Evaluating a Structural Model Forecast:

Decomposition Approach

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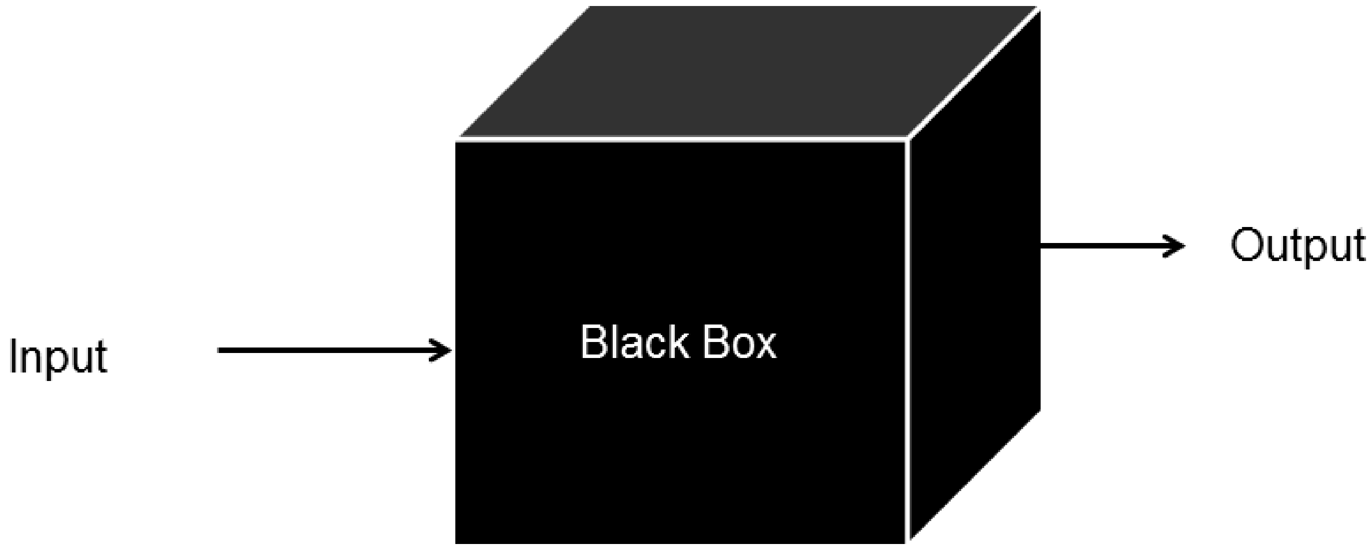
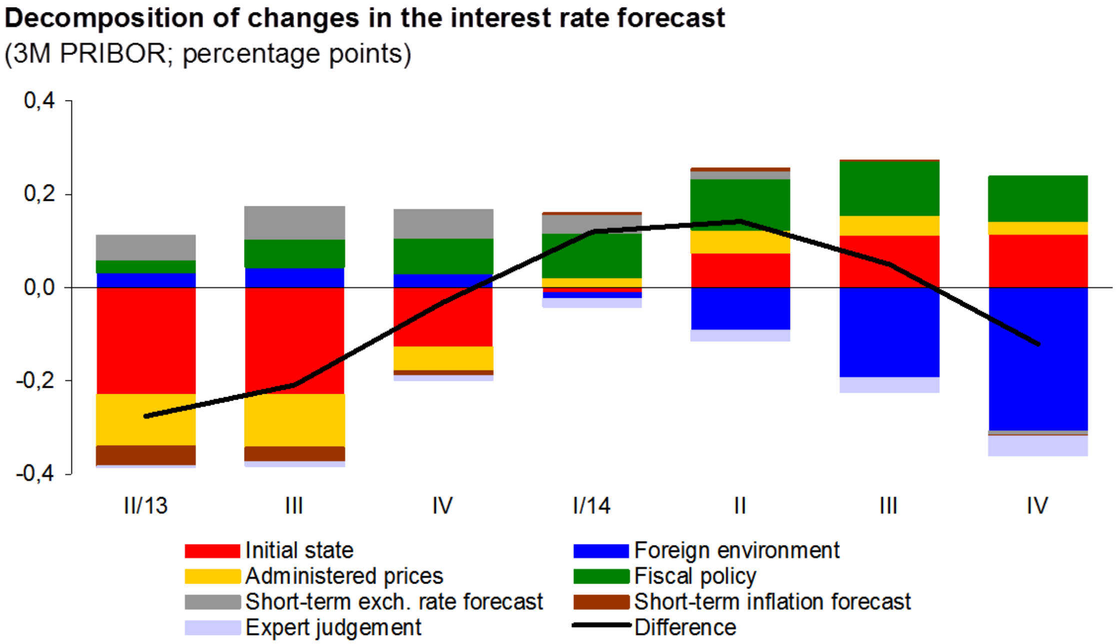
This is how many people see forecasting at central bank:



Is Forecast a Black Box?

- Our goal is transparent presentation of forecast updates
- Forecast presentation is supported by graphs like this:



- Inflation Report:
  - Analysis of two subsequent forecasts: Support for policy making decisions
  - Scenario comparison
- Forecast evaluation: Analysis of forecast–data difference

History of Decomposition Development

- Prehistoric state:
  - QPM model
  - Decompositions were created by inserting information additively
  - Problem: The size of effects conditional on ordering of information inclusion
- Recent history state:
  - g3 model
  - More advanced decompositions still based on inserting information additively
  - Fragmented methodologies: Each decomposition was based on special set of assumptions
  - Incompatibility: Specialized tools for different types of decomposition
- Present state:
  - Ability to compute the size of effects by use of model's elasticities - based on the impulse response function
  - Improvement: General framework for decomposition
  - Single engine for computation of decompositions
  - Types of decompositions are defined by grouping of effects

General Forecasting Model

General form of the model for forecasting:

- Measurement equation (linking data and model):

$$Y_t = \mathbf{C}X_t + \mathbf{D}\xi_t,$$

- System equation (model of economy):

$$X_t = \mathbf{A}X_{t-1} + \mathbf{B}\varepsilon_t,$$

where

- X_t ... vector of state/transition variables
- Y_t ... vector of observable/measurement variables
- ε_t, ξ_t ... structural shocks/measurement errors and $E(\varepsilon_t \varepsilon_t') = Q$, $E(\xi_t \xi_t') = R$,

1. Position in cycle?: Initial state identification
   - Ingredients: Data, model and expert judgement
   - State space representation of augmented model:

$$\begin{bmatrix} Y_t \\ Y^J_{t|T} \end{bmatrix} = \begin{bmatrix} \mathbf{C}X_t \\ \mathbf{\Gamma}_{t|T}X_t \end{bmatrix} + \begin{bmatrix} \mathbf{D}\xi_t \\ \mathbf{\Delta}_{t|T}\nu_t \end{bmatrix}$$
$$X_t = \mathbf{A}X_{t-1} + \mathbf{B}\varepsilon_t.$$

   - Tuning the data, state variables and shocks
   - Identification is based on Kalman smoother application

2. Where are we heading?: Forecast creation

- Ingredients: Initial state, model, outlooks (variables values) and expert judgement (shocks)
- Deciding on nature of a shock: anticipated or unanticipated - model agents' view
- State space representation of augmented model:

$$
\begin{aligned}
Y_t &= \mathbf{C}X_t + \mathbf{D}\xi_t \\
X_t &= \mathbf{A}X_{t-1} + \mathbf{B}\varepsilon_t + \mathbf{B}\overline{\varepsilon}_t \\
\mathbf{Z}_{t|T}X_t &= R_{t|T} + \mathbf{\Lambda}_t\mu_t \\
\overline{\mathbf{Z}}_{t|T}X_t &= \overline{R}_{t|T} + \overline{\mathbf{\Lambda}}_{t|T}\overline{\mu}_{t|T},
\end{aligned}
$$

- Equivalence between insertion of shocks and trajectories
- Forecasting problem solution: If system is underdetermined, to discriminate among the outcomes, likelihood maximization is used to find an unique solution

Forecasting Framework Features

- Expert judgment inclusion:
  - One-to-One relation: One variable–One structural shock on the forecast trajectories
  - No such relation over the identification phase
- Shock anticipation mode selection:
  - Anticipated shocks: Judgment will immediately affect the forecast from its beginning
  - Unanticipated shocks: Effect will occur from its effective period

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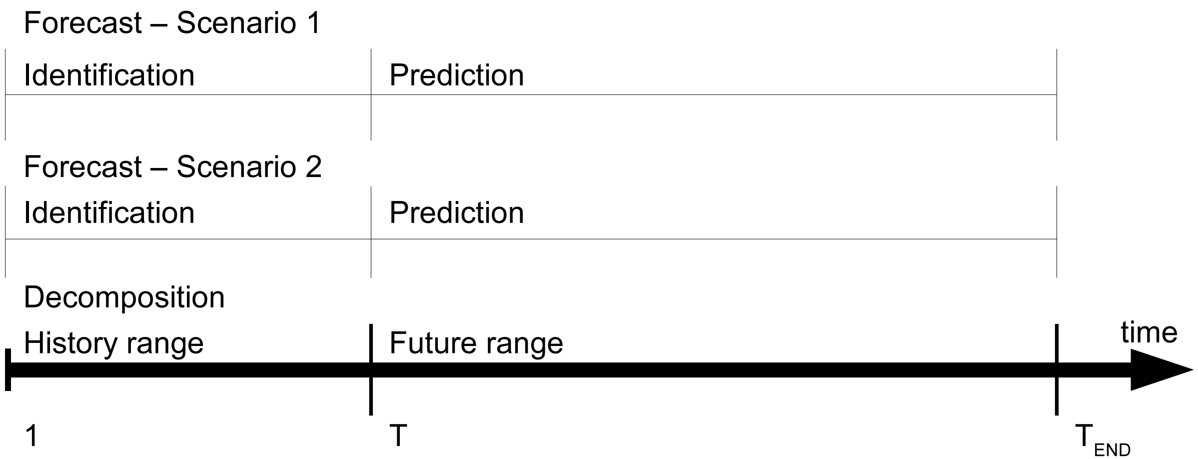
We need numbers to fill our tables and plot graphs:

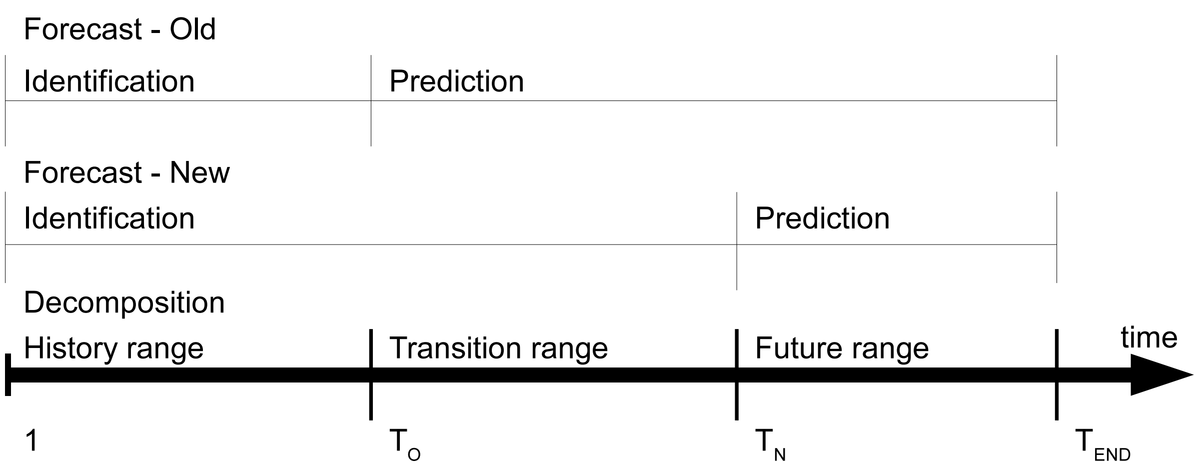
- Forecasting:
  - Advanced forecast function that allows for conditioning
  - Solution method: Forward expansion of model solution
  - Treating anticipated and unanticipated shocks as "layers" of information
  - Simulation engine: Kalman filter - The best linear estimator
- Properties:
  - Forecast can be re-simulated with shocks only
  - Allows to recover path based on the anticipated judgment only
  - Separates the unanticipated judgement, so it can be applied as additional layer

Timing in Decomposition Analysis

- Scenario comparison: Identification and Prediction ranges are the same



- Forecast evaluation, Forecast update analysis: Time shift in the starting point of the prediction phase is present



Elements of Decomposition

General principles of decomposition methodology:

- Each forecast is associated with information set
- Decomposition moves from New to Old forecast
- Remove information in layers: Information sets
- Analyze effects by creation of supporting forecasts X^S :

$$X^N - X^O = (X^N - X^S) + (X^S - X^O),$$

- Time shift needs to be handled
- Decomposition exploits equivalence between filtering and forecasting is used when no judgement/conditioning is present

Peeling Onion I

Supporting decompositions:

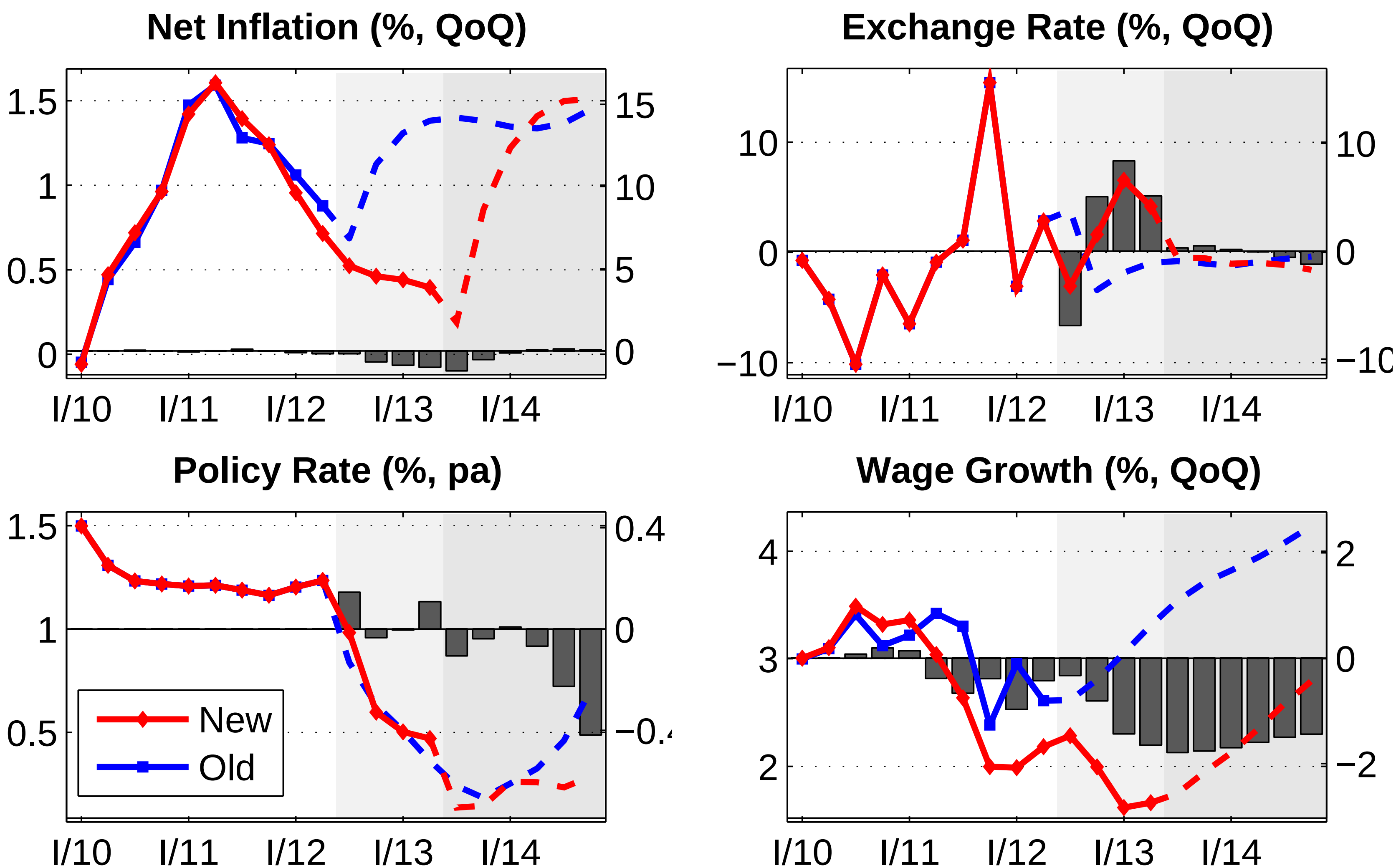
1. New forecast − judgment on future = New forecast without expert judgments from prediction phase
2. New forecast without expert judgments on future − new judgment on transition range = New forecast without expert judgments on future and transition range
3. New forecast without expert judgments on future and transition range − new judgment on history range = New forecast without expert judgments on future and history
4. New forecast without expert judgments on future and history − newly released data = New forecast without expert judgments on future and history without newly released data

Peeling Onion II

5. New forecast without expert judgments on future and history without newly released data is equivalent to Old forecast without expert judgments on future and history
6. Old forecast without expert judgments on future and history + old judgment on history = Old forecast without expert judgments on future
7. Old forecast without expert judgments future range + old judgment on future = Old forecast

Forecast Evaluation: Forecast–Data

- Comparing trajectories of the New and Old Forecast



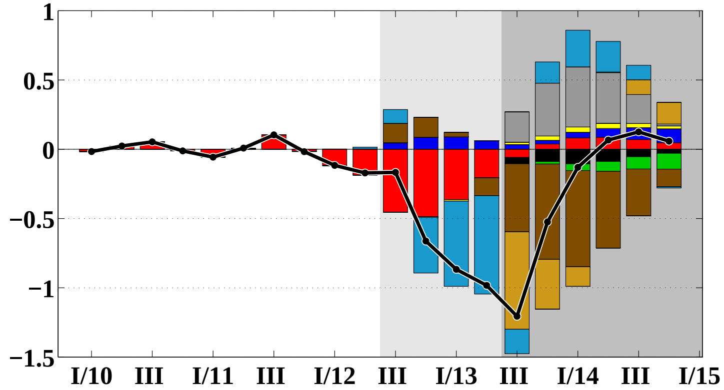
Forecast Evaluation: Two Views

- Running the decomposition identifies contributions of shocks
- This somehow resembles PUEs
- After the decomposition is computed, effects have to be grouped
- Two views: Variables vs Shocks

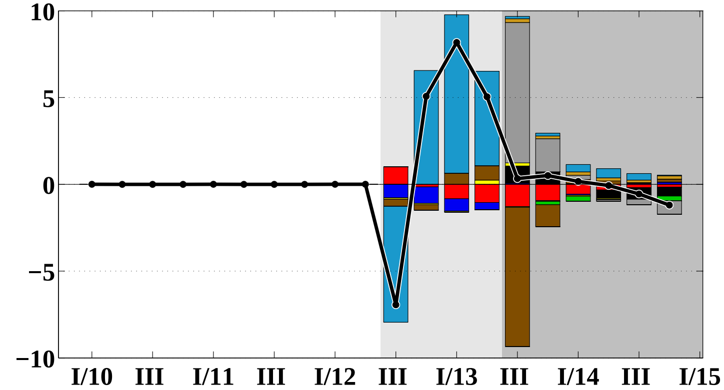
Forecast Evaluation: Variables View

- Forecast evaluation: Contributions of the information set updates

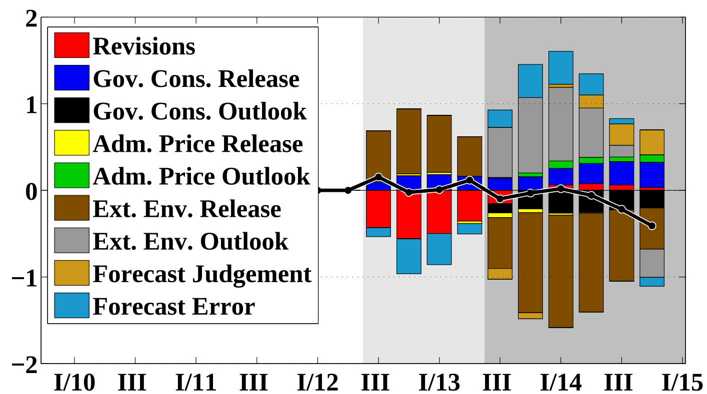
Net Inflation (%, QoQ)



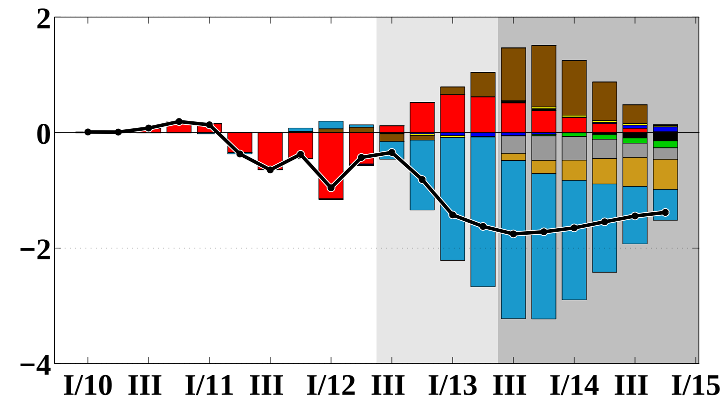
Exchange Rate (%,QoQ)



Policy Rate (%, pa)



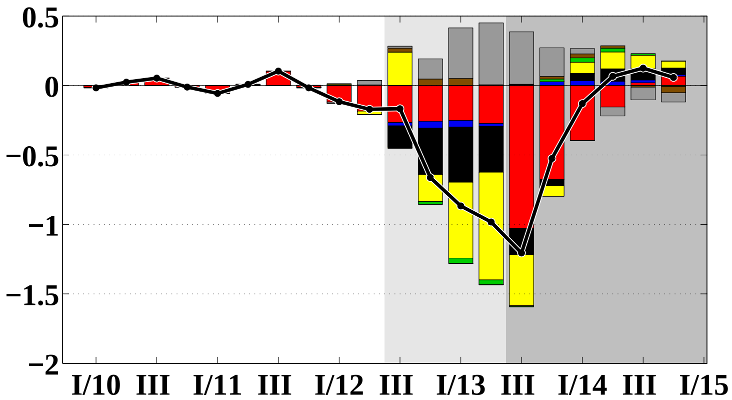
Wage Growth (%, QoQ)



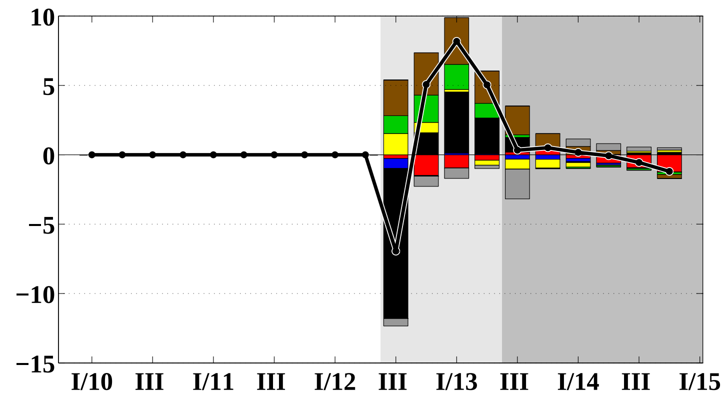
Forecast Evaluation: Shock View

- Missing structural shocks: Identification of shocks contributing to the difference between the prediction with updated information and the "true" data

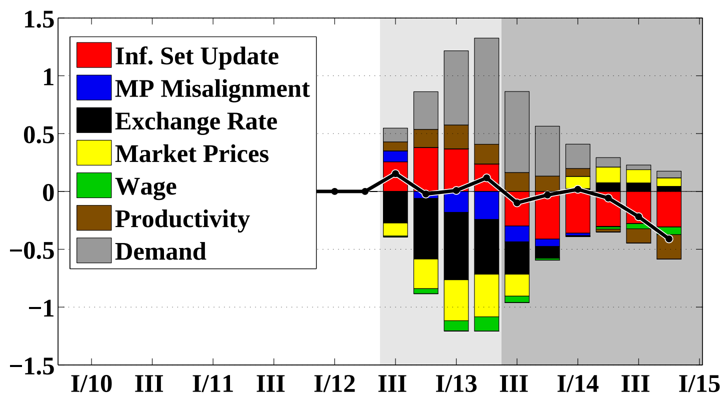
Net Inflation (%, QoQ)



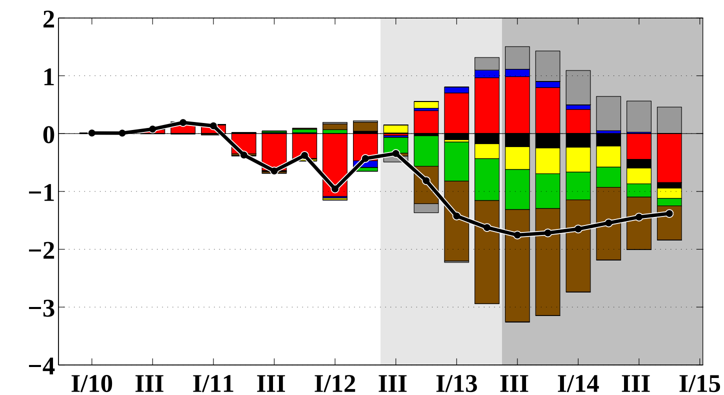
Exchange Rate (%,QoQ)



Policy Rate (%, pa)



Wage Growth (%, QoQ)



Where Did We Get?

- What we can learn from forecast update analysis?
  - Structural model in forecasting: Driving forces of trajectories
  - Learning about model and data properties
  - Hints on data revisions properties
  - Learning about experts' errors
- Drawbacks of decomposition tools were removed:
  - One purpose tools
  - Incompatible methodologies and different assumptions on update decompositions - complicates presentation of results
  - Tools face restrictions in terms of too strong underlying assumptions and limited flexibility
- How did we got here?
  - Generalize framework for decomposition

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Structural Model

- g3 model: [Andrle et al., 2009]
- Model used in regular forecasting exercises
- Linear model in state space:
  - Measurement block: Linking observed data and state variables, measurements shocks
  - Transition block: Structural model of the small open economy, structural shocks
- Model features:
  - Non-stationary observed variables, stationary unobserved variables, models of trends
  - Forward looking monetary policy, Administered price
  - Exogenous processes for foreign economy

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- Even more general formulation:

$$\mathbf{A}X_{t+1|t} + \mathbf{B}X_t + \mathbf{C}\varepsilon_t = 0$$

- Forward expansion:

$$X_t = \mathbf{D}X_{t-1} + \mathbf{R_0}\varepsilon_t + \sum_{k=1}^{\infty} \mathbf{R_k} E[\varepsilon_{t+k}|t]$$

- Some remarks:
  - X_0 calculated by running Kalman smoother
  - Distributions of the future shocks ε_t conditional on time t upon have zero means, and covariance matrices $\Omega_t = COV[\varepsilon_t|0]$

- After more iterations of

$$X_t = \mathbf{D}X_{t-1} + \mathbf{R_0}\varepsilon_t + \sum_{k=1}^{\infty} \mathbf{R_k} E[\varepsilon_{t+k}|t]$$

- Each value of X_t is linear function of
  - Initial condition
  - Realizations of past shocks
  - Complex structure of expectations of the future shocks:

$$\begin{aligned} E[\varepsilon_2,1], E[\varepsilon_3,1], E[\varepsilon_4,1], \ \ldots, \ & E[\varepsilon_\infty,1] \\ E[\varepsilon_3,2], E[\varepsilon_4,2], \ \ldots, \ & E[\varepsilon_\infty,2] \\ & \vdots \\ E[\varepsilon_t+1,t], \ \ldots, \ & E[\varepsilon_\infty,t] \end{aligned}$$

- Such solution is based on works by [Blanchard and Kahn, 1980], [Doan et al., 1983] and [Waggoner and Zha, 1999]

Even More Linearity

Three important observations about the solution:

- In a Gaussian model, the likelihood-maximising path for X_t subject to a linear constraint $Z_t|X_t = R_t$ will coincide with the mean of the distribution of X_t conditional upon that linear constraint $E[X_t|0, Z_t|X_t = R_t]$.
- Along the likelihood-maximising path, there will be no change in the conditional expectations:$E[\varepsilon_t|1] = E[\varepsilon_t|2] = \cdots = E[\varepsilon_t|k-1]$, and these expectations will equal the realisations of ε_t.
- Solution depends on the infinite sum, however expectations can be set to 0 after the forecast horizon (end of conditioning information)
- Re-formulate the problem of finding a judgmentally adjusted forecast as the likelihood maximisation, restrictions are augmented over the time dimension
- It gives solution for finding values of εs and by plugging in the generalized form, the system can be simulated