Econometrics with System priors

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Disclaimer

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The views expressed herein are those of the authors and should not be attributed to the International Monetary Fund, its Executive Board, or its management.

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The views expressed here are those of the authors and do not necessarily reflect the position of the Czech National Bank.

Aims and scope

- To provide more nuanced and more general introduction to system priors (devised by Andrle and Beneš within the DSGE context)
- To demonstrate the generality of principles and its wide range of application
- To illustrate the use of system priors with a simple but practically relevant example
- ... to invite fellow researchers to jump on the bandwagon

What are system priors?

- Economically-meaningful priors about high-level model properties
  - impulse-response functions
  - variance error decompositions
  - frequency-domain properties
  - policy scenarios (sacrifice ratios, delayed MP response)
  - ...anything that can be computed with the model (forecasts...)
- Two layer approach that facilitates formulation of priors on both the parameter and model level
- Complement rather than substitute for traditional Bayesian setup

Why and when one should use system priors?

- In many complex models individual parameters are difficult to interpret.
- Reasonable priors for individual parameters may lead in sum to highly erratic priors about the overall model behavior.
  - Even „non-informative“ priors can be implicitly very informative in a highly undesirable way
  - Prior predictive analysis – which parameter priors “bite”?
- Policy makers only hold firm views about economic behavior.
  - Communication channel between modelers and policy makers

First glance at system priors

- Traditional Bayesian setup

$$p(\theta | Y;M) \propto L(Y|\theta;M) \times p_m(\theta)$$

- System priors setup

$$p(\theta | Y;M) \propto L(Y|\theta;M) \times [\, p_s(h(\theta);M) \times p_m(\theta)]$$

  - $p_m(\theta)$ – priors on individual parameters
  - $p_s(h(\theta);M)$ – system priors „add-on“
  - $[p_s(h(\theta);M) \times p_m(\theta)]$ – composite prior enabling to implement views on elements in both layers

How to understand system priors I

- **(Non-conjugate) dummy observation prior**
  - Instead of inserting dummy observations into the dataset, create a dummy/artificial likelihood for the auxiliary model that summarizes the information in the dummy observations
- $[\, p_s(h(\theta);M) \times \; p_m(\theta)] \equiv likelihood \times prior\ on\ parameters$
  - Structure of the auxiliary model corresponds to the high-level property of interest: $h(\theta;M)$ + error term (set of stochastic restrictions)
- Posterior inference is obtained by updating priors on individual parameters twice:
  - first with artificial likelihood of the auxiliary model (system priors)
  - second with real likelihood based on observed data

How to understand system priors II

- **Penalized likelihood problem**
  - Taking logs of the RHS...

$$p(\theta\,|\,Y;M) \propto L(Y|\theta;M) \times [\,p_s(h(\theta);M) \times p_m(\theta)]$$

  - ... one obtains

$$\log(L(Y|\theta;M)) + \log\left(p_m(\theta)\right) + \log(p_s(h(\theta);M)$$

- Finding the mode of the posterior distribution is a traditional maximum likelihood approach with additional penalties that “regularize” the problem

- Penalty terms are nothing new in econometrics
  - ridge regression
  - lasso
  - many others...

Related literature I

- A desire for priors on model properties in not new, however most of the existing attempts only solve *ad hoc* problems
  - priors only solve specific a problem at hand (e.g. steady-state priors – Villani, 2005; priors on impulse responses – Dwyer, 1998, Kocięcki, 2012; long-run priors – Giannone et al., 2016; priors on frequencies – Planas et al., 2008)
  - priors only take specific form (usually gaussian priors)

- More general approaches
  - *Feature of interest priors*: Hollifield et al. (2003) – this approach is conceptually identical to system priors
  - *Priors on observables*: Jarociński and Marcet (2013)

Related literature II

Comparison of our approach with that of Jarociński and Marcet

- Both approaches can be used to solve similar problems, however they differ in concept (and flexibility & versatility).

- Both approaches need to solve the inverse problem:

- Jarociński and Marcet
  - Priors on high-level features -> Priors on observables -> Fredholm equation/fixed point solution -> implied priors on individual parameters -> bayesian update (likelihood) -> posterior distribution

- System priors
  - Priors on individual parameters -> bayesian update (artificial likelihood) -> bayesian update (likelihood) -> posterior distribution

Illustrative example

- Stationary AR(2) process with additional belief that most of its variance is generated by business-cycle frequencies
  - AR(2) is a very simple case, but the process can exhibit non-trivial dynamics
- We use the example only as an illustration, however it can be quite useful for empirical work
  - output gaps are frequently modelled as the AR(2) process: (see e.g. Watson, 1986, Clark, 1987, Kuttner, 1994, Planas et al., 2008, Jarociński and Lenza, 2016 and many others)
  - the same goes for inflation gaps (Clark and Doh, 2014)
  - ...or unemployment gaps (Chan et al., 2016)

Illustrative example

- Full (but still simple) model may look something like this...

$$
\begin{aligned}
(\pi_t - \tau_t^{\pi}) &= \rho_t^{\pi} \left(\pi_{t-1} - \tau_{t-1}^{\pi}\right) + \lambda_t \left(u_t - \tau_t^{u}\right) + \varepsilon_t^{\pi} \\
(u_t - \tau_t^{u}) &= \rho_1^{u} \left(u_{t-1} - \tau_{t-1}^{u}\right) + \rho_2^{u} \left(u_{t-2} - \tau_{t-2}^{u}\right) + \varepsilon_t^{u} \\
\tau_t^{\pi} &= \tau_{t-1}^{\pi} + \varepsilon_t^{\tau\pi} \\
\tau_t^{u} &= \tau_{t-1}^{u} + \varepsilon_t^{\tau u} \\
\rho_t^{\pi} &= \rho_{t-1}^{\pi} + \varepsilon_t^{\rho\pi} \\
\lambda_t &= \lambda_{t-1} + \varepsilon_t^{\lambda}
\end{aligned}
$$

Source: A Bounded Model of Time Variation in Trend Inflation, NAIRU and the Phillips Curve
Chan, Koop and Potter (2016)

Illustrative example

If the AR(2) is used to capture some "business-cycle gap" variable, what are the options to consider?

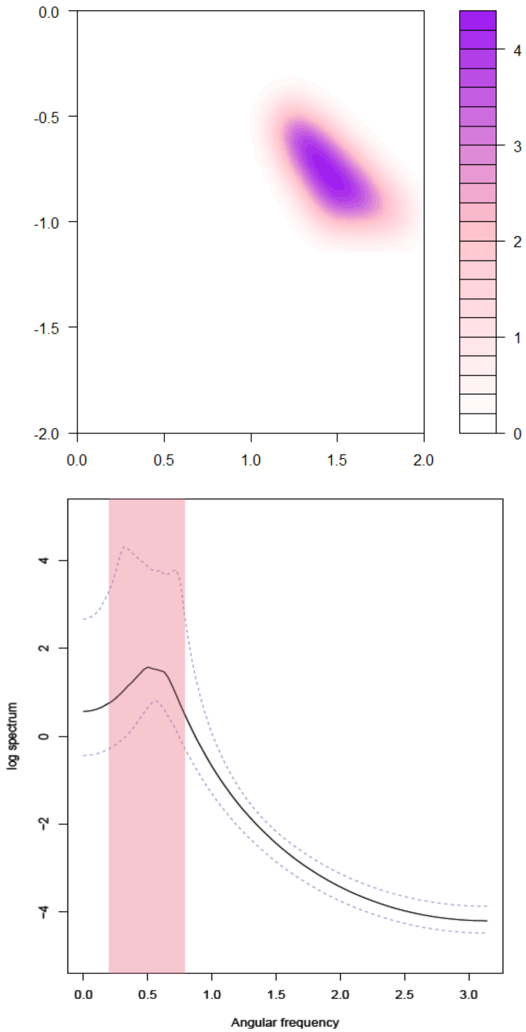
- basic Gaussian option: Chan and Grant(2017); earlier versions of Chan et al. (2015)
- model reparametrization: Planas et al. (2008)
- more refined Gaussian option: Chan et al. (2015); Grant and Chan (2017); Lenza and Jarociński (2016)
- System priors based on the business-to-total-variance ratio
  - at least 60% of variance comes from business cycle frequencies
  - the ratio follows some distribution [Be(15,5) is used in the paper]

$$ratio = \int_a^b S_y(w)\mathrm{d}w \Big/ \int S_y(w)\mathrm{d}w,$$

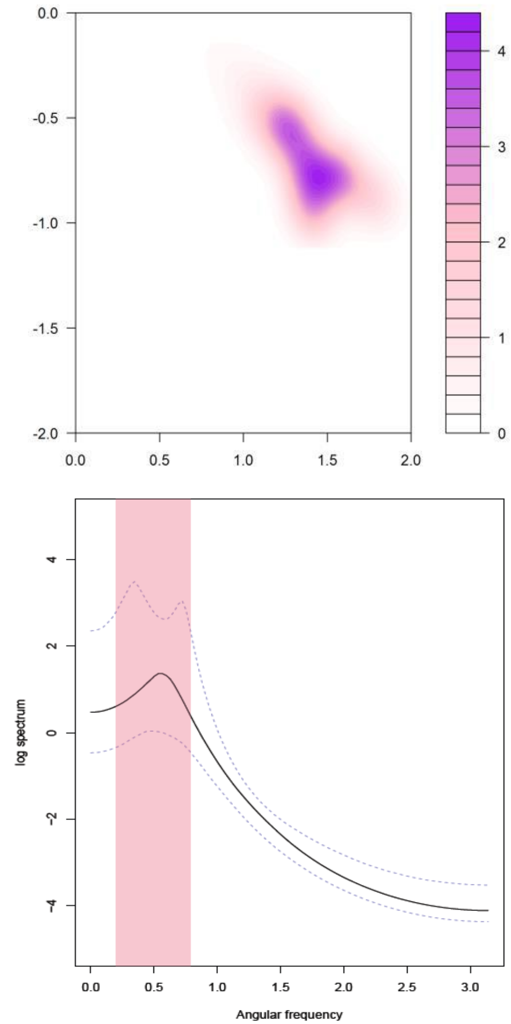
$S_y(w)$ – spectral density of the process

a,b – limits for business cycle frequencies

System priors



At least 60 %



Be(15,5)

Computational Aspects

- Integration of system priors into existing Bayesian toolkit is straightforward.
- General-purpose samplers can be used:
  - Standard Metropolis-Hastings algorithm (low-dimensional problems)
  - Sequential Monte Carlo samplers (Herbst and Schorfheide, 2014)
  - Dynamic striated Metropolis-Hastings algorithm (Waggoner, Wu and Zha, 2016)
  - Homotopy optimization methods (for finding a mode of posterior distribution)

Software implementation and current use of system priors

- SW
  - IRIS [Matlab, Octave]
  - Dynare [Matlab, Octave]
  - Yada [Matlab]
  - R Project – Dynamic Striated Metropolis-Hastings algorithm, SMC
- Applications
  - IMF
  - CNB, Financial Stability Dept. (a forecasting model for residential property prices – “Trend-Cycle VAR”)
  - ECB: NAWM – The New-Area Wide Model of the Euro Area

Conclusions

- System priors represent a flexible way of incorporating economically meaningful information.
- They are very general and can be easily implemented within existing Bayesian toolkit.
- The paper places emphasis on the elements and mechanics of system priors' application.
- Implementation of system priors was illustrated using second-order autoregressive process and constraints on stationarity and frequency-domain properties.

Thank you for your attention

- Q & A section
  - Questions and comments are more than welcome!

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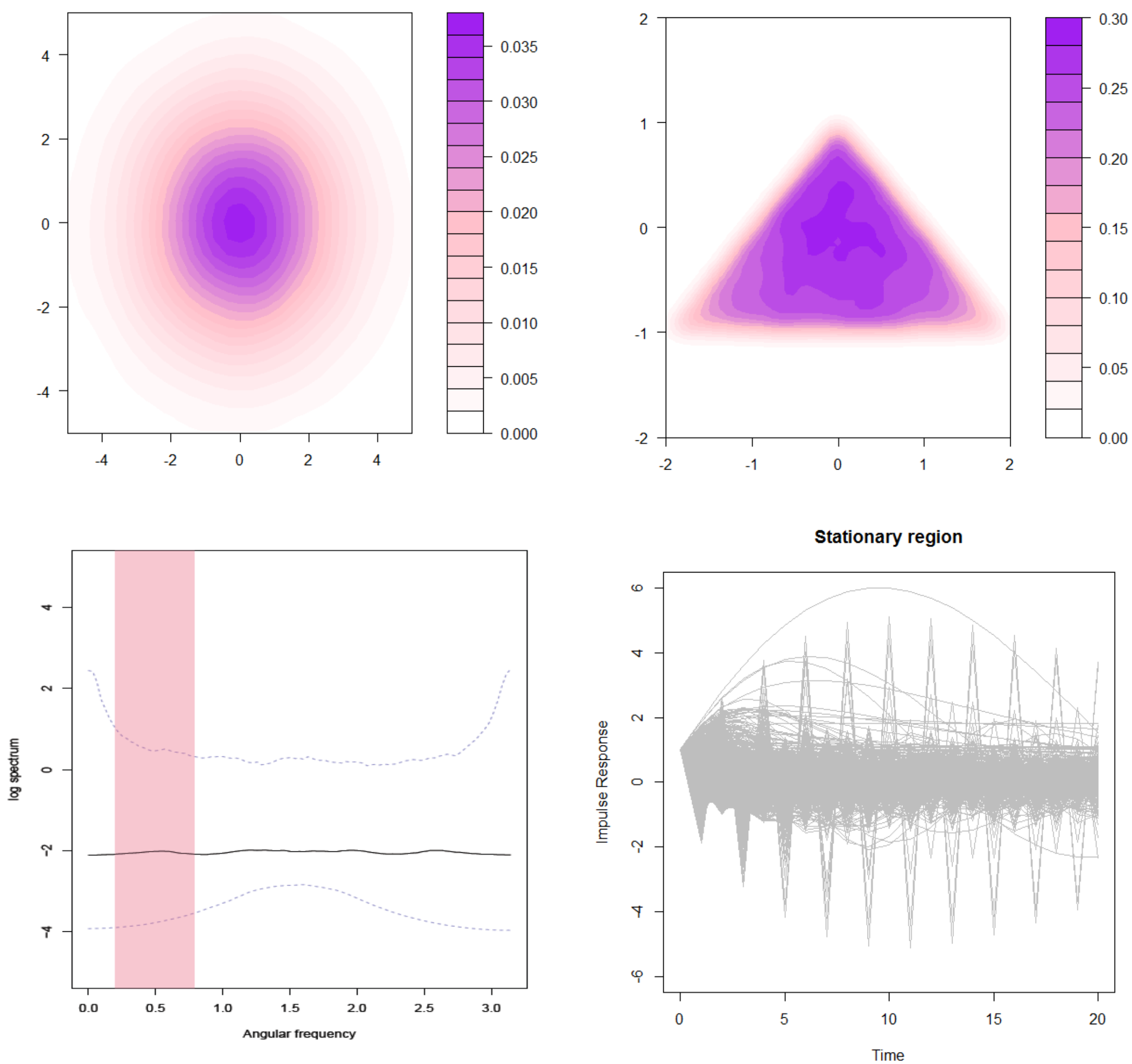
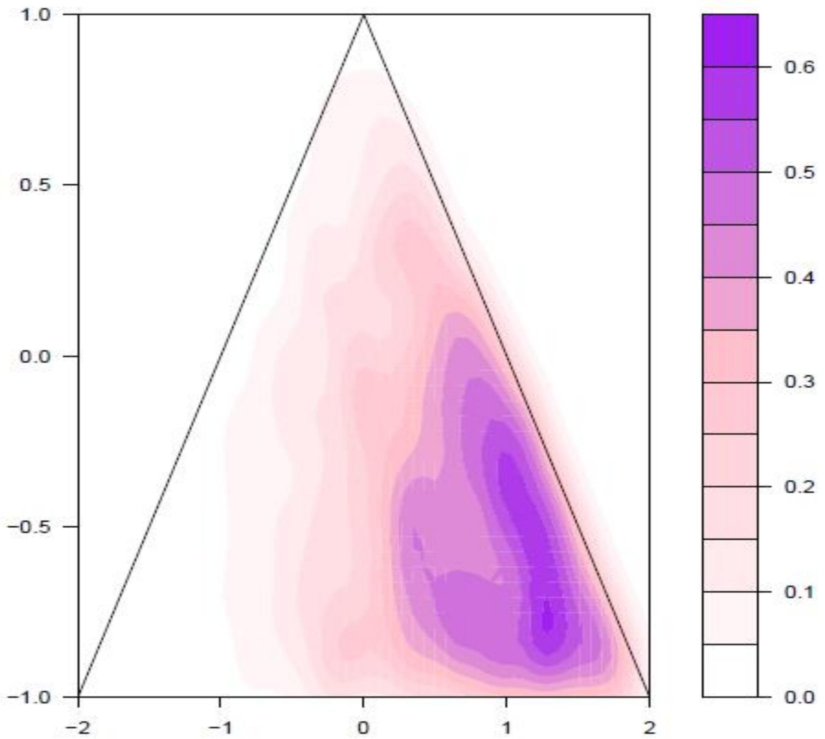
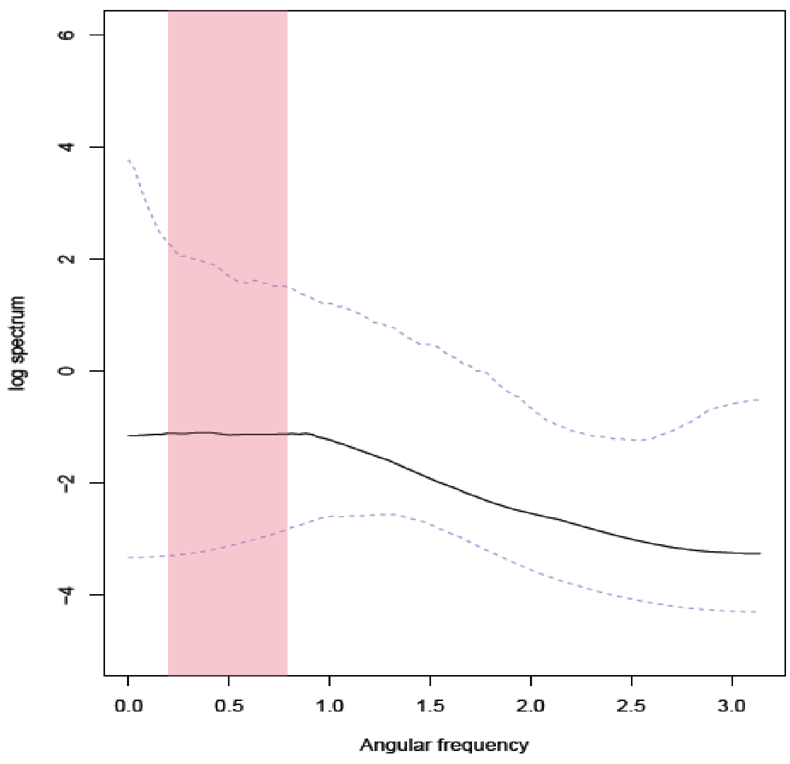
Back-up slides: Basic Gaussian option



Back-up slides: Refined Gaussian option I

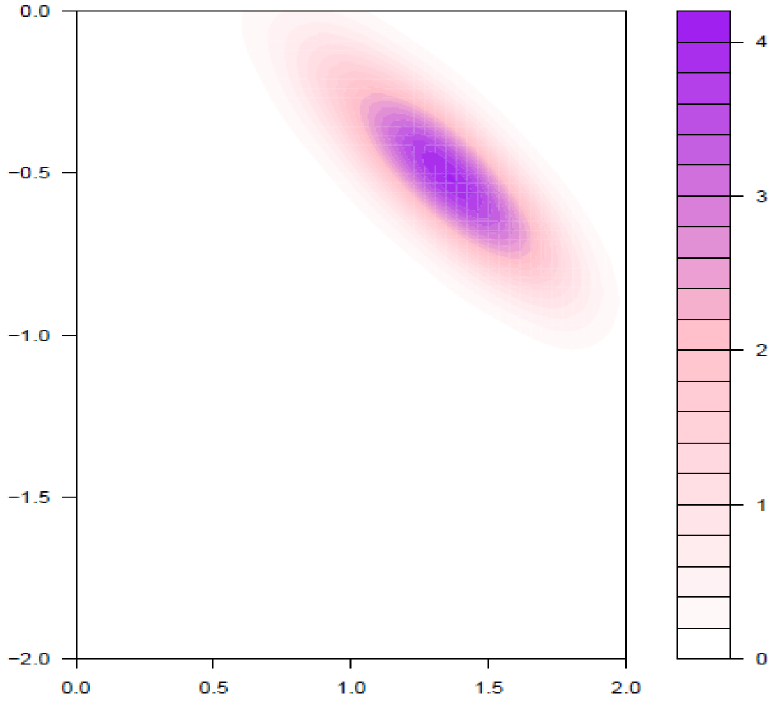
Grant and Chan (2017): $N\left(\begin{pmatrix}1.3\\-0.7\end{pmatrix}, I(2)\right)$

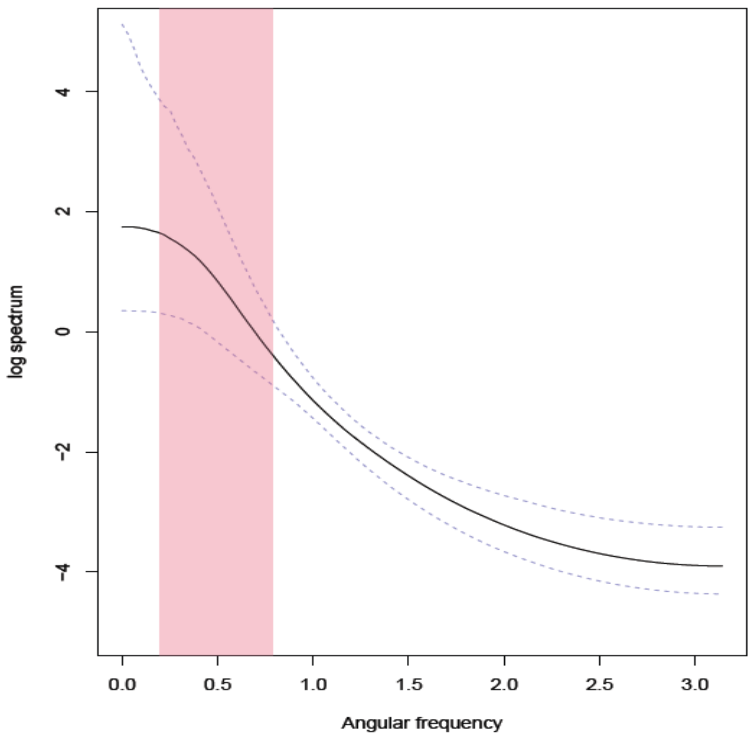




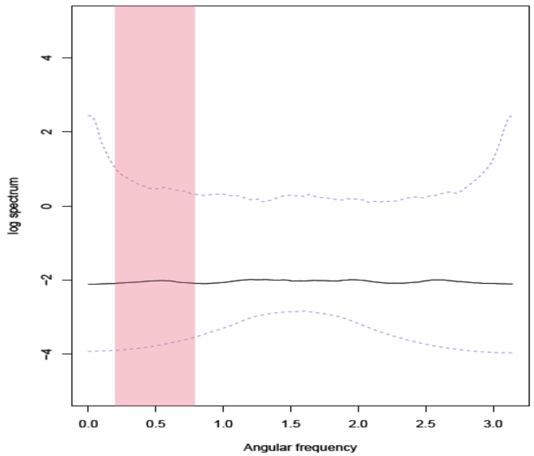
Back-up slides: refined Gaussian option II

Lenza and Jarociński (2016): $N\left(\begin{pmatrix}1.352\\-0.508\end{pmatrix},\begin{bmatrix}0.0806 & -0.0578\\-0.0578 & 0.0464\end{bmatrix}\right)$

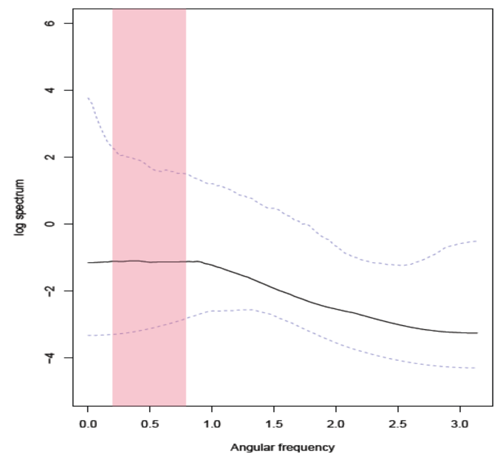




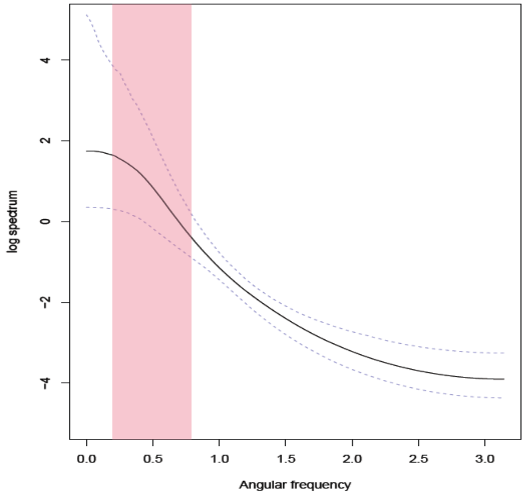
Back-up slides 1



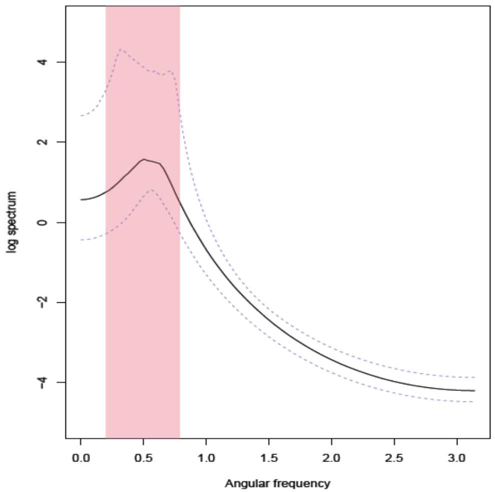
Stationary AR(2)



Chan et al. (2015)

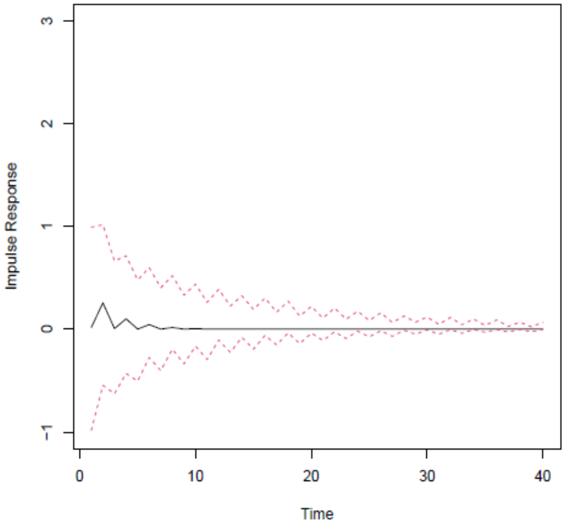


Lenza and Jarociński (2016)

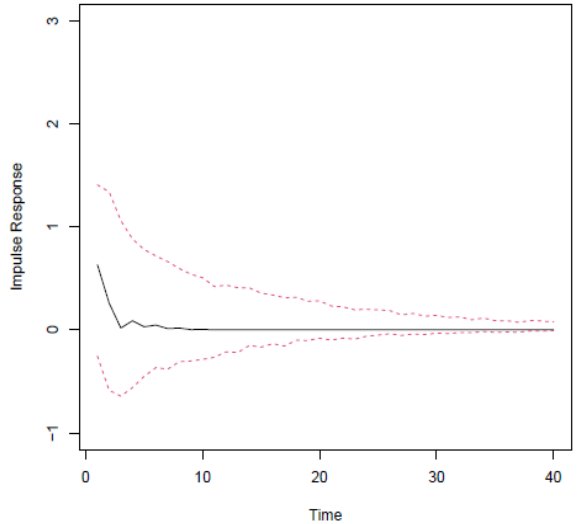


System priors, at least 60%

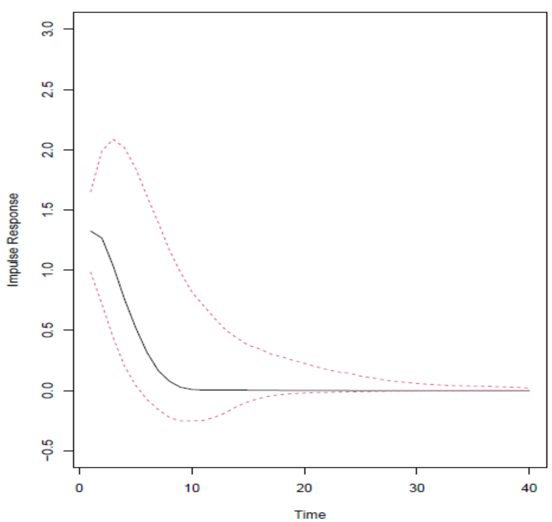
Back-up slides 2: Impulse response functions



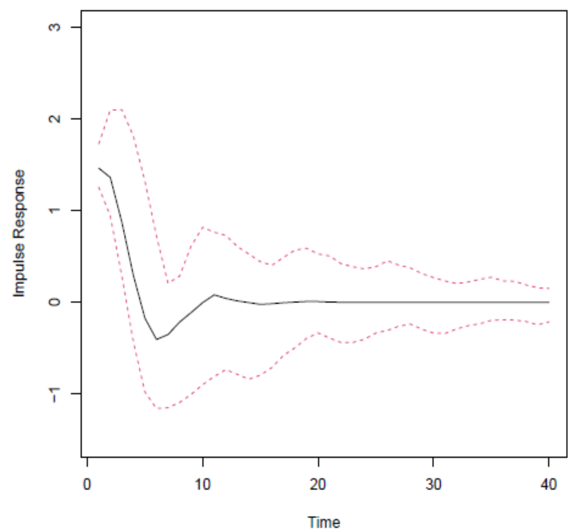
Stationary AR(2)



Chan et al. (2015)



Lenza and Jarociński (2016)



System priors, at least 60%

Back-up slides 2: System priors – alternatives