# Central Banks Balance Sheet Policies Without Rational Expectations

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# Central Banks Balance Sheet Policies

## Examples

- QE (long-term public and private assets purchases)
- FX interventions

"The problem with QE is that it works in practice,

but it does not work in theory."

Ben Bernanke (2014)

# Empirics

## QE

- Gagnon-Raskin-Remache-Sack (2011), Krishnamurthy-Vissing-Jorgensen (2011), Hancock-Passmore (2011), Di Maggio-Kermani-Palmer (2016), Chakraborty-Goldstein-MacKinlay (2016), Fieldhouse-Mertens-Ravn (2018), Stroebel-Taylor (2012),Greenlaw-Hamilton-Harris-West (2018)

## FX interventions

- Dominguez-Frankel (1990, 1993), Dominguez (1990, 2006), Catte-Galli-Rebecchini (1994), Kearns-Rigobon (2005), Blanchard-Adler-de Carvalho (2014), Fratzscher-Gloede-Menkhoff-Sarno-Stohr (2015) Beine-Benassy-Quere-Lecourt (2002)

# Theory

The irrelevance result if

1. people can freely trade targeted assets
2. symmetric info between policy maker and markets
3. people correctly predict future effects of policies

Prominent channels

1. Portfolio balance channel (segmented markets)
2. Signaling channel (asymmetric info or limited commitment)

This paper: bounded rationality channel

- Beliefs about future deviate from rational expectations
- Agents do not fully understand future effects of the policies

# Deviations from Rational Expectations

## Induction

- *Idea*: beliefs about future are refined over time through a process of induction from observed outcomes.
- *Econometric learning*: Evans-Honkapohja; Shleifer; etc.

## Eduction

- *Idea*: agents understand the model and form expectations from it about future outcomes through the process of reflection
- ***Level-k thinking***

# Level-$k$ Thinking

## Literature

- Stahl-Wilson (1994,1995); Nagel (1995); Crawford (2013)

## Idea

- Agents know the game; rationally respond to beliefs; form beliefs about opponents actions recursively

## Result

- Level-$k$ thinking better approximates experimental results in strategic games (more so in new games)

# Simple Model

Infinitely-lived households solve

$$\max_{\{x_{t+1},b_{t+1},c_t\}} \widetilde{\mathbb{E}}_0 \left[ -\frac{1}{\gamma} \sum_{t=0}^{\infty} e^{-\rho t - \gamma c_t} \right]$$

$$\text{s.t.:} \quad c_t + b_{t+1} + q_t x_{t+1} \leq W_t - T_t + R b_t + (D_t + q_t) x_t$$

Dividends: $D_t = \overline{D} + \epsilon_t^x, \ \epsilon_t^x \sim \mathcal{N}(0, \sigma_x^2)$

Assumption: beliefs about future endogenous variables

$$\widetilde{q}_{s+1} = \alpha_{q,s} + \beta_{q,s} \epsilon_{s+1}^x$$
$$\widetilde{T}_{s+1} = \alpha_{T,s} + \beta_{T,s} \epsilon_{s+1}^x$$

Risky-asset demand

$$x(q_t; \{\widetilde{q}_{t+s}, \widetilde{T}_{t+s}\}) = \frac{\overline{D} + \widetilde{\mathbb{E}}_t q_{t+1} - q_t R}{\gamma \frac{R-1}{R} \sigma_x^2} + \beta_{T,t}$$

# Government

- Central bank announces path $\{X_{t+1}, B_{t+1}\}$

- Government budget constraint is

$$q_t X_{t+1} + RB_t = (D_t + q_t) X_t + B_{t+1} + T_t$$

# Temporary Equilibrium (TE)

Idea: TE takes as given a sequence of beliefs and imposes that markets clear in every period (Hicks; Lindahl; Grandmont)

Definition For $\{\widetilde{T}_t, \widetilde{q}_t\}$, a TE is $\{X_{t+1}, B_{t+1}, T_t; q_t; b_{t+1}, x_{t+1}, c_t\}$ s.t. $\{x_{t+1}, b_{t+1}, c_t\}$ are optimal,

$$\frac{\overline{D} + \widetilde{\mathbb{E}}_t q_{t+1} - q_t R}{\gamma \frac{R-1}{R} \sigma_x^2} + \beta_{T,t} = \overline{X} - X_{t+1}$$

and

$$T_t = -\left[\left(D_t + q_t\right) X_t - R B_t\right] + q_t X_{t+1} - B_{t+1}$$

REE

# Level-$k$ thinking Belief Formation

Status quo $\{\widetilde{q}_{t+s}, \widetilde{T}_{t+s}\} = \{q^*, 0\}$ (REE before intervention)

$$\left.\begin{array}{ll} \text{Level-1} & x(q_t^1; \{q^*, 0\}) = \overline{X} - X_{t+1} \\ \text{Thinking} & T_t^1 = -\left[\left(D_t + q_t^1\right) X_t - RB_t\right] + q_t^1 X_{t+1} - B_{t+1} \end{array}\right\} \Rightarrow \{T_t^1,$$

$$\left.\begin{array}{ll} \text{Level-2} & x\left(q_t^2; \{q_{t+s}^1, T_{t+s}^1\}\right) = \overline{X} - X_{t+1} \\ \text{Thinking} & T_t^2 = -\left[\left(D_t + q_t^2\right) X_t - RB_t\right] + q_t^2 X_{t+1} - B_{t+1} \end{array}\right\} \Rightarrow \{T_t^2,$$

Level-$k$ Thinking $\{q_t^k, T_t^k\} = \Psi(\{q_t^{k-1}, T_t^{k-1}\}; \{X_{t+1}, B_{t+1}\})$

REE $\{q_t^*, T_t^*\} = \Psi(\{q_t^*, T_t^*\}; \{X_{t+1}, B_{t+1}\})$

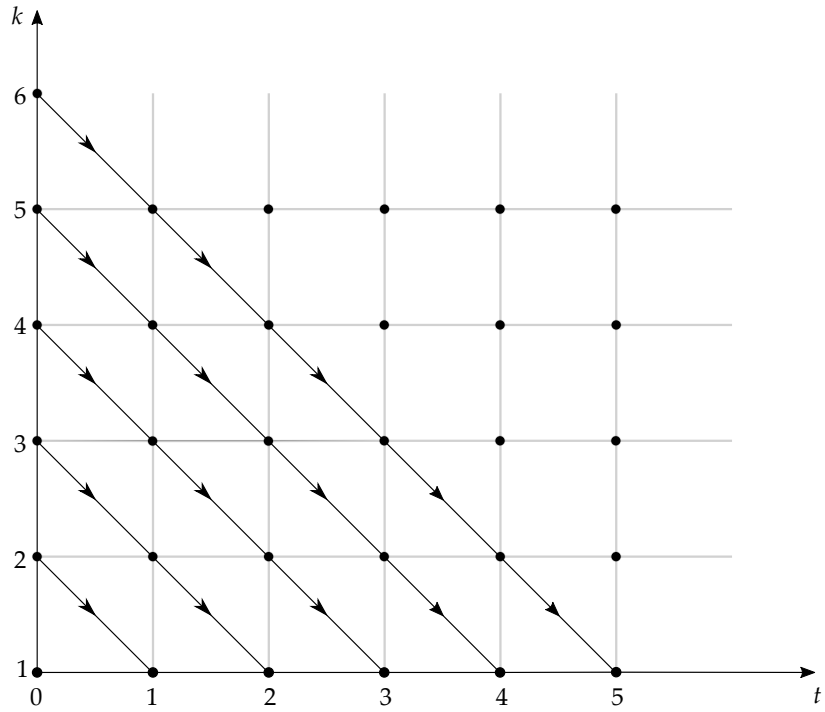
# Level-k thinking Belief Formation

$$q_t^k = \begin{cases} \dfrac{\overline{D} + q^* - \gamma \frac{R-1}{R} \sigma_x^2 \left( \overline{X} - X_{t+1} \right)}{R}, & k = 1 \\ \dfrac{\overline{D} + q_{t+1}^{k-1} - \gamma \frac{R-1}{R} \sigma_x^2 \overline{X}}{R}, & k > 1 \end{cases}$$

# Diagonal Iteration

$$q_t^k = \frac{\overline{D} + q_{t+1}^{k-1} - \gamma \frac{R-1}{R} \sigma_x^2 \overline{X}}{R},$$

$$q_{t+k-1}^1 = \frac{\overline{D} + q^* - \gamma \frac{R-1}{R} \sigma_x^2 \left( \overline{X} - X_{t+k} \right)}{R}$$




Endogenous discounting

# Reflective Equilibrium

Idea: agents form beliefs according to level-$k$ thinking, the economy is populated by agents with different $k$ with pdf $f(k)$

In case of exponential $f(k)$ with average $\overline{k}$

$$q_t = q^* + \gamma\sigma_x^2 \frac{R-1}{R} \cdot \frac{\sum_{k=1}^{\infty} \left(\frac{\overline{k}-1}{\overline{k}}\right)^{k-1} \frac{X_{t+k}}{R^k}}{\overline{k}}$$

Higher $\overline{k}$

1. reduces the direct effect of interventions
2. makes the price react more to expected future interventions

# Does QE affect output?

## Simple Model

- Endowment economy
- Balance sheet policies affect prices and taxes only

## A New-Keynesian model with risk

- Output is "demand determined" (rigid prices)
- Risky assets are claims on part of output
- Shocks to discount factor

Details

# Does QE affect output?

Rational expectations equilibrium: no effects of interventions

$$q^* = \frac{1}{R-1} \cdot \frac{\delta}{\overline{X}} \left( \overline{Y} - \frac{\sigma_x^2}{\gamma} \right)$$

$$Y_t^* = \overline{Y} - \frac{1}{\gamma} \epsilon_t^x$$

Level-1 equilibrium: first vs. second order effects

$$q_t^1 - q^* = \Gamma_q \frac{X_{t+1}}{\overline{X}}; \qquad Y_t^1 - Y_t^* = \Gamma_Y \left( \frac{X_{t+1}}{\overline{X}} \right)^2$$

Level-$k$ equilibrium: first vs. second order effects

# Extensions and Empirics

## Extensions

1. Long-term public bonds purchases (+nominal variables)
   - Model with level-1 thinkers resembles Vayanos-Vila (2009)

2. Foreign exchange interventions (+nominal variables)

3. Learning to play equilibrium
   - Existing policy effect disappears over time
   - New policies are less effective

4. Presence of rational expectations agents
   - Does not change qualitative results
   - Can amplify effects due to level-$k$ thinking

## Empirics

- Asset prices forecast errors are predictable

# Conclusion

1. Bounded rationality channel of balance sheet policies
   - Interventions have a first-order effect on prices
   - . . . but only a second-order effect on output
2. Testable predictions
   - Forecast errors respond to interventions
   - Evidence from mortgage rate forecasts errors

# Rational Expectations Equilibrium

Definition: REE is a TE such that

$$\widetilde{T}_t = T_t,\ \widetilde{q}_t = q_t$$

Specifically

$$\underbrace{\alpha_{T,t} + \beta_{T,t}\epsilon_t^x}_{\text{tax beliefs } \widetilde{T}_t} \overset{\text{REE}}{=} \underbrace{q_t X_{t+1} - B_{t+1} + RB_t - X_t(\overline{D} + q_t) - X_t\epsilon_t^x}_{\text{realized taxes } T_t}$$

Risky assets market in $t$

$$\frac{r^x + \mathbb{E}_t q_{t+1} - q_t R}{\gamma \frac{R-1}{R}\sigma_x^2} + \beta_{T,t+1} = \overline{X} - X_{t+1}$$

$\Rightarrow$ Balance sheet policy does not affect price $q_t$ in REE!

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# A Model with Endogenous Output

## Households

$$\max_{\{x_{t+1},b_{t+1},c_t\}} \widetilde{\mathbb{E}}_0 \left[ -\frac{1}{\gamma} \sum_{t=0}^{\infty} e^{\sum_{s=0}^{t} \epsilon_{s-1} - \epsilon_{-1} - \rho t - \gamma c_t} \right]$$

$$\text{s.t.:} \quad c_t + b_{t+1} + q_t x_{t+1} \leq W_t - T_t + R b_t + (D_t + q_t) x_t$$

## Total Income/Output $Y_t$ distributed as

- $W_t = (1-\delta) Y_t$ – labor (non-traded) income
- $D_t \overline{X} = \delta Y_t$ – dividends

## What determines output? goods market clearing (in TE)

$$Y_t = C\left[W_t(Y_t) - T_t(Y_t), D_t(Y_t), q_t, \{W^e_{t+s} - T^e_{t+s}, D^e_{t+s}, q^e_{t+s}\}\right]$$

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# Testable Predictions

## Forecast errors

- Agents make predictable forecast errors
- The errors can differentiate the model from other theories
  (segmented markets, signaling channel)

## Forecast errors in the data

- Forecasts of future taxes?
- Forecasts of asset prices

## Forecast errors in the model

$$\text{Individual: } u^k_{t+s} \equiv q_{t+s} - \widetilde{\mathbb{E}}^k_t q_{t+s}$$

$$\text{Average: } \bar{u}_{t+s} \equiv \sum_{k=1}^{\infty} f(k) u^k_{t+s} = \mu^s \frac{\gamma \sigma_x^2 \frac{R-1}{R} X_{t+1}}{\overline{k}[(R-\mu)\overline{k} + \mu]}$$

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# Empirics

## Fieldhouse-Mertens-Ravn (2018, QJE)

- Monthly data on GSEs mortgage purchases: 1967-2006
- "Unexpected exogenous" purchases narrative identification
- Result: mortgage yield reacts significantly to interventions

## Forecast errors

- Blue Chip conventional mortgage rate forecasts: 1982-2006
- Project median forecast errors on "exogenous" purchases

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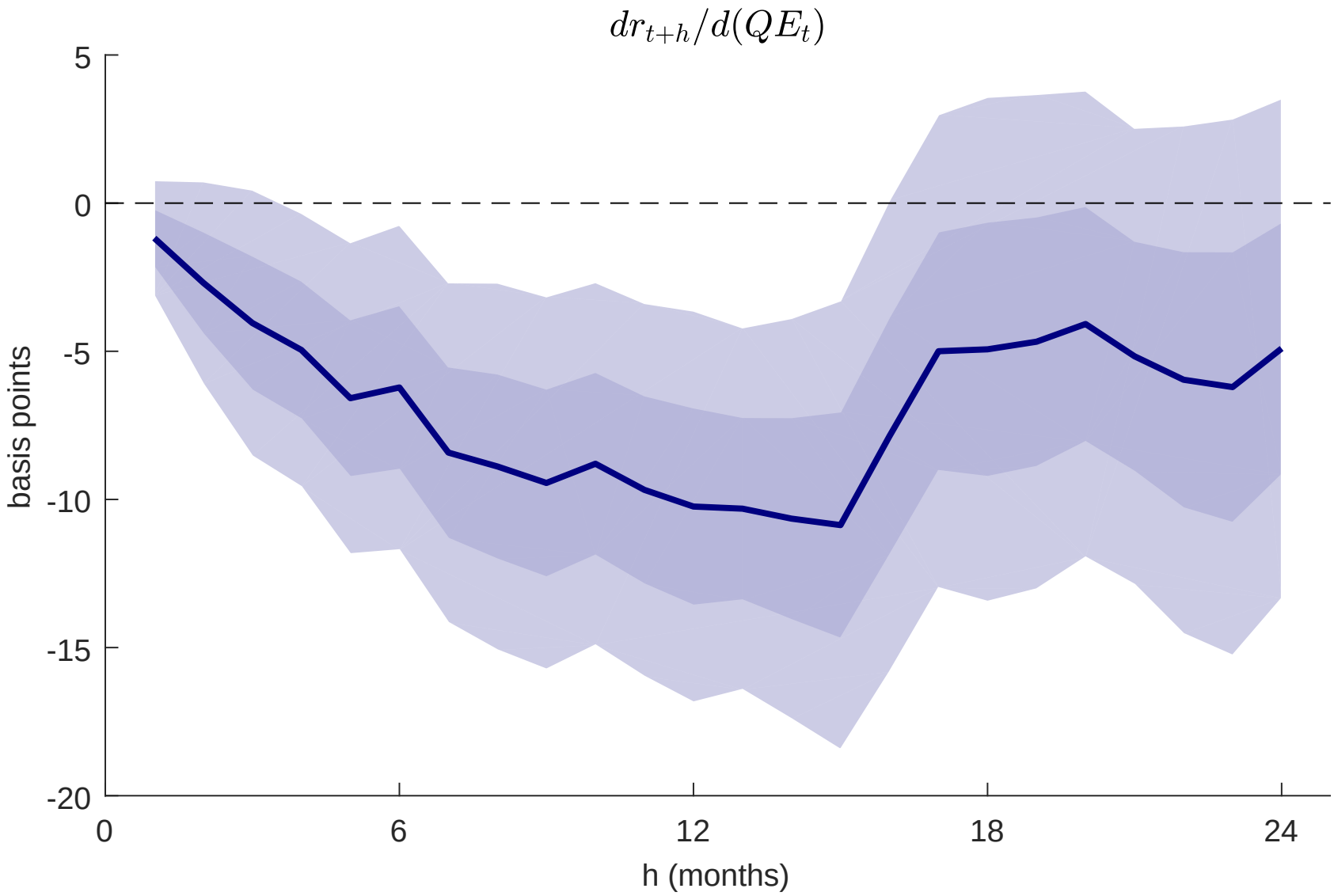


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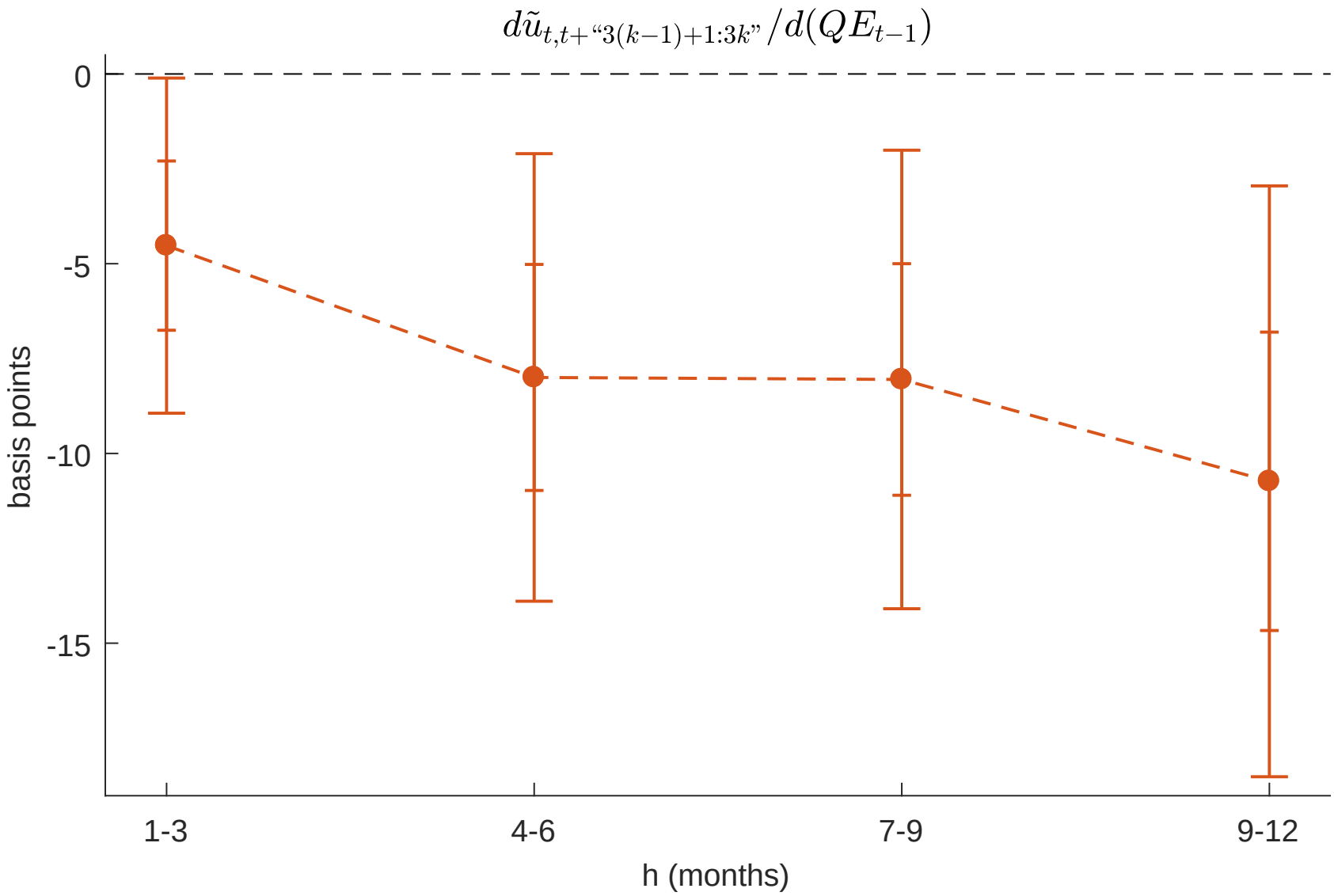
$d\tilde{u}_{t,t+\text{“}3(k-1)+1:3k\text{”}}/d(QE_{t-1})$
0
-5
-10
-15
basis points
1-3
4-6
7-9
9-12
h (months)

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