

Nevertheless, investors' attitudes toward risk might also be distinct under the two scenarios. In this case, higher time value of the options in the depreciation scenario would reflect increased risk aversion rather than actually expected volatility and the 'dynamic' approach would seem more appropriate.

The remaining group representing 10%-20% of turnover encompasses strangles and various spreads (butterfly, calendar, currency). A long strangle contract is a combination of long positions in two options: an out-of-the-money (OTM) call and OTM put. It is quoted as the average of the volatilities of its components above ATMF volatility. Strangle contracts reflect how concentrated expectations around the forward are. The higher the strangle value, the higher likelihood which market attributes to extreme values of returns. Thus, the value of strangles might be associated with kurtosis[8] of the

[6] It should be emphasised that a change in the risk reversal, holding other things equal, does not alter the mean of the distribution of expected returns. Thus, a somewhat lower likelihood of significant koruna depreciation might be offset, for example, by the higher likelihood of mild depreciation of koruna.

[7] Skewness is the third central moment of a probability distribution, normalised by the third power of its standard deviation.

[8] Kurtosis is the fourth central moment of a distribution, normalised by

distribution of returns.

ATMF implied volatilities are often quoted several times a day, but risk reversal and strangle prices change less frequently. For example, it seems sufficient to sample risk reversals only once in two days for the koruna market. However, this lower frequency does not necessarily mean low informational value of the data. Indeed, even for the dollar - yen, which is the most liquid and most heavily traded currency pair, a 25-delta risk reversal often stays constant for several days.

To see why this makes sense, consider the difference in meaning between ATMF-implied volatility and other prices. While ATMF implied volatility measures an overall level of uncertainty and makes the essence of option prices, risk reversals and strangles determine only the shape and curvature of the volatility smile[9]. They serve as first and second order adjustments of the Black Scholes model, which would imply a flat volatility smile and run. In this sense, a change in the risk reversal or strangle amounts to the model change. A certain analogy might be found between volatility smiles and yield curves. Changes of the ATMF-implied volatility resemble parallel shifts of some yield curve and risk reversal, and strangle changes are similar to the changes of the yield curve spread and curvature. Indeed, spread and curvature of yield curves tend to change more slowly than the interest rate level.

2.2 Implied risk neutral distributions: A tool for interpreting option prices

It is possible to interpret directly quoted option prices along the lines described above; or alternatively, one might want to construct some embracive characteristic. Risk neutral distributions (RNDs) turn out to be such a summarising tool, encapsulating all available information contained in market prices. This comfortable feature of RNDs stems from the theoretical result that under the assumption of no arbitrage and frictionless markets, the price of a traded se-

the fourth power of its standard deviation.

[9]Volatility as a function of delta.

curity can be expressed as an expected discounted security payoff, where the expectation is taken with respect to an appropriate RND (Cox, Ingersoll and Ross [11], Ross [16]). Breeden and Litzenberger [2] showed that if the RND is a continuous one, then its discounted density f is equal to the second derivative of the European call option price c with respect to the strike price X. Denoting by r and S the appropriate domestic interest rate and the current spot price and by S_T the random spot price at option's maturity T, we may write formally

$$c\left(S, X; r, T\right) = e^{-rT} \int_0^\infty \max\left(S_T - X, 0\right) f(S_T) dS_T \qquad (1)$$

$$f\left(X\right) = e^{rT} \frac{\partial^2}{\partial X^2} c\left(S, X; r, T\right). \qquad (2)$$

Equations (1) and (2) directly or indirectly underlie all methods of constructing RNDs from option prices. In this paper, the Malz [15] estimation method, which consists in the quadratic interpolation of OTC volatility quotes, was employed. The method is also described in the Appendix 5.2. More thorough treatment of the implied distributions and their estimation could be found in Rubinstein [17], Bahra [1] or Soederlind and Svensson [18].

Bahra [1] and Chang and Melick [7] summarise issues relating to the usefulness of implied RNDs and their interpretation. And, Clews, Panigirtzoglou and Proudman [9] describe methods used at the Bank of England for estimating RNDs, which enter as a regular input to its Monetary Policy Committee briefings. In this paper, a practical useof the implied distributions for monitoring market sentiment changes is also illustrated below in the section 3.3, which deals with market reaction to forex interventions of the CNB.

3 The Czech koruna in 1997-2000

In this section, we make three brief case studies of the development of the Czech koruna from the option prices point of view. First in 3.1, we observe that option price behaviour before the currency crisis in 1997 indicated their rational forward-looking nature as they predicted the break of the currency peg. Also, in the section 4 we investigate this rationality issue more formally, where we test whether option prices predict actual future volatility of the koruna exchange rate. Second, the part 3.2 shows that a significant change in the level of prices of risk-reversals in the beginning of 1999 marks a break in the FX market structure and the composition of market participants. Options, as a quantitative indicator, may help to understand this change, which had been otherwise known only on the level of rumours. And third, we investigate how forex interventions have been perceived by option markets and show how probabilistic distributions implied by option prices might be used for monitoring market sentiment.

The data set we used consists of time series of mid-rate quotes[10] for Czech koruna-German mark (euro) option contracts with one-month maturity. For ATMF straddles and 25-delta risk reversals, we use time series over the

[10] CSOB trading desk database.

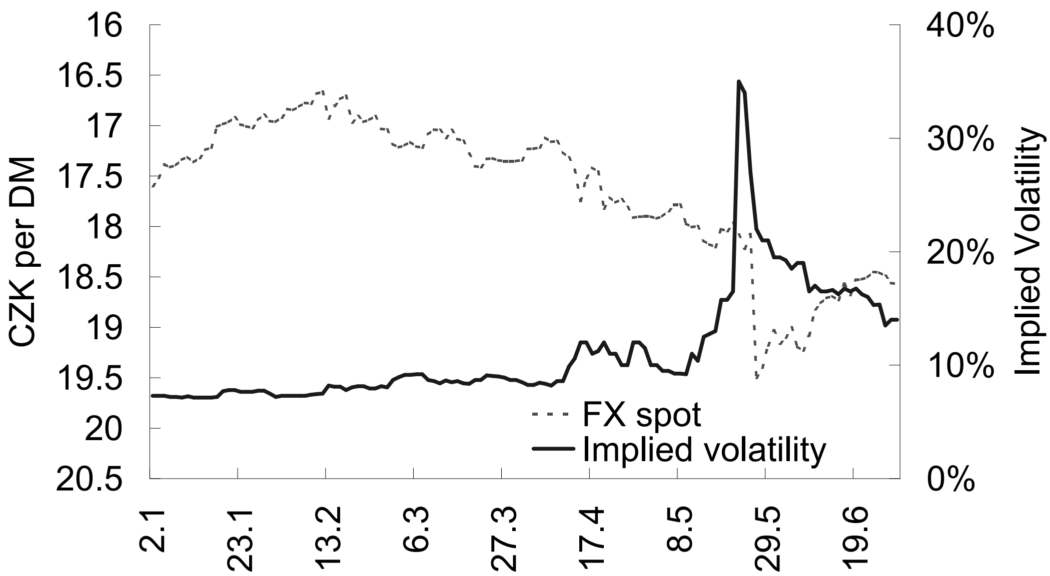
period January 1997 - April 2001, and data for 25-delta strangles have been available since 1998. Whenever we needed spot exchange rates and interest rates, we used the close of business interbank quotes.

3.1 Options look forward: The currency crisis in May 1997

One of the most prominent events in the recent history of the Czech koruna was the currency crisis in May 1997. Šmídková et.al [19] explicate its roots and give a detailed description of its course and the reaction of the central bank.

Figure 1

Exchange rate and implied volatility 1997



The current account deficit and other problems of the Czech economy were highly publicised, and the issue of devaluation was discussed in the media a long time before the actual crisis occurred. However, as benign developments of the spot rate, implied volatility and risk reversals suggest (Figures 1 and 2), the market seemed to be unruffled until spring 1997. Indeed, in the middle of February, the koruna was close to the appreciation side margins of its $\pm 7.5\%$ band, and the CNB spokesman had to declare that the central bank was concerned about the strength of the currency.

Figure 2

ATMF-implied volatility and risk reversals before the crisis in 1997

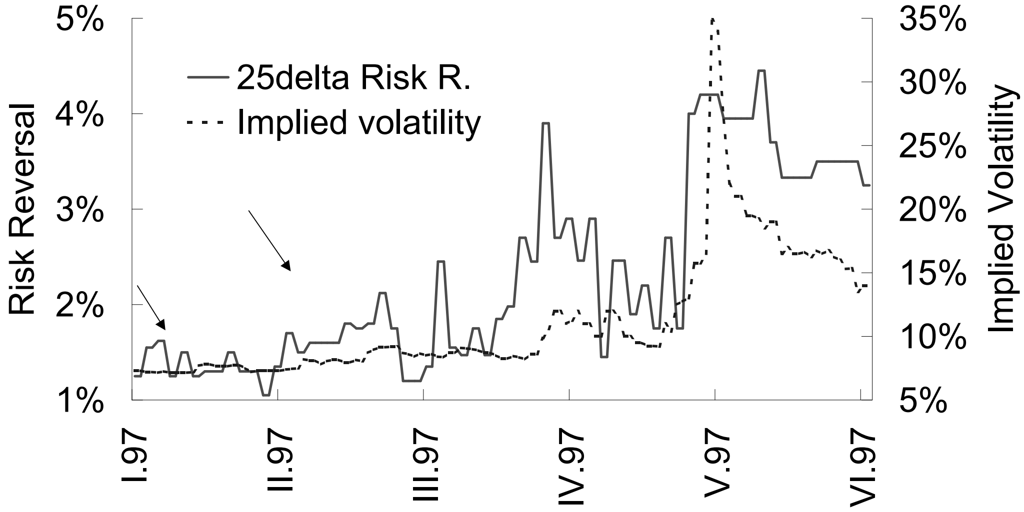


Figure 1 demonstrates that ATMF implied volatility significantly increased already in April 1997, well before the crisis actually happened. However, risk reversals suggest that the market began to worry as early as February. The elevated likelihood of large depreciation as it was perceived by market participants was manifested in higher risk reversal prices. Figure 2 illustrates that risk reversals tended to react before ATMF implied volatility, and as such, they were an earlier and more sensitive indicator of potential future problems. One might even argue that since investors operating on the option market are quite sophisticated and since option contracts enable high leverage, high option prices might have revealed speculators' positioning before they started attacking the koruna exchange rate regime.

The forward-looking information contained in options before the crisis was also studied by Campa and Chang [6]. They constructed arbitrage-based tests[11] of credibility of the target zone based on the forward rates and on the

[11] Their tests consist in constructing bounds for option prices that should not be violated if the target zone were perfectly credible. Consequently, from violating such a bound one could infer non-perfect credibility of the arrangement. It may be proved that option-based tests are more powerful

option prices, and they showed that the koruna exchange rate band lost its credibility weeks before the actual devaluation.

In the final part of the article, we econometrically support the notion that not only do options mirror current market perception, but that this sentiment is highly rational and that option prices carry information about future volatility of the spot. Therefore, the behaviour of option prices during the pre-crisis period may be viewed as an illustration of this option forward-looking nature.

3.2 Options help to identify and understand structural breaks: The outflow of hot capital in 1999

Most observers of the koruna market agree that the relative importance of reasons for currency trades changed substantially in early 1999. The wide interest rate differential was an important impetus for capital flows throughout 1998, but afterwards, as Czech inflation and interest rates dropped, it was replaced by privatisation and direct investment flows, and the structure of market participants changed accordingly. As some traders noted, this change was also reflected in a structural shift in market behaviour: since the second quarter of 1999, they described the currency market as being relieved in comparison with the previous period. Nevertheless, beyond option prices there is lack of directly observable indicators that might have shown this shift[12].

For an interpretation of how this structural shift was manifested in the option market, it is useful first to recall the economic environment in 1998. The Czech koruna was quite strong at that time. According to market sources[13], it was the world's third-best-performing currency against the dollar and mark

than simple forward rate tests.

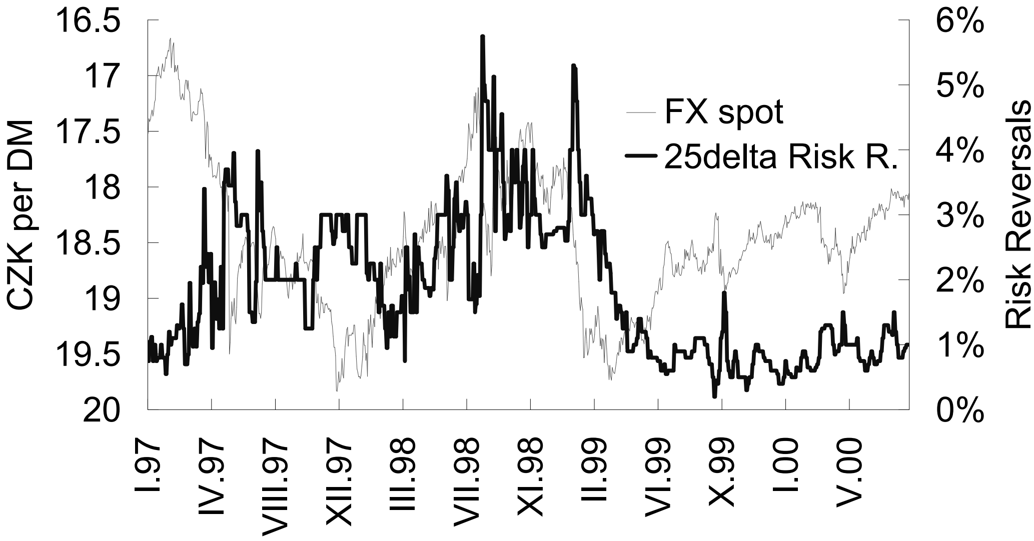
In particular, they report that a convexity test based on options signalled a credibility loss already 29 days before the breakdown of the regime. In comparison, the simple Svensson test of credibility based on the forward rate level indicated a credibility loss only 11 days in advance, which just coincides with the beginning of the speculation wave against the currency.

[12]Also, surveys of the Czech National Bank showed that the turnover of koruna spot, forward and FX swaps somewhat decreased from the high levels experienced in 1998.

[13]Bloomberg.

Figure 3

Koruna risk reversals 1997-2000



in 1998. Its strength stemmed from the interest rate differential, which was about twelve percentage points above the mark in terms of one-month deposits during the first months of the year. Domestic interest rates were kept relatively high by the Czech National Bank, which had adopted direct inflation targeting in January 1998 and set an interval of about six per cent for the net inflation target for the end of the year[14]. During 1998, the Czech economy experienced fast disinflation, which resulted from a combination of subdued domestic and foreign demand, declining world commodity prices and strengthening of the currency. The Czech National Bank reacted by a series of interest rate cuts in the second half of the year[15].

In these circumstances, the koruna exchange rate was influenced by conflicting forces. From the point of view of the real economy, the strength of the currency was not justified by the competitiveness of domestic producers

[14]Although many considered the inflation target to be too ambitious, after several months it became increasingly clear that, not only could the target be hit, but that it would even be undershot.

[15]The Czech National Bank's policy was a source of controversy. Many blamed the conjunction of monetary and fiscal restriction for causing the recession or making it deeper. Others claimed that the recession was a result of structural problems related to inefficient institutions in the market economy.

(e.g. Cincibuch and Vávra [10]), and also the interest rate cuts would have normally led to currency weakening. However, the rate cuts actually provided support for the koruna as they led to soaring bond prices along the entire length of the yield curve, and investors were attracted by capital gains. Such an obviously unstable situation seemed to mirror the learning process of capital markets during the transition to a low inflation environment. The longer end of the yield curve had not reflected expectations of such fast inflation and the ensuing interest rate decline and tended to react only ex-post and to move together with the short end. Clearly, interest rate cuts and the consequential capital gains could not have gone on forever, and it was only a matter of time before the currency would lose its last supporting pillar. Actually, it happened in January and February 1999 when depreciation of almost ten per cent was triggered by the Brazilian currency crisis, bad trade data and further interest rate cuts[16].

The option market correctly reflected this uncertainty. Figure 3 shows high levels of at-the-money-forward, implied volatility and, in particular, of risk reversals[17] in 1998, and their dramatic decline in early 1999 documents the structural shift and the end of the hot-capital period.

As it was discussed above in the section 2.1, higher positive quotes of risk reversals may indicate market worries about the sustainability of the currency's strength. However, it seems that high risk reversals were also a by-product of hedging short term speculative positions. Indeed, the very presence of mercurial capital increases the vulnerability of the market. As short-term investors were being replaced by others with a medium- or long-term investment horizon, the value of risk reversals went down.

[16]The eurobond market may serve as an illustration of this development. Investors tended to redeem koruna eurobonds issued by foreign institutions. In 1999, the outstanding amount of these instruments decreased by more than 26%.

[17]Risk reversal quotes of approximately 3%-5% are quite high in comparison with developed markets where values less than 2% are typically observed.

3.3 Options help to monitor market sentiment: Forex interventions

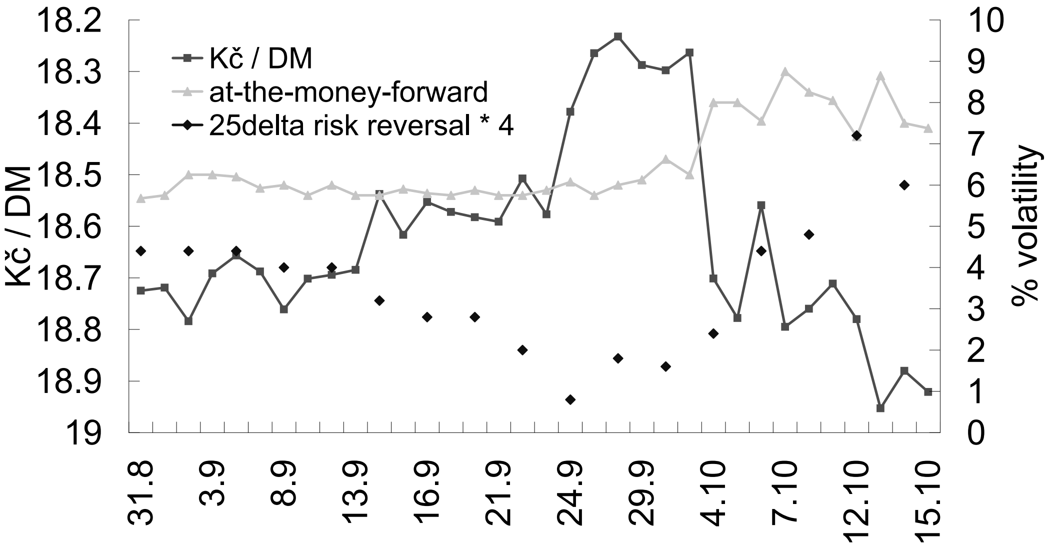
Forex interventions naturally provide a good opportunity for observing changes in market sentiment. Therefore, on the example of interventions pursued by the Czech National Bank, we illustrate how options might be used for sentiment monitoring.

Let's first recall the macroeconomic environment in the summer of 2000. Since 1999, yield differentials have played a smaller role, and the effects connected with longer-term investment flows have become the most important factor affecting the koruna exchange rate. Note that foreign direct investment flows accounted for 9.6% of GDP in 1999 and 7.9% of GDP in the first three quarters of 2000. In comparison, the current account deficit was much smaller, the capital account was negligible, and thus, as the balance of payments identity stipulates, this inflow had to be offset by induced outflows in other parts of the financial account or possibly by an increase in the Czech National Bank's international reserves. In this situation, the abundance of foreign exchange had an impact on the exchange rate and led to relatively quick appreciation.

In August and September 1999, a series of news caused the market sentiment towards the koruna to be very optimistic. Reports showed that the economy was recovering from the recession earlier than had been expected and that annual inflation rose for the first time in 17 months. This led market participants to the belief that the central bank would forbear further interest rate cuts. Moreover, the external position of the economy improved: the current account was in surplus in the second quarter of the year and the August trade deficit was lower than expected. Most importantly, strong foreign capital inflow was expected for privatisation, real economy projects, as well as new issues of government eurobonds. All of these factors together with the low level of overall indebtedness were cited in the market as a reason for further strengthening of the koruna with minimal downside risk. Figure 4 shows how the option market reflected these changes. The decreasing price of 25-delta risk reversal contracts during the first two decades of September indicated that appreciation of the koruna was not a random aberration within the usual market volatility, but might have developed in a strong trend.

Figure 4

Exchange rate and option prices in September and October 1999



The central bank was worried that some parts of the economy might have had problems with adjusting efficiently to the continued fast pace of appreciation. Moreover, expected substantial inward investment seemed to inspire secondary speculation. Therefore, at the end of the month, one of the members of the central bank's board tried to talk the koruna down. He stressed the readiness of the bank to stem undue nominal appreciation through sterilising[18] the inflow of privatisation capital. Moreover, he did not exclude the possibility of further rate cuts. This verbal intervention resulted in some increase of risk reversals.

The situation changed further on September 4, when the central bank sold koruna on the market and also cut the official repo rate. In response, the koruna weakened, and the price of risk reversals increased. Also, the main contract of the option market, the ATMF straddle, began to rise already

[18]The government established a special account with the central bank for privatisation proceeds in foreign currency. Later and according to the market situation, the central bank would either buy this foreign currency for its reserves or convert it to koruna on the market.

before the intervention as a result of CNB comments. In the wake of intervention, its implied volatility rose reflecting the uncertainty which the central bank introduced in the market to stem the view that koruna was a one-way bet. However, market fears waned, and the skewness soon fell again in late October. The currency also began to appreciate once again. Nevertheless, the intervention seemed to have effectively slowed appreciation, because the exchange rate reached the intervention level only in January of the following year.

Figure 5

Implied distributions of one month returns

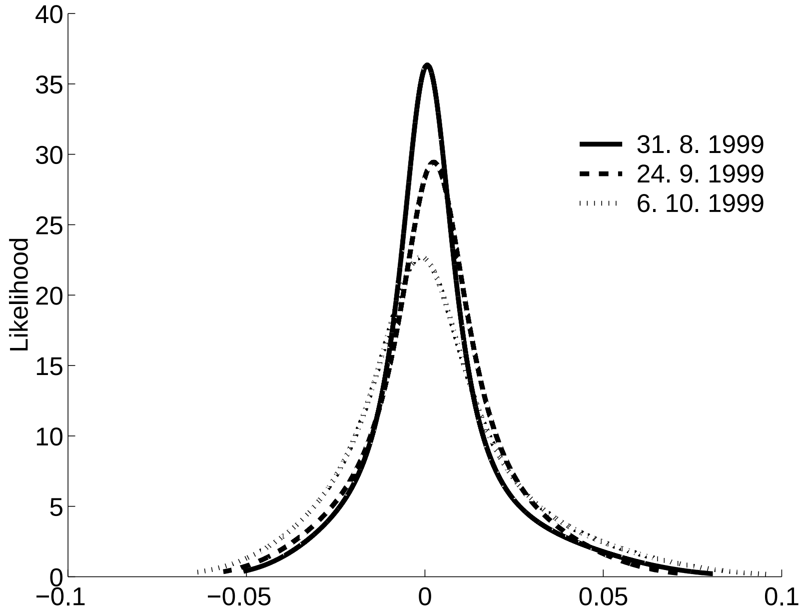


Table 2

Summary statistics for implied distributions of monthly returns

	31-Aug-99	24-Sep-99	6-Oct-99
Downside risk (>3%)	8.38%	8.24%	12.29%
Upside risk (>3%)	3.37%	4.77%	7.21%
Mean	0.31%	0.31%	0.27%
Mode	0.07%	0.25%	-0.10%
Median	0.14%	0.27%	0.05%
Standard deviation	1.84%	1.89%	2.40%
Skewness	73.99%	18.43%	57.36%
Pearson statistic	9.77%	2.32%	9.39%
Kurtosis	188.37%	87.35%	98.80%

This development is documented also in the behaviour of risk neutral distributions estimated[19] from market option prices graphed in Figure 5 and characterised in Table 2. The distribution from September 24 in comparison with the distribution from the end of August shows an elevated upside risk[20] and a reduced downside risk, which reflects the market's optimism and its shift towards a one-track view of the exchange rate. Also, a lower distribution asymmetry measured by its skewness and Pearson statistics[21] supports this interpretation. Higher variance and asymmetry observed for distributions estimated after intervention reflected higher market uncertainty.

[19] Risk neutral distributions for the Czech koruna are estimated from straddles, risk reversals and strangles via the Malz method [15].

[20] Upside (downside) risk is a risk neutral probability that the currency will appreciate (depreciate) 3% or more.

[21] Pearson statistics is the difference between the mean and median normalised by standard deviation.

4 Predictive power of implied volatility

In the section 3.1 dealing with the period before the currency crisis in 1997, we pointed out, in consensus with other authors, that option prices carried rational forward-looking information. Here, we attempt to test the hypothesis of the rational market more formally over a longer time span. We follow authors who estimated how well implied volatility can predict actual realised volatility for various option markets including those in main currency pairs.

Two regression specifications were investigated in the literature. The first specification is a conventional test of market efficiency

$$\sigma^A_{t,k} = \alpha_0 + \alpha_i \sigma^I_{t,m} \tag{3}$$

and then the second is an extended "encompassing" one

$$\sigma^A_{t,k} = \alpha_0 + \alpha_i \sigma^I_{t,m} + \alpha_h \sigma^A_{t,-l}. \tag{4}$$

In the equations, the letter m denotes the maturity of the option, the symbol $\sigma^i_{t,m}$ denotes implied volatility observed at date t and referring to the option with the maturity date $t+m$. Actual volatility is defined as a standard deviation[22] of the daily exchange rate returns over a given period of time.

[22]Sometimes volatility is measured as a sample standard deviation, e.g.

Specifically, the symbol $\sigma^A_{t,k}$ denotes the volatility between dates t and $t+k$:

$$\sigma^A_{t,k} = \sqrt{\frac{1}{k-1}\sum_{i=t}^{t+k-1}\left(r_i - \bar{r}_{t,k}\right)^2}, \tag{5}$$

where $r_t = \log\left(S_{t+1}\right) - \log\left(S_t\right)$ represent daily exchange rate returns and $\bar{r}_{t,k} = \frac{1}{k}\sum_{i=t}^{t+k-1} r_i$ their appropriate average. To denote historical volatility, we use the shortcut $\sigma^A_{t,-k} \equiv \sigma^A_{t-k,k}$.

The rationale for regressions, which use the Black-Scholes implied volatility as an explanatory variable even when it is not believed that assumptions of the Black-Scholes model hold, was given by Heynen, Kemna and Vorst [13]. They showed that the Black-Scholes implied volatility is an accurate forecast of average expected volatility over the remaining life of the option for various GARCH, EGARCH or mean reverting stochastic processes governing the development of the underlying security.

Implied volatility was found to be a good predictor for actual future volatility for stock indices and main currencies. Although, Canina and Figlewski [5] estimated equations (3) and (4) for S&P 100 index options[23] over the period 1983-1987 and claimed that implied volatility was a poor forecast of subsequent realised volatility and that implied volatility had less predictive power than the historical one, other works qualified their results. In examining S&P 100 index options for the longer time span of 1983-1995, Christensen and Prabhala [8] observed that implied volatility predicted future realised volatility and that it also outperformed past volatility. Moreover, they found a structural shift that happened around the market crash in 1987, after which implied volatility seemed to have more predicting power. This partially explains the inconsistency of their results with Canina and Figlewski's results. As another source of difference, Christensen and Prabhala cite their different and safer sampling procedure as they use non-overlapping monthly sampling, while Canina and Figlewski used daily data, corrected for their serial correlation.

Christensen and Prabhala [8]; other authors use the average of squared returns without adjustment for the mean [14]. However, differences between these definitions should not lead to distinct qualitative results.

[23]Chicago Board Options Exchange OEX options.

Table 3

Predictive power of implied and historical volatility for main currencies (Jorion [14])

	Information content coeficients for				Predictive power coeficients for			
	const.	implied vol.	historical vol.	R^2	const.	implied vol.	historical vol.	R^2
DM	-0.080	0.852^*		0.052	0.323^*	$0.547^{*\dagger}$		0.156
	-0.095	0.785^*	0.085	0.052	0.401^*	0.622^*	-0.173	0.160
JY	-0.006	0.783^*		0.036	0.327^*	$0.496^{*\dagger}$		0.097
	-0.076	0.668^*	0.210	0.037	0.042	0.421^*	0.474	0.105
SF	-0.041	0.854^*		0.041	0.392^*	$0.520^{*\dagger}$		0.145
	-0.040^*	0.858^*	-0.005	0.041	0.526^*	0.609^*	-0.240	0.149

* Significantly different from zero at the 5 percent level
† Significantly different from unity at the 5 percent level

Jorion [14] analysed the forecasting ability of options on currency futures traded at the Chicago Mercantile Exchange for the German mark, Japanese yen and Swiss franc. He distinguished between the informational content and the predictive power of volatility implied by option prices. While the predictive power measures the ability to forecast average volatility over the period before the contract's maturity, the informational content is less demanding and measures how good the implied volatility is in forecasting one-day actual volatility. In our notation, investigating the predictive power and informational content would mean to estimate equations (3) and (4) for $k = m$ and $k = 1$, respectively. As Jorion's results presented in the Table 3 show, in both cases and for all three currencies, options contain a significant amount of information about future volatility and that they have more forecasting power than time series models.

4.1 The sampling procedure and estimation results

We investigated whether Jorion's results hold also for the exchange rate of the Czech koruna. Concerning sampling methodology, we took into account Christensen and Prabhala's [8] warning suggesting that an improper econometric approach towards the overlapping data might lead to biased estimates.

To diminish the possible problems stemming from possibly correlated data in overlapping samples, we constructed non-overlapping samples and estimated Equations 3 and 4 on these restricted data sets. Specifically, for a given number of trading days k over which actual future volatility is calculated, we used only a subset of observations that would allow calculation of the actual volatilities from disjoint sets of spot rates. Formally, if one data point for Equation 3 consists of $\sigma^A_{j,k}, \sigma^I_{j,m}$ then the next one in a subsample consists of $\sigma^A_{j+k,k}, \sigma^I_{j+k,m}$. Similarly for regression 4, if one data point consists of $\sigma^A_{j,k}, \sigma^I_{j,m}, \sigma^A_{j,-k}$ then the next one in our sample consists of $\sigma^A_{j+k,k}, \sigma^I_{j+k,m}, \sigma^A_{j+k,-k}$. It follows that for a given k, we may obtain k such non-overlapping subsamples.

We chose three values of $k \in \{5, 10, 20\}$, and using $l = k$, we ran ordinary least squares regressions for all $j \in \{1, 2, ..., k\}$. As we used one-month options $m \approx 1M$, estimating the equations for $k = 20$ was quite close to measuring predictive power, while for $k = 5$, we moved closer to Jorion's informational content. For the sake of brevity, we present estimation results in Tables 4 and 5 only for some js.

Table 4

Estimation results for the market efficiency equation

k:j	α_0	t-stat.	5% c.i.	α_i	t-stat	5% c.i.	R^2
5:1	0.014	1.40	(-0.006;0.033)	0.792	7.74	(0.59, 0.99)	0.21
5:2	0.002	0.24	(-0.015;0.019)	0.921	10.87	(0.75, 1.09)	0.35
5:3	0.001	0.14	(-0.016;0.018)	0.932	10.51	(0.76, 1.11)	0.34
5:4	0.003	0.32	(-0.016;0.022)	0.912	9.05	(0.71, 1.11)	0.27
5:5	0.005	0.50	(-0.014;0.024)	0.907	8.86	(0.71, 1.11)	0.26
10:1	0.024	1.85	(-0.002;0.050)	0.781	5.80	(0.51, 1.05)	0.24
10:3	0.024	1.86	(-0.002;0.050)	0.776	5.84	(0.51, 1.04)	0.24
10:5	0.017	1.29	(-0.009;0.044)	0.862	6.25	(0.59, 1.14)	0.27
10:7	0.006	0.63	(-0.014;0.026)	0.957	9.65	(0.76, 1.15)	0.47
10:9	0.004	0.32	(-0.019;0.026)	1.010	8.65	(0.78, 1.24)	0.41
20:1	0.030	1.88	(-0.002;0.062)	0.786	4.85	(0.46, 1.11)	0.31
20:5	0.028	1.57	(-0.008;0.065)	0.813	4.28	(0.43, 1.19)	0.26
20:9	0.021	1.20	(-0.014;0.056)	0.902	4.85	(0.53, 1.27)	0.31
20:13	0.035	1.98	(-0.000;0.071)	0.706	3.99	(0.35, 1.06)	0.23
20:17	0.026	2.10	(0.001;0.050)	0.797	6.92	(0.57, 1.03)	0.48

In general, our results confirm Jorion's findings also for the Czech koruna, an example of an emerging market currency. The results in Table 4 show that Czech koruna options carry a significant amount of information and that they predict well the character of the currency movements. In all cases, the estimates of the coefficient α_i are significantly different from zero and virtually in all cases the estimates are not significantly different from unity. It is not surprising that estimated coefficients on implied volatility tend to be higher for a shorter forecasting horizon k, because option dealers are natuarlly less uncertain in short-term volatility predictions, while for longer term predictions extrapolations sometimes might be used.

Table 5

Estimation results for the encompassing equation

k:j	α_0	t-stat.	5% c.i.	α_i	t-stat	5% c.i.	α_h	t-stat	5% c.i.	R^2
5:1	0.013	1.33	(-0.006;0.033)	0.709	5.85	(0.47, 0.95)	0.094	1.33	(-0.05, 0.23)	0.22
5:2	0.001	0.10	(-0.016;0.018)	0.883	9.05	(0.69, 1.08)	0.052	0.84	(-0.07, 0.18)	0.35
5:3	0.000	0.01	(-0.017;0.017)	0.861	8.32	(0.66, 1.07)	0.088	1.36	(-0.04, 0.21)	0.34
5:4	0.002	0.23	(-0.017;0.022)	0.817	7.04	(0.59, 1.05)	0.110	1.66	(-0.02, 0.24)	0.28
5:5	0.004	0.43	(-0.015;0.024)	0.808	6.75	(0.57, 1.04)	0.110	1.62	(-0.02, 0.24)	0.27
10:1	0.023	1.80	(-0.002;0.049)	0.737	4.08	(0.38, 1.09)	0.047	0.42	(-0.18, 0.27)	0.24
10:3	0.023	1.79	(-0.002;0.049)	0.665	3.70	(0.31, 1.02)	0.112	0.99	(-0.11, 0.34)	0.25
10:5	0.017	1.27	(-0.010;0.044)	0.801	4.38	(0.44, 1.16)	0.059	0.54	(-0.16, 0.28)	0.27
10:7	0.007	0.72	(-0.013;0.028)	1.015	8.55	(0.78, 1.25)	-0.071	-0.84	(-0.24, 0.10)	0.47
10:9	0.004	0.32	(-0.019;0.027)	1.064	7.43	(0.78, 1.35)	-0.055	-0.60	(-0.23, 0.13)	0.42
20:1	0.030	1.90	(-0.002;0.062)	0.555	2.01	(0.00, 1.11)	0.203	1.04	(-0.19, 0.60)	0.33
20:5	0.030	1.60	(-0.007;0.066)	0.699	2.12	(0.04, 1.36)	0.088	0.42	(-0.33, 0.51)	0.26
20:9	0.021	1.18	(-0.015;0.057)	0.876	2.98	(0.29, 1.47)	0.022	0.12	(-0.34, 0.39)	0.31
20:13	0.033	1.85	(-0.003;0.069)	0.616	2.44	(0.11, 1.12)	0.097	0.56	(-0.25, 0.44)	0.24
20:17	0.023	1.66	(-0.005;0.050)	0.782	5.92	(0.52, 1.05)	0.037	0.32	(-0.19, 0.27)	0.49

More importantly, when implied volatility is pitted against historical volatility, estimates show that implied volatility is a better instrument for forecasting actual volatility than past volatility. The estimates of the coefficient α_i are only marginally lower than for the previous regression, which contrasts with estimates of the historical volatility coefficient which are not significantly different from zero.

5 Appendix

5.1 OTC market conventions

This section details OTC market conventions for quoting currency options (see also Malz [15]).

The price of a European currency option is determined by market forces and is mainly affected by the spot price of the underlying security S, the strike price X, the option's maturity T, the domestic and foreign interest rates r and r^*, and by the level of future uncertainty. To model the uncertainty, the Black-Scholes model introduces a volatility parameter σ. Although this model does not reflect reality completely, it is used for its very convenient properties as a tool for quoting prices. Thus, market participants in the OTC market quote prices in volatility terms. Deviations from the benchmark model are reflected in the fact that the quoted volatility is not constant across strike prices and maturities. Furthermore, to concentrate on the most option-like parts of the option price and abstract from erratic changes in the currency spot rate, the OTC market developed a convention of measuring the moneyness of options by the option's delta rather than by the variables S and X. And finally, prices of combinations like risk reversals and strangles are quoted, rather than prices of plain vanilla calls or puts.

5.1.1 Transformation from option-strike space to volatility-delta space

Let $c \equiv c\,(X; S, T, r, r^*)$ and $p \equiv p\,(X; S, T, r, r^*)$ denote the prices of European call and put options with a strike price X, respectively, and let other parameters in the brackets be defined as usual. To describe the transformation from price-strike space to volatility-delta space, it is convenient to define a European call implied volatility function, $\sigma^{impl}(c, X; S, T, r, r^*)$. The function $\sigma^{impl} \equiv \sigma^{impl}(c, X; ...)$ is implicitly defined as a solution to the Black-Scholes formula:

$$c = Se^{-r^*T} N\left(d_1\left(\sigma^{impl}\right)\right) - Xe^{-rT} N\left(d_2\left(\sigma^{impl}\right)\right), \text{ where} \tag{6}$$

$$d_1\left(\sigma^{impl}\right) = \frac{\ln\left(\frac{S}{X}\right) + \left[r - r^* + \frac{1}{2}\sigma^{impl2}\right]T}{\sigma^{impl}\sqrt{T}} \text{ and} \tag{7}$$

$$d_2\left(\sigma^{impl}\right) = d_1\left(\sigma^{impl}\right) - \sigma^{impl}\sqrt{T}. \tag{8}$$

Given S, T, r and r^*, the function σ^{impl} transforms the market price of a call option with a strike price X to the corresponding implied volatility. Formally, let us write the mapping from price-strike space (c, X) to volatility-delta space (σ, δ) as

$$\sigma = \sigma^{impl}(c, X; S, T, r, r^*) \tag{9}$$

$$\delta = \Delta_c\,(X, \sigma; S, T, r, r^*) \equiv e^{-r^*T} N\left(d_1\left(X\right)\right), \text{ where} \tag{10}$$

$$d_1\left(X\right) = \frac{\ln\left(\frac{S}{X}\right) + \left[r - r^* + \frac{1}{2}\sigma^2\right]T}{\sigma\sqrt{T}} \tag{11}$$

The function $\Delta_c\,(X, \sigma; S, T, r, r^*)$ defined in equation(10) is the well-known formula for the Black-Scholes delta of a European call.

A similar mapping is defined for put options. Note that the delta of a European put can be written in terms of the delta for a call $\Delta_p\,(X, \sigma; S, T, r, r^*) = \Delta_c\,(X, \sigma; S, T, r, r^*) - e^{-r^*T}$. Also, due to put-call parity, a put and a call with the same strike price X imply the same volatility. The mapping from (p, X) to (σ, δ) is then given by

$$\sigma = \sigma^{impl}(p + Se^{-r^*T} - Xe^{-rT}, X; S, T, r, r^*) \tag{12}$$

$$\delta = \Delta_p\,(X, \sigma; S, T, r, r^*) + e^{-r^*T}. \tag{13}$$

Thus both transformations from (c, X) to (σ, δ) and from (p, X) to (σ, δ) map the interval $(0, \infty) \times (0, \infty)$ to the interval $(0, \infty) \times \left(0, e^{-r^{*}T}\right)$.

5.1.2 Transformation from volatility-delta space to option-strike space

To describe the inverse transformation from volatility-delta space to call-strike space, it is convenient to define the function $X^{impl} \equiv X^{impl}(\sigma, \delta_c; S, T, r, r^{*})$, which generates the appropriate (implied) strike price of a call option from the values of σ and δ_c. It is defined as a solution of

$$\delta_c = \Delta_c\left(X^{impl}, \sigma; S, T, r, r^{*}\right). \tag{14}$$

Thus, the mapping from volatility-delta space to price-strike space, $(\sigma, \delta_c) \to (c, X)$, is given by

$$c = Se^{-r^{*}T} N\left(d_1\left(\sigma\right)\right) - Xe^{-rT} N\left(d_2\left(\sigma\right)\right) \tag{15}$$

$$X = X^{impl}(\sigma, \delta_c; S, T, r, r^{*}) \tag{16}$$

While the Newton method is very suitable for the numerical implementation of the function σ^{impl}, one has to be more cautious when numerically calculating the function X^{impl}. Note that the second derivative $\frac{\partial^2}{\partial x^2}\Delta_c\left(x, \sigma; S, T, r, r^{*}\right)$ changes sign once for $x \in (0, \infty)$ with inflexion point $X^{*} = Se^{\left[r - r^{*} - \frac{1}{2}\sigma^2\right]T}$. However, it is still possible to use this numerical approach. The function $\Delta_c\left(X; ...\right)$ is concave for $X \leq X^{*}$ and convex for $X \geq X^{*}$ Using this property, the Newton method might be amended. Then it is likely to be faster than a more general numerical method.

The following market conventions are used. A call option with delta δ_c is referred to as $100\delta_c$, i.e. a 25-delta call is a call option with $\delta_c = 0.25$. Similarly, a put option with delta δ_p is referred to as $-100\delta_p$, i.e. a 25-delta put is a put option with $\delta_p = -0.25$. Moreover, since for short maturity options the term $e^{-r^{*}T}$, which facilitates transformation between the delta of a put and the delta of a call, is close to one, a call counterpart to the 25-delta put is often referred to as a 75-delta call instead of a $100\left(e^{-r^{*}T} - 0.25\right)$delta-call. Another abbreviation is used for an at-the-money-forward (ATMF) call. Denote the market quotes of ATMF volatility as $(\sigma_{atmf}, \delta_{atmf})$. Sometimes, people refer to it as a 50-delta call, i.e.$(\sigma_{atmf}, 0.5)$. In fact, it is easy to show that, given a market quote of volatility σ_{atmf}, the delta of a call option with $X = F = Se^{(r-r^*)T}$ is $\delta_{atmf} = \Delta_c(F, \sigma_{atmf}; S, T, r, r^*) = e^{-r^*T} N\left(\frac{1}{2}\sigma_{atmf}\sqrt{T}\right)$. For example $\Delta_c(F, \sigma_{atmf}; S, 0.5.6\%, 4\%)$ =0.52.

Actually, the market convention of mapping the (σ, δ_c) space to the (c, X) space seems to be somewhat odd. Note that for a given call delta (e.g. $\delta_c = 0.25$), a change in quoted volatility also represents a change in the implied strike, as the function $X^{impl}(\sigma, \delta_c; S, T, r, r^*)$ indicates. Therefore, it is theoretically possible to have two (or many) volatility-delta pairs that are transformed to one strike only. However, it apparently does not pose a problem for the market. The reason is that prices are not quoted close to each other and also that 'perverse' quotes would break no-arbitrage conditions for the volatility function[24].

5.1.3 Usually quoted contracts

In the over-the-counter (OTC) market, currency options are usually quoted in terms of at-the-money forward (ATMF) implied volatilities, risk reversals and strangles. Both risk reversals and strangles are combinations of call and put options, which are equally out-of-the-money (OTM). Their moneyness is measured by their delta. While a buyer of a risk reversal acquires a long position in an OTM call option and a short position in an OTM put option, a buyer of a strangle buys both of them, i.e. gets long positions in OTM calls and puts. The price of a risk reversal is quoted in volatility terms as a difference between implied volatilities of an appropriate call and put. Similarly, strangle prices are quoted as an average volatility premium paid for the strangle components above the ATMF implied volatility. Let σ_{atmf}, $rr(.)$ and $str(.)$ denote market quotes of ATMF implied volatility, risk reversal and strangle. The parameter in brackets refers to the delta of the call of the constituents.

[24] We thank Allan Malz for clarifying this point.

Malz [15] shows how to back out pairs (σ, δ_c) from these market quotes. For convenience, denote these pairs as a function$\sigma(\delta_c)$. Then the following relationships hold:

$$rr(\delta_c) = \sigma(\delta_c) - \sigma\left(e^{-r^*T} - \delta_c\right) \tag{17}$$

$$str(\delta_c) = \frac{1}{2}\left[\sigma(\delta_c) + \sigma\left(e^{-r^*T} - \delta_c\right)\right] - \sigma_{atmf}. \tag{18}$$

So, for example a 25-delta risk reversal $rr(0.25)$ is the difference between the implied volatility of a call option with a delta of 0.25 and the implied volatility of a put option with a delta of -0.25. A call option that has the same volatility as the 25-delta put is the one with a delta of $(e^{-r^*T} - 0.25)$. Note that sometimes the strangle is quoted without subtraction of σ_{atmf} in equation (18).

Equations (17) and (18) are easy to invert, and one obtains

$$\sigma(\delta_c) = rr(\delta_c) + \frac{1}{2}rr(\delta_c) + \sigma_{atmf} \tag{19}$$

$$\sigma\left(e^{-r^*T} - \delta_c\right) = str(\delta_c) + \frac{1}{2}rr(\delta_c) + \sigma_{atmf} \tag{20}$$

Thus, equations (19) and (20) show how to get from a market quote of σ_{atmf}, $rr(\delta_c)$ and $str(\delta_c)$ to $\sigma(\delta_c)$ and $\sigma\left(e^{-r^*T} - \delta_c\right)$.

5.2 Malz method for estimating risk neutral distributions

Malz [15] suggested using interpolation of observed implied volatilities of call options, i.e. $\sigma(\delta_{0.25})$, σ_{atmf} and $\sigma\left(e^{-r^*T} - \delta_{0.25}\right)$ derived via Formulas (19) and (20) by a quadratic function $\tilde{\sigma} = \tilde{\sigma}(\delta_c) = a\delta_c^2 + b\delta_c + c$ Therefore, it amounts to calculating of such a, b and c that $\tilde{\sigma}(\delta_c) = \sigma(\delta_c)$ for observed δ_c. Then the function $\tilde{\sigma}(\delta_c)$ is transformed into the call price - strike price space using formulas (15) and (16). Finally, the formula (2) can be used to obtain risk neutral density.

References

[1] Bahra, B., 1997. Implied Risk Neutral Probability Density Functions from Option Prices: Theory and Application. Working Paper, Bank of England, ISSN 1368-5562.

[2] Breeden, D. T., Litzenberger, R. H., 1978. Price of State Contingent Claims Implicit in Option Prices. Journal of Business 51 (1978), 621-651.

[3] Central Bank Survey of Foreign Exchange and Derivatives Market Activity 1998. Bank of International Settlements, Basle, 1999.

[4] Czech National Bank Survey of Foreign Exchange Market Turnover. www.cnb.cz

[5] Canina, L., Figlewski, S., 1993. The Informational Content of Implied Volatility. Review of Financial Studies 6 (3), pp. 659-681

[6] Campa, J. M., Chang, P.H.K., 1998. Target Zone Credibility When Pegging to a Basket: The Czech Koruna vs. the Dollar and the Mark. Unpublished paper.

[7] Chang, P.H.K., Melick, W.R., 1999. Background note. Proceedings of the BIS workshop on implied PDFs, Bank of International Settlements, Basle.

[8] Christensen, B.J., Prabhala, N.R., 1998. The Relation between Implied and Realized Volatility. Journal of Financial Economics 50, pp. 125-150.

[9] Clews, R., Panigirtzoglou, N., Proudman, J., 2000. Recent Developments in Extracting Information from Options Markets. Bank of England Quarterly Bulletin 40(1) February 2000, 50-57.

[10] Cincibuch, M., Vávra, D., 2000. Na cestě k EMU: Potřebujeme kursovou flexibilitu? Finance a úvěr 6/2000.

[11] Cox, J., Ross, S., 1976. The Valuation for Alternative Stochastic Processes. Journal of Financial Economics 3, pp. 145-66.

[12] Gemmill, G., 1996. Did Option Traders Anticipate the Crash ? Evidence from Volatility Smiles in the U.K. with U.S. Comparisons. Journal of Futures Markets 16 (8), pp. 881-897.

[13] Heynen, R., Kemna, A, Vorst, T., 1994. Analysis of the Term Structure of Implied Volatilities. Journal of Financial and Quantitative Analysis, 29 (1), pp. 31-56.

[14] Jorion, P., 1995. Predicting Volatility in the Foreign Exchange Market. Journal of Finance 50 (2), pp. 507-528.

[15] Malz, A.M., 1997. Estimating the Probability Distribution of the Future Exchange Rate from Option Prices. The Journal of Derivatives, Winter 1997, pp. 18-36.

[16] Ross, S., 1976. Options and Efficiency. Quarterly Journal of Economics 90, pp. 75-89.

[17] Rubinstein, M., 1994. Implied binomial trees. Journal of Finance, 49

[18] Soederlind, P., Svensson, L., 1997. New Techniques to Extract Market Expectations from Financial Instruments. Journal of Monetary Economics 40. pp. 383-429.

[19] Šmídková, K., (et.al),. 1998. Koruna Exchange Rate Turbulence in May 1997. ČNB working paper #2