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Ivan Sutóris*

Abstract

When consumers optimize intertemporally, a true cost of living index will depend on changes in both current and future prices as well as rates of return on financial assets. This paper aims to construct a measure of such "dynamic inflation" for the Czech Republic from a solution to the household's intertemporal consumption-saving problem. Dynamic inflation is derived to be a function of current movements in consumption and house prices as well as revisions to forecasts of the future paths of inflation and interest rates. The resulting series constructed from Czech data roughly follows CPI inflation, but is more volatile and less persistent. Housing booms can cause persistent upward deviations, while changes in expected interest rates have a stabilizing effect. In addition, the intertemporal cost of living can also potentially be affected by low-frequency structural shifts in the economy.

Abstrakt

Když spotřebitelé optimalizují napříč časem, jejich skutečný index životních nákladů závisí nejen na změnách současných, ale i budoucích cen, a také na výnosech finančních aktiv. Tento článek má za cíl zkonstruovat takový ukazatel „dynamické inflace" pro Českou republiku na základě řešení intertemporální optimalizace spotřeby a úspor domácností. Dynamická inflace je odvozena jako funkce současných změn cen spotřebních statků a nemovitostí a také revizí predikcí budoucí trajektorie inflace a úrokových sazeb. Výsledná časová řada se vyvíjí zhruba podobně jako inflace měřená indexem CPI, je však volatilnější a méně perzistentní. Boom cen nemovitostí může způsobit trvalejší odchylky směrem nahoru, zatímco změny očekávaných úrokových sazeb mají spíše stabilizující efekt. Kromě toho mohou mít na intertemporální životní náklady potenciální vliv také dlouhodobé strukturální změny v ekonomice.

JEL Codes: C43, D15, E31.

Keywords: Cost of living, CPI, dynamic inflation, intertemporal optimization.

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1. Introduction

One of the main goals of measuring inflation is to track the purchasing power of money and, more specifically, the costs necessary to maintain a constant standard of living. Wage bargaining, indexation of social benefits, and computation of the real return on savings all depend on the reported inflation rate capturing changes in the cost of living. However, in practice the task of measuring inflation is far from being simple. A particular concern – one which is still a matter of some controversy – is whether an index of consumer prices should in some way account for prices of financial claims or long-lived durable goods such as housing. A common view is that asset prices should not enter the index because the underlying assets are traded for purposes other than immediate consumption. On the other hand, some argue that more costly houses or financial assets will negatively affect the budgets of the households who purchase them regardless of their intended purpose, and thus ignoring them in official statistics will provide biased evidence about the true costs of living.

What does economic theory say about the issue? Conceptually, the ideal cost of living index (Konüs, 1939) can be constructed from a solution to the utility maximization problem of an individual consumer. One can simply feed a sequence of prices into the indirect expenditure function, obtain a sequence of incomes necessary to attain a constant level of utility, and define inflation as the growth rate of these hypothetical incomes. However, the underlying optimization problem is traditionally a static one in which the household chooses optimal consumption independently in each period. Yet if we are to take the economic approach to measuring the cost of living seriously, we should account for the fact that the true decision problem of the consumer is inherently dynamic. People do not simply receive and spend their income each period; they can shift their consumption over time by saving or dissaving. As observed in the seminal paper by Alchian and Klein (1973), what really matters for a consumer who can allocate consumption over multiple periods is the whole sequence of prices, not just prices at a point in time.

From this perspective, the measurement of inflation becomes a forward-looking problem. An increase in prices of nondurable goods that is expected to persist for a long time will affect utility more than a short, temporary fluctuation, and should command correspondingly higher compensation. Since the capability to substitute consumption over time depends on the returns offered by financial markets, the index should also depend on those returns, or equivalently on prices of financial assets. In addition, the dynamic framework provides a theoretically consistent way of dealing with durable goods such as housing which combine elements of both nondurables and assets. This alternative, unified perspective could thus offer a complementary insight into cost of living calculations and allow us to find out whether our traditional static price indices are missing some part of the picture.

The goal of this paper is to construct an approximate "dynamic" price index for the Czech economy and compare its properties with those of the CPI, the most prominent existing inflation measure. In practice, of course, we do not know the true dynamic optimization problem or the sequence of future prices, and thus an actual dynamic price index will have to be constructed based on approximations from observed data (just like static indices are). In terms of methodology, we follow Reis (2005) by solving for a value function of a relatively simple intertemporal optimization problem of a representative household which takes as given the sequence of nondurable and housing prices and financial returns, and makes optimal consumption and saving decisions. The resulting inflation series, computed by comparing the value function between subsequent periods, can be expressed analytically and consists of a static term reflecting current changes in prices and extra forward-looking terms capturing revisions of the expected discounted sums of future prices and returns. In practice, these can be proxied by revisions to forecast paths over time, either obtained from external sources or estimated from a time series model. The methodology is thus quite tractable and can easily be applied without the need for repeated numerical solving of the underlying optimization problem.

In the empirical implementation, expectations about future nondurable prices and financial returns are proxied by forecasts from the Czech National Bank's Inflation Reports, while prices and forecasts of housing inflation are constructed based on Czech housing prices. The resulting measure of dynamic inflation behaves qualitatively similar to CPI inflation. In year-on-year terms, dynamic inflation is about one percentage point higher than CPI inflation over the sample period, but the two measures co-move to some extent (with a correlation coefficient of 0.71). On the other hand, dynamic inflation does seem to be more volatile and less persistent, in line with its forward-looking nature. During some periods, it also deviates upward from CPI inflation more persistently due to house price booms. Changes in expected interest rates, on the other hand, act in the opposite direction to contemporary price developments, since lower (higher) rates caused by a monetary policy response push dynamic inflation up (down) by making future consumption more (less) expensive. Comparing alternative versions of the index reveals that both house prices and forward-looking news contribute to the deviation between the two series, albeit differently in different periods. Finally, although most of the analysis focuses on the cyclical dynamics of relevant variables, a stylized example suggests that long-term movements in the economy, such as changes in the natural real rate of interest, can also have potentially important effects on the intertemporal cost of living.

In terms of related literature, as has already been mentioned, the first contribution pointing out the dynamic implications of a utility-based cost of living index is due to Alchian and Klein (1973). Further theoretical analysis can be found in Pollack (1975) and more recently in Reis (2005), who derives more formally the theoretical properties of a dynamic price index and extends it to cover durable goods. On the empirical side, Shibuya (1992) and Shiratsuka (1999) compute a simplified version of the index for the Japanese economy, where the divergence between traditional inflation and asset prices had been especially large. Reis (2005) constructs a dynamic price index for the United States and finds it to be more volatile than the CPI and responding substantially to housing and bond prices. Aoki and Kitahara (2010) propose an alternative approach to computing the index based on forecasts of the aggregate consumption process.

More broadly, this paper is also related to the discussion about the merits of including asset prices in inflation measures and monetary policy responses. Goodhart (2001) argues in favor of responding to asset prices, especially when they predict future inflation, and Bryan (2002) includes asset prices in a factor model that tries to isolate pure price level changes. More skeptical views are presented in Vickers (2002) and Diewert (2002), while Frait and Komárek (2007) discuss the connection between asset prices and monetary policy in the context of new EU member states. Another related debate considers the treatment of housing in the consumer price index, as reviewed, for example, in Diewert and Nakamura (2009). In the Czech context, the issue has been of some interest to the central bank, as discussed in Hampl and Havránek (2017), and in fact, the Czech National Bank has recently started to publish an experimental variant of the CPI with a component linked to housing prices (Růžička and Šnobl, 2017).

The paper is organized as follows. Section 2 presents the theoretical model and derives the resulting formulas for dynamic inflation. Section 3 describes the data and forecasts used to construct the index, while section 4 contains the empirical results and is followed by a conclusion.

2. Model

This section presents the decision problem of an intertemporally optimizing household that serves as the basis for the construction of the dynamic inflation measure. The model is a simplified version of a more general setup analyzed in Reis (2005) with one nondurable consumption good and one durable good, henceforth interpreted as housing. Time is discrete, and each period an infinitely-lived household allocates its wealth between purchases of nondurable goods, housing, and financial savings, with the latter two determining its wealth in the following period. The household takes as given current and future prices, which will thus affect its lifetime utility. As time moves forward, the sequence of prices may change, and dynamic inflation is obtained by comparing the welfare of two hypothetical households facing two such different sets of prices.

More formally, a household that starts with nominal wealth W at the beginning of period t decides about the sequence of current and future nondurable consumption $\{C_\tau\}_{\tau=t}^{\infty}$, durable (housing) consumption $\{H_\tau\}_{\tau=t}^{\infty}$, and financial savings $\{A_\tau\}_{\tau=t}^{\infty}$ while taking as given the sequences of current and future nondurable prices $\{P_\tau^c\}_{\tau=t}^{\infty}$, durable (housing) prices $\{P_\tau^h\}_{\tau=t}^{\infty}$, and financial (nominal) returns $\{R_{\tau+1}\}_{\tau=t}^{\infty}$. For simplicity, we will abstract from uncertainty and assume the household has (or believes it has) perfect foresight about future exogenous variables. Its goal is to maximize the discounted sum of the logarithmic utilities over the Cobb-Douglas composite of nondurables and housing, with the weight on nondurables α and the discount rate β. The maximized value function is thus defined as

$$V_t(W) = \max_{\{C_\tau, H_\tau, A_\tau, W_{\tau+1}\}} \sum_{\tau=t}^{\infty} \beta^{\tau-t} \log(C_\tau^\alpha H_\tau^{1-\alpha}). \tag{1}$$

The optimization is subject to a budget constraint stating that current wealth is spent on nondurable and housing expenditures and financial savings:

$$\tau \geq t\colon\ P_\tau^c C_\tau + P_\tau^h H_\tau + A_\tau = W_\tau, \tag{2}$$

and a wealth accumulation constraint which says that next-period wealth is the sum of financial savings, including returns thereon, and the next-period value of housing less proportional depreciation at rate δ:

$$\tau \geq t\colon\ W_{\tau+1} = R_{\tau+1} A_\tau + (1-\delta) P_{\tau+1}^h H_\tau, \tag{3}$$

while the initial wealth is given as $W_t = W$. The notation emphasizes that the value function depends on initial wealth, but of course the maximized objective is also a function of the whole sequence of prices and returns from time t onward. For notational convenience, this dependence is captured by a time index on the value function.

Assuming we have solved for the sequence of value functions $\{V_t\}_{t=1}^{\infty}$, dynamic inflation Π_t^d between periods $t-1$ and t will be defined as the implicit solution to the equation

$$V_{t-1}(W) = V_t(\Pi_t^d \cdot W), \tag{4}$$

so that it can be interpreted as the rate of growth in wealth required for a household in period t to attain an identical level of utility to a household with given wealth in period $t-1$. Note that with a homothetic utility function (such as the one used here) dynamic inflation will not depend on the initial value of wealth, only on external economic conditions.

The fact that we have presented dynamic inflation in terms of value functions defined over levels of wealth affects its interpretation. Current capital gains or losses will not enter into consideration, because the calculation is based on comparing the welfare of two hypothetical households that are inserted into the economy in the two periods with the respective levels of wealth, and not on comparing the welfare of a single actual household that chooses a particular mix of housing and savings and may experience capital gains or losses on those assets. Another issue that deserves comment is that we abstract from labor income. Thus the model, if taken literally, describes a rentier living off his savings, but this is of course mainly due to analytic convenience rather than realism. A better way of thinking about the model would be to interpret wealth broadly as including the present value of labor income, i.e., human capital. An increase in wealth would then mean a proportionate rise not only in financial resources, but also in labor income earned in the current and future periods.

To solve for the value function, we first need to find the optimal consumption and saving decisions for the household. After eliminating A_t using the budget constraint, the household aims to maximize objective (1) subject to a sequence of constraints

$$\tau \geq t: W_{\tau+1} = R_{\tau+1}(W_\tau - P_\tau^c C_\tau - P_\tau^h H_\tau) + (1-\delta)P_{\tau+1}^h H_\tau. \tag{5}$$

It will be useful to define housing user cost as

$$U_t = P_t^h - \frac{(1-\delta)P_{t+1}^h}{R_{t+1}}, \tag{6}$$

i.e., the difference between the current price of a house and the present value of the depreciated remainder of the house next period. The user cost represents the expense of buying a house, consuming its services for one period, and then immediately selling next period. In a frictionless market, such a strategy would be equivalent to renting the house, so the user cost can be understood as an implied rent. In addition, the constraints can be simplified when written in terms of the user cost:

$$\tau \geq t: \frac{W_{\tau+1}}{R_{\tau+1}} = W_\tau - P_\tau^c C_\tau - U_\tau H_\tau. \tag{7}$$

The Lagrange function corresponding to the optimization problem can be written as

$$L = \sum_{\tau=t}^{\infty} \beta^{\tau-t} \left[\log(C_\tau^\alpha H_\tau^{1-\alpha}) - \lambda_\tau \left(\frac{W_{\tau+1}}{R_{\tau+1}} - (W_\tau - P_\tau^c C_\tau - U_\tau H_\tau) \right) \right], \tag{8}$$

and the necessary conditions for the optimal solution require that for all $\tau \geq t$ we have $\frac{\partial L}{\partial C_\tau} = \frac{\partial L}{\partial H_\tau} = \frac{\partial L}{\partial W_{\tau+1}} = \frac{\partial L}{\partial \lambda_\tau} = 0$. As a result, we obtain first-order conditions

$$\forall \tau \geq t:,\ \frac{\alpha}{C_\tau} = \lambda_\tau P_\tau^c,\ \frac{1-\alpha}{H_\tau} = \lambda_\tau U_\tau,\ \lambda_\tau = \beta R_{\tau+1}\lambda_{\tau+1}. \tag{9}$$

The first two conditions show that the shadow value of wealth, the multiplier λ_τ, equals the marginal utility from spending an additional unit of currency on nondurable consumption, and imply that the consumption mix will satisfy a constant ratio of expenditure shares when written in terms of the housing user cost: $\frac{P_\tau^c C_\tau}{U_\tau H_\tau} = \frac{\alpha}{1-\alpha}$. The last condition represents the standard consumption smoothing that links marginal utility growth with the discounted return.

With log utility, it turns out that the household will save a fixed proportion of its wealth in financial assets, and the optimal consumption choice will involve constant expenditure shares on nondurables and housing when the latter is evaluated with the user cost instead of the house price. The resulting choices, which can be easily verified as optimal by plugging into the conditions above, can be written as:

$$C_t = \frac{1}{P_t^c}\alpha(1-\beta)W_t,\quad H_t = \frac{1}{U_t}(1-\alpha)(1-\beta)W_t,\ \lambda_t = \frac{1}{(1-\beta)W_t}. \tag{10}$$

As a result, the evolution of wealth is given by $W_{t+1} = \beta R_{t+1} W_t$.

Plugging the expressions for nondurable and housing consumption back into the household objective function yields the expression

$$V_t = \sum_{\tau=0}^{\infty} \beta^\tau [w_{t+\tau} - \alpha p_{t+\tau}^c - (1-\alpha)u_{t+\tau}], \tag{11}$$

where lower-case variables denote logarithms and we have disregarded constant terms (which will not play a role when value functions are compared between periods). The value function is thus, up to a constant, the discounted sum of the current and future log real wealth levels, deflated by a combination of nondurable prices and housing user costs. However, it will be beneficial to work with growth rates rather than levels. By expressing each future price or wealth level as the current level plus the sum of the growth rates from now up to that point, and changing the order of summation, we can express the value function in terms of current wealth and prices and in terms of the discounted sums of future inflation rates $\pi_t^c = \Delta p_t^c$, $\pi_t^u = \Delta u_t$ and returns:

$$V_t = \frac{1}{1-\beta}[w_t - \alpha p_t^c - (1-\alpha)u_t + \omega_t^r - \alpha\omega_t^c - (1-\alpha)\omega_t^u], \tag{12}$$

with the discounted sum terms defined as follows:

$$\omega_t^c = \sum_{i=1}^{\infty} \beta^i \pi_{t+i}^c,\ \omega_t^u = \sum_{i=1}^{\infty} \beta^i \pi_{t+i}^u,\ \omega_t^r = \sum_{i=1}^{\infty} \beta^i r_{t+i}. \tag{13}$$

The returns appear in the expression because they contain the growth rate of wealth (recall that up to a constant, $\Delta w_t = r_t$).

Finally, using the expression for the value function (12) with the definition of dynamic inflation in equation (4) yields a comparison

$$w_{t-1} - \alpha p_{t-1}^c - (1-\alpha)u_{t-1} + \omega_{t-1}^r - \alpha\omega_{t-1}^c - (1-\alpha)\omega_{t-1}^u = w_{t-1} + \pi_t^d - \alpha p_t^c - (1-\alpha)u_t + \omega_t^r - \alpha\omega_t^c - (1-\alpha)\omega_t^u, \tag{14}$$

so that the resulting expression for (log) dynamic inflation reads:

$$\pi_t^d = \alpha\pi_t^c + (1-\alpha)\pi_t^u + \alpha\Delta\omega_t^c + (1-\alpha)\Delta\omega_t^u - \Delta\omega_t^r. \tag{15}$$

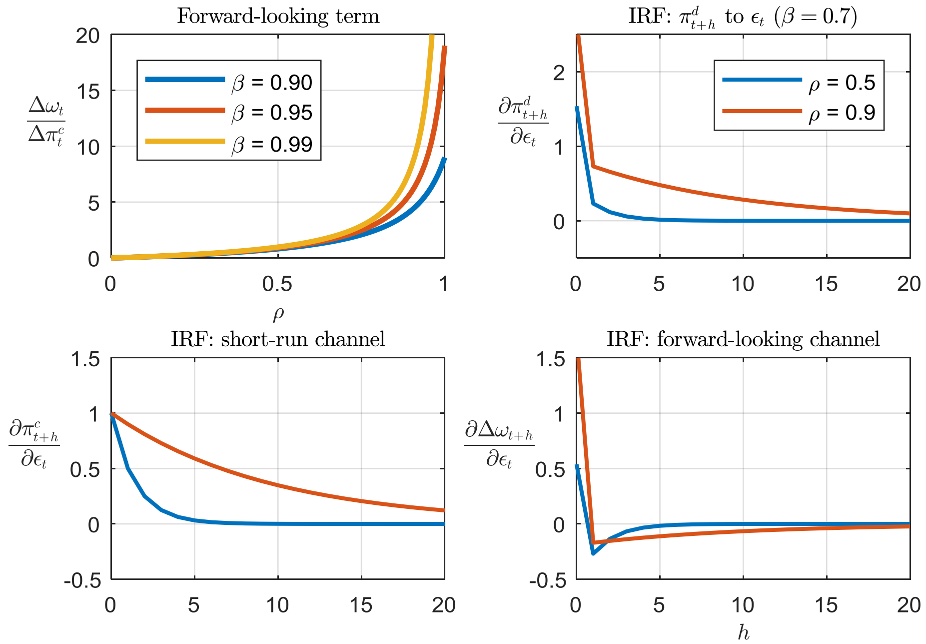
The first two terms are a weighted average of static price changes,[1] just like in conventional inflation measures. The next two terms represent changes in the discounted sums of future inflation rates, and enter positively, so that news about higher future inflation will increase current dynamic inflation. The last term stands for the discounted sum of future financial returns, and enters negatively, because news about higher future returns effectively makes future consumption cheaper.[2] Note that with the timing assumptions used here, this term will include the return between the current and the following period (i.e., the current interest rate), but not the realized return between the last and the current period, in line with the treatment of capital gains discussed earlier.

It may be instructive to consider the behavior of dynamic inflation in a simpler setting without housing ($\alpha = 1$) and where nondurable inflation follows the AR(1) process $\pi_t^c = \rho\pi_{t-1}^c + \epsilon_t$. For simplicity we shall also assume that the nominal interest rate is constant (and thus changes in inflation are reflected in the real rate of return). Then it is easy to see that $\omega_t^c = \Sigma_{i=1}^{\infty}\beta^i E_t[\pi_{t+i}^c] = \left[\frac{\beta\rho}{1-\beta\rho}\right]\pi_t^c$, ω_t^r is constant, and thus $\pi_t^d = \pi_t^c + \frac{\beta\rho}{1-\beta\rho}\Delta\pi_t^c$, meaning the deviation between dynamic and static inflation is proportional to the current change in nondurable inflation. An increase in static inflation will impact dynamic inflation in excess of the direct effect by changing the forecasts of future inflation. This effect is stronger if the inflation process is more persistent (ρ is high) and if the household is more forward-looking (β is high), as illustrated in the first panel of Figure 1. Dynamic inflation would, in this case, follow an ARMA(1,1) process and typically would be more volatile and less autocorrelated than the underlying nondurable inflation process. The second panel of the figure plots an example of the impulse response of dynamic inflation for two values of the persistence parameter. The response, as shown in the bottom panels, can be decomposed into the static contribution, which just tracks the mean-reverting response of nondurable inflation, and the forward-looking contribution, which dominates the initial reaction.

[1] Although the notation does not reflect it explicitly, we should clarify that the forward-looking terms are to be constructed from the expectations prevailing in each period, which may change over time in response to news. We may interpret this as the household always believing it has perfect foresight, but then receiving an unexpected shock at the beginning of each period that will change its forecasts of future prices. Alternatively, the expressions derived above could be understood as linear approximations to the true solution under uncertainty, in which expectations are replaced with point forecasts.

[2] When interpreting the expression for dynamic inflation, one should keep in mind that financial returns are in nominal terms. Thus, an increase in future goods inflation without a corresponding change in nominal returns is de facto a decrease in future real returns and affects the household’s value function. It is possible to express the definition equivalently in terms of current inflation and future real returns.

Figure 1: Behavior of Dynamic Inflation in AR(1) Example



Note: Upper left panel: sensitivity of forward-looking term to persistence parameter; upper right panel: impulse response to nondurable inflation shock. Bottom panels: static and forward-looking components of impulse response.

3. Data

In order to construct a measure of dynamic inflation, one needs to calibrate parameters α and β and obtain measures of inflation for nondurable goods, housing user costs, and returns on financial assets. Moreover, to construct the forward-looking discounted sums, it is also necessary to obtain or construct forecasts, starting from each period, of future values for each of these variables. It is also not entirely clear what would be the best observable proxy for the theoretical concept of user costs used in the derivation of the model. Nevertheless, once these issues are dealt with, dynamic inflation can easily be computed according to expression (15) independently from the specific parametric or nonparametric assumptions made when constructing the forecasts.

The discount rate parameter is set to a standard value $\beta = 0.99$. The share of nondurables in utility is chosen as $\alpha = 0.8$, which is approximately consistent with the share of housing expenditures in household consumption in the national accounts of about one fifth.[3] The data series on inflation, the interest rate, and house prices in quarterly terms are obtained from the CNB's ARAD database or from the Czech Statistical Office and cover the period from mid-2004 to mid-2018. The return on financial assets is proxied by the interest rate on the Czech interbank market (3-month PRIBOR), while nondurable inflation is measured by change in the consumer price index. The treatment of housing user costs is described in more detail later in this section.

The choice of proxies for nondurable inflation and financial returns deserves some discussion. Although the interbank rate may not be the rate of return most directly relevant for households

[3] According to the Czech Statistical Office Database of National Accounts, in 2017 household expenditure on housing (COICOP category 04 excluding 04.5, i.e., energy and fuel) constituted 18.6% of all expenditure; this figure includes imputed rent for owner-occupied housing.

(that would be the overall return on household wealth, including the return on human capital, which is not directly observable), what really matters for computing dynamic inflation are changes in current and future interest rates. To the extent that the spreads between the various rates are stable over time, using the interbank rate seems like a sufficient approximation. As for the choice of the CPI, a relevant question is whether it already accounts for housing prices. In the Czech case, the CPI includes the cost of owner-occupied housing with a weight of less than 10% of the overall consumption basket. This component has typically been based mostly on maintenance costs and has only recently started to include a small part directly derived from prices of new dwellings (Hampl and Havránek, 2017). In the overall basket, however, this weight is quite small, so the inclusion of house prices in the CPI is unlikely to affect the results.

Forecasts of the interbank rate and CPI inflation are taken from the Czech National Bank's Inflation Reports, which are published within each quarter and present the bank's official forecasts for multiple quarters ahead (based on data up to and including the previous quarter). For the period 2004–2007, the data come from internal staff forecasts (Situation Reports), which are released publicly with a six-year lag. Given the forecasts, ω_t^c and ω_t^r are defined as the truncated discounted sums of the forecasts (made based on data available at time t, i.e., from the report published within quarter $t+1$) over the nearest eight quarters.[4] In cases where the forecast was made for a smaller number of periods, the remaining quarters (up to the eighth quarter) were filled with the furthest value provided. Since dynamic inflation depends on changes in ω_t^c and ω_t^r, the truncation effectively corresponds to assuming that the forecasts of the interest rate and inflation more than two years ahead are time-invariant constants, which is roughly consistent with the central bank's self-described monetary policy horizon of two years, after which inflation is expected to return to the steady state. On the other hand, the truncation effectively abstracts from low-frequency structural changes that may have implications for the economy further in the future. Given the difficulty in accurately forecasting (or even measuring) such changes, this paper focuses mainly on the short and medium term dynamics, although the following section also briefly discusses the potential importance of long-run movements.

Compared to inflation and interest rates, the treatment of housing user costs is more challenging, since the theoretical concept of user costs that includes expected appreciation has no directly observable counterpart. The user cost can be decomposed as $U_t = P_t^h \psi_t$, where the second term, the relative user cost, corresponds to the ratio of user costs to the current house price and is given by

$$\psi_t = 1 - (1-\delta)\frac{1}{R_{t+1}}\frac{P_{t+1}^h}{P_t^h} \approx \delta + r_{t+1} - \Delta p_{t+1}^h, \tag{16}$$

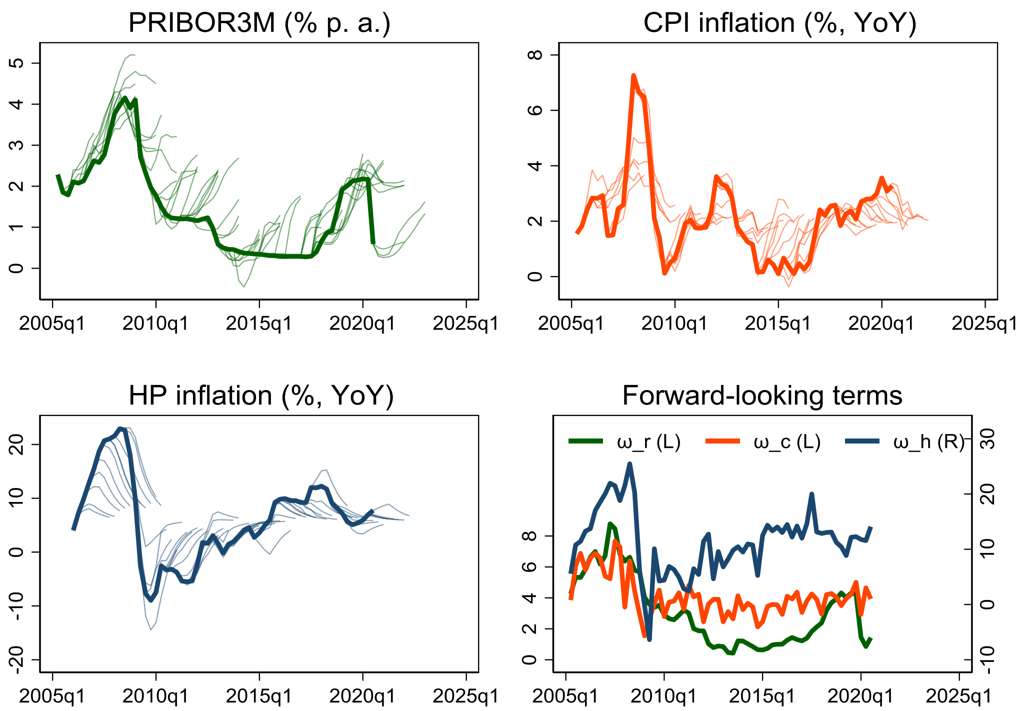
i.e., it is approximately equal to the depreciation rate plus the financial opportunity cost of owning the house minus the (expected) growth rate of house prices. User cost inflation, in log terms, can then be decomposed as $\pi_t^u = \pi_t^h + \pi_t^\psi$, i.e., inflation in house prices and in relative user costs. In principle, one could make an estimate of relative user costs conditional on forecasts of house price growth and use it to construct a time series for π_t^ψ. This, however, is problematic in practice, since

4 This truncation means that only the first eight terms in the sum are considered and the rest are ignored. Alternatively, if one assumed that the series will stabilize on its last forecasted value, it would be possible to impute the remainder of the sum from a geometric series. However, such an assumption does not seem particularly realistic and the resulting values of forward-looking terms would be too volatile in practice, so this approach is not followed here.

swings in expected house price appreciation will cause large relative changes in relative user costs, or may even drive them below zero. In the theoretical model discussed previously, such variation would be reflected in large and counterfactual changes in housing quantities, or even in arbitrage opportunities. The model abstracts from transaction and adjustment costs and thus is not sufficiently realistic for the concept of user costs to be approached literally in the data.

Another possible alternative would be to proxy user cost inflation by change in actual rents, or equivalently to proxy relative user cost inflation by growth in the rent-to-price ratio, since rents and user costs should be the same according to the model. This, too, is less than satisfactory, since in reality rental and owner-occupied housing are traded on distinct markets, and there is evidence that changes in rents are disconnected from the dynamics of estimated user costs (Verbrugge, 2008). Moreover, in the Czech Republic the share of households living in a property they own is almost 80%, i.e., quite high.[5] In the end, a pragmatic choice that is followed in this paper is to focus only on the first term, house price inflation π_t^h, while implicitly assuming that relative user costs are constant. In a somewhat different context, a similar logic is followed by the net acquisition approach, which is recommended by Eurostat for future treatment of owner-occupied housing in the HICP. Housing user costs are therefore proxied by the index of offered apartment prices published by the Czech Statistical Office. Forecasts of house price inflation are then obtained from an AR(2) model based on this series.

Figure 2: Input Data and Forward-Looking Terms



Note: Inflation rates show growth in prices compared to same quarter of previous year. Thin lines show corresponding forecasts from CNB Inflation Reports (PRIBOR, CPI) or AR(2) model (HP). Forward-looking terms are discounted sums of forecast trajectories (using quarter-to-quarter inflation rates).

[5] Data from EU-SILC (as reported by Eurostat) for 2017 show 78.5% of Czech households owning their dwelling; the average for the EU28 is 69.3%.

In its first three panels, Figure 2 plots the three main series used for constructing dynamic inflation (in the case of the CPI and house price (HP) inflation, the plots are on a year-on-year basis, i.e., summed over the last four quarters; however, all the computations are done on a quarter-on-quarter basis) and also shows the forecasts (from Inflation Reports or the AR(2) model) used for constructing the forward-looking terms. The last panel shows the forward-looking terms constructed as discounted sums of each forecast trajectory. Given the higher volatility of house prices, the corresponding term ω_t^h undergoes larger swings than the other two variables and is thus plotted on separate y-axis.

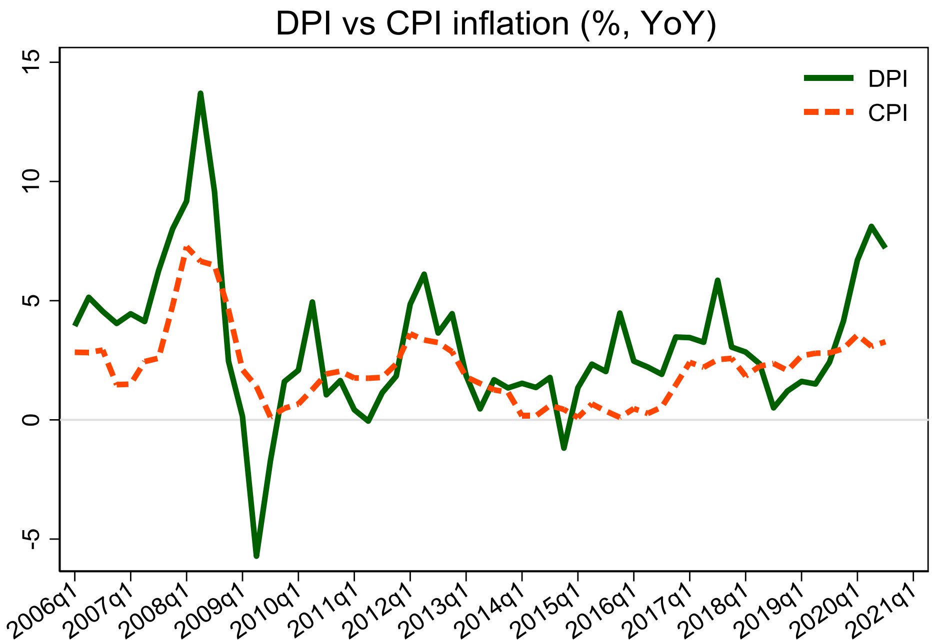
4. Results

Figure 3 plots the dynamic inflation series computed as in expression (15), together with headline CPI inflation, both in yearly terms (i.e., with respect to the same quarter of the previous year) to smooth out possible seasonal patterns. While the two series seem to evolve similarly in the sense that they mostly rise and fall at the same time, it is also clear that dynamic inflation is more volatile and undergoes larger swings than CPI inflation. This is especially noticeable in 2008–2009, when dynamic inflation initially exceeded 10% before falling to -5%. In addition, there seem to be a few episodes when dynamic inflation was systematically higher than CPI inflation: first in 2005–2007 during the pre-crisis boom, second during 2015–2018, and finally at the end of the sample, when dynamic inflation jumped up during the COVID-19 pandemic. Table 1 follows up by reporting some basic time-series moments of both series, which confirm that dynamic inflation is about twice as volatile and less autocorrelated than headline inflation. It is also about 1 percentage point higher in annual terms, averaging 3% over the sample period. Nevertheless, the two series remain positively correlated, and in year-on-year terms their correlation is quite high at 0.72.

Table 1: Moments of DPI and CPI Inflation

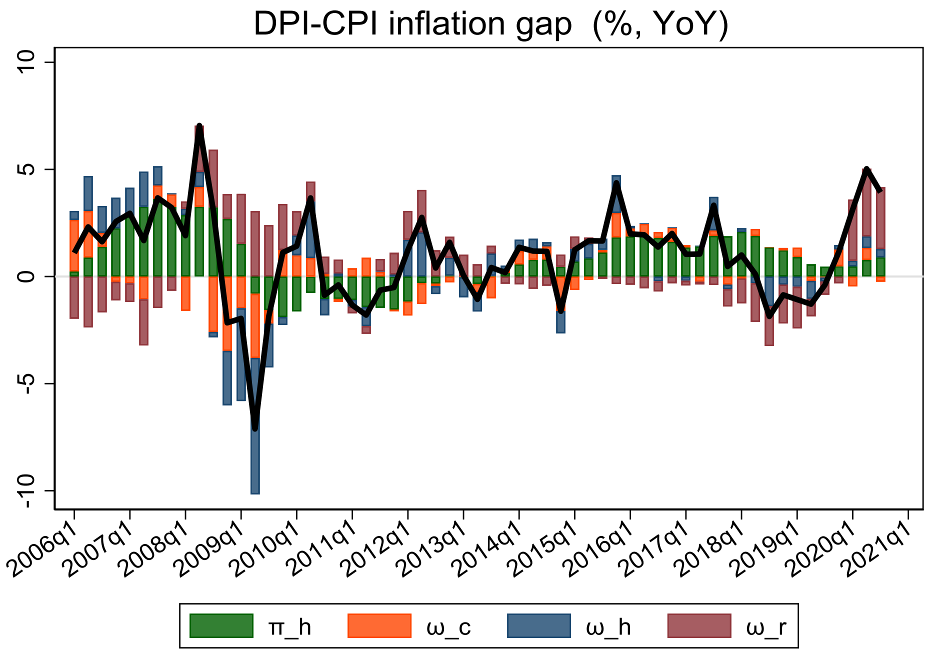
	sample	mean	st. dev.	autocorr(1)	autocorr(2)	correlation
DPI inflation (%, QoQ)	2005q2–2020q3	0.80	1.52	0.04	-0.09	0.25
CPI inflation (%, QoQ)		0.55	0.75	0.07	-0.05	
DPI inflation (%, YoY)	2006q1–2020q3	3.14	3.01	0.71	0.38	0.71
CPI inflation (%, YoY)		2.15	1.53	0.86	0.64	

Figure 3: DPI and CPI Inflation



In order to better understand the gap between dynamic and CPI inflation, Figure 4 plots the observed differential together with its decomposition into contributions from housing inflation (more specifically, the difference between the housing and CPI inflation rates) and from changes in the forecasts of consumption, housing prices, and financial returns. In 2005–2007, the persistent positive deviation of dynamic inflation was dominated by the static contribution of rapidly increasing house prices. The positive spike in 2008 was due to a coincidental combination of all contributing factors, while the following negative swing was driven by large downward revisions in the forecasts of future prices. On the other hand, the fall in expected interest rates provided a partially offsetting positive contribution to dynamic inflation, as households suffered from an effective increase in prices of future goods. In the subsequent couple of years, the gap was relatively small and transitory, oscillating between plus and minus one percentage point. In about 2015, dynamic inflation started to deviate systematically higher again due to another boom in housing prices. Interest rate hikes during 2018 began to work in the opposite direction, as households could benefit from greater saving opportunities offered by higher returns, although this trend was quickly reversed when the interest rate was quickly lowered again in 2020.

Figure 4: Decomposition of DPI/CPI Inflation Gap

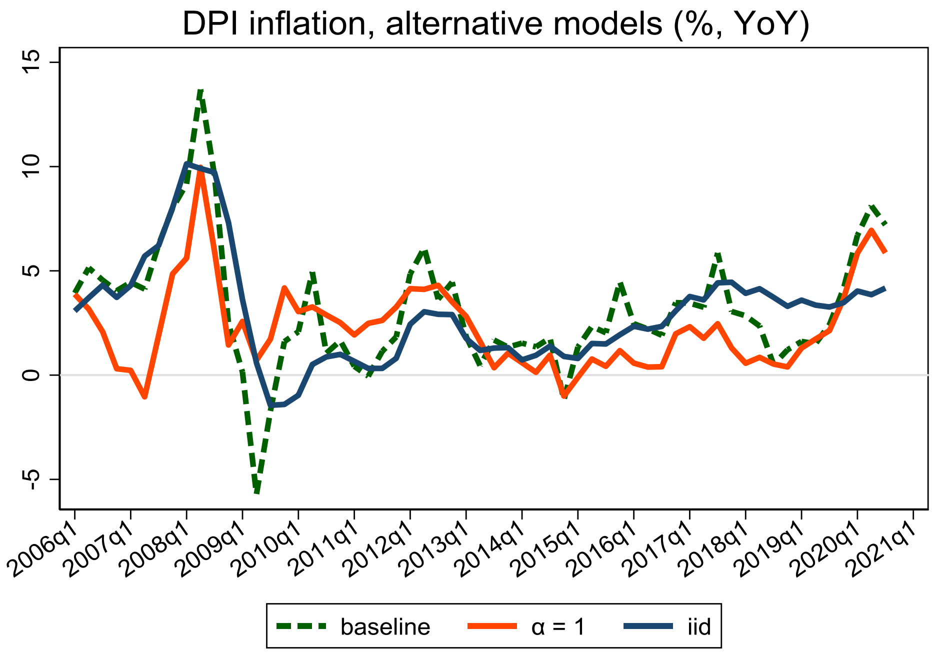


Note: Difference between DPI and CPI inflation over previous four quarters and corresponding contributions from short-run gap between housing and nondurable inflation (π_h) and from changes in forward-looking terms (ω^c, ω^h, ω^r).

Since the gap between the two inflation measures is due both to the inclusion of house prices and to forecast revisions, it might be interesting to consider the role that each of these two elements plays. Figure 5 plots two alternative versions of dynamic inflation, one that disregards housing ($\alpha = 1$) and one that includes housing but ignores the forward-looking terms, as if inflation and interest rates were completely uncorrelated over time (iid).[6] The figure suggests that the "iid" series follows baseline DPI inflation more closely, especially at the beginning and in the middle of the sample, where dynamic inflation was systematically high, and thus indicates the importance of accounting for contemporary housing price movements. However, disregarding expectations about future prices and returns would on the other hand cause us to miss some of the short-run fluctuations in dynamic inflation and understate its volatility, as well as to ignore the counteracting effects of the interest rate changes in 2009–2010 and 2018–2020. Therefore, neither of the two components of dynamic inflation should be disregarded.

[6] Another related issue is how much the DPI calculation is robust to smaller changes in parameters. Redoing the computation for a range of values of α from 0.7 to 0.9, and of β from 0.95 to 0.999, yields series (not shown) very close to the original one. Parameter α affects the weight of the housing terms in a linear way, so small changes do not affect the result much. Parameter β controls the discount rate in the forward-looking terms and could thus potentially have a larger effect for values close to 1. However in the current application, where we consider only the nearest eight quarters, its impact also turns out to be small.

Figure 5: DPI Inflation – Alternative Models



Note: "$\alpha = 1$" is model without housing. "iid" is model without forward-looking terms (as if expected inflation and return were constant).

Is it possible to obtain an approximation of dynamic inflation from static price indices? Table 2 shows the coefficients from a regression of dynamic inflation on the consumption and housing inflation rates. There is no constant term and the coefficients are constrained to sum to 1, so they can be interpreted as weights. The first column shows that the best approximation assigns a 30% weight to housing prices, which is more than their weight in the model given by parameter $1 - \alpha = 0.2$. This indicates that housing prices are more important in determining dynamic inflation, which can be intuitively explained by their greater persistence. As discussed previously, the static measure of nondurable inflation is the CPI, which does include a house price component within the category of imputed rents. The second column thus repeats the estimation with Eurostat's HICP measure for the Czech Republic, since the HICP does not cover imputed rent. The third column uses a variant of the HICP that excludes housing-related expenditures altogether. The results are almost identical in both cases. In terms of fit, however, all the regressions achieve R^2 of between 24% and 28%, suggesting that there is still a large amount of variation in dynamic inflation attributable to other sources, namely, interest rates and forecast revisions.

Table 2: Relationship between Dynamic Inflation and Static Indices

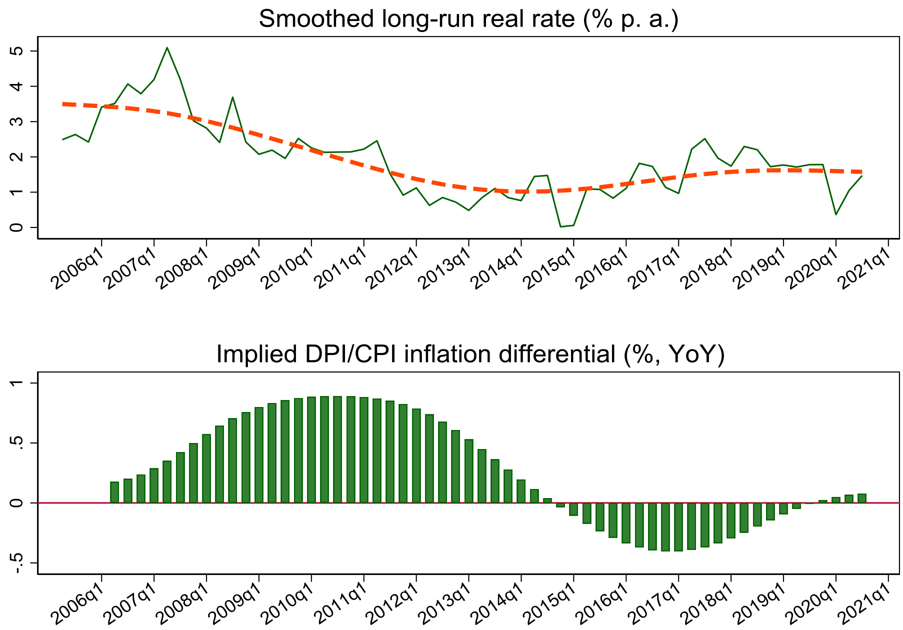
	(1) pi_d	(2) pi_d	(3) pi_d
pi_c	0.694 (0.072)		
pi_hicp		0.695 (0.071)	
pi_hicpxh			0.706 (0.068)
pi_h	0.306 (0.072)	0.305 (0.071)	0.294 (0.068)
R2	0.234	0.248	0.285
N	62	62	62

The preceding results and discussion were mostly focused on cyclical movements in prices and returns. Since the computation of forward-looking terms was truncated at the two-year horizon due to the availability of Inflation Report forecasts, the dynamic inflation measure will not capture revisions in expectations about the more distant future. This may potentially cause us to miss important effects of slowly evolving structural changes, such as shifts in the natural interest rate (see Hlédik and Vlček, 2018) or the perceived inflation target. Needless to say, such changes are not easy to observe in real time (or even in hindsight) and are much more difficult to predict reliably. Nevertheless, we can use the presented model to produce an illustrative estimate about the potential importance of such events. For simplicity, consider the model without housing ($\alpha = 1$), in which case it is easy to show that the DPI/CPI inflation gap can be written as

$$\pi_t^d - \pi_t^c = \Delta\omega_t^{r*},\ \omega_t^{r*} = \sum_{i=1}^{\infty} \beta^i (r_{t+i} - \pi_{t+i}^c), \tag{17}$$

that is, as a change in expectations about the (nontruncated) future path of real returns. Focusing on long-run expectations about the real return, Figure 6 plots the difference between the expected interest rate and CPI inflation eight quarters ahead, together with its smoothed trend obtained from the Hodrick-Prescott filter. The smoothed long-run real return slowly oscillates around 2%, with the time distance between the peak and crossing the average level being perhaps five years. The bottom panel shows the implied DPI/CPI inflation gap if households were subject to this smoothed rate of return and at every point expected it to decay slowly toward the 2% steady-state level. More specifically, the figure assumes that households predict geometric convergence with autocorrelation 0.9, implying that about 90% of the current deviation would be undone in five years. We see that even under such relatively mild and less-than-permanent movements in the long-run real rate, the implied differential reaches almost 1% per year and its cumulative effects on the cost of living would be substantial over multiple years. While these costs might eventually be reversed over an even longer timescale of decades, these results suggest that long-lasting structural changes in the economy can also potentially imply substantial impacts on the cost of living of intertemporally optimizing households.

Figure 6: Effect of Movements in Long-Run Real Rate of Return



Note: Long-run real rate is difference between expected interest rate and CPI inflation eight quarters ahead. Smoothed series is obtained with Hodrick-Prescott filter ($\lambda = 1600$). Lower panel shows implied gap between DPI and CPI inflation in model without housing and where households expect slow convergence of real rate from current smoothed trend toward 2%.

5. Conclusion

Inflation is one of the most important macroeconomic indicators, but the debate about how it should account for house or asset prices is still ongoing. The computation of a cost of living index for an intertemporally optimizing consumer offers a natural setting for treating these issues. The goal of this paper has therefore been to construct a measure of inflation that explicitly accounts for changes in household purchasing power caused by variations in prices of both current and future consumption. The model derived for dynamic inflation combines, in a tractable way, contemporary movements in prices with news about the future paths of prices and interest rates. In the case of the Czech Republic, such inflation measure does not dramatically diverge, at least most of the time, from headline CPI inflation, but there are still noticeable differences between the two series. By giving explicit weight to housing prices, dynamic inflation can deviate from CPI inflation during housing booms or crashes. Responding to news about future prices introduces additional volatility but also more forward-looking behavior into the dynamic inflation series. On the other hand, news about interest rates often seems to push dynamic inflation in the opposite direction than current price movements. Finally, the impact of long-term expectations opens the way to potential large effects of even slow but persistent shifts in the economy.

Of course, the measure put forward in this paper is not perfect. The underlying model of consumption-saving optimization is very simple and abstracts from important features, such as explicit treatment of labor income, nondivisibilities and adjustment costs associated with housing transactions, uncertainty, and risk premia. More research on the treatment of housing user costs in a manner consistent with the underlying theory would be especially desirable, although that may require us to enrich the household model at the cost of less tractability. Another possibly interesting direction would be to consider more general forms of utility functions that allow, for example, for a varying degree of intertemporal substitution, thus possibly leading to different intertemporal tradeoffs considered by the household. Finally, the potential implications of long-term changes in the economy on the intertemporal cost of living, which may have important generational effects given the relevant timescales, also deserve further attention.

References

ALCHIAN, A. A. AND B. KLEIN (1973): "On a Correct Measure of Inflation." *Journal of Money, Credit and Banking 5 (1)*, pp. 173–191.

AOKI, S. AND M. KITAHARA (2010): "Measuring a Dynamic Price Index Using Consumption Data." *Journal of Money, Credit and Banking 42 (5)*, pp. 959–964.

BRYAN, M. F., S. G. CECCHETTI, AND R. O'SULLIVAN (2002): "Asset Prices in the Measurement of Inflation." NBER Working Paper 8700.

DIEWERT, W. E. (2002): "Harmonized Indexes of Consumer Prices: Their Conceptual Foundations." *Swiss Journal of Economics and Statistics 138 (IV)*, pp. 547–637.

DIEWERT, W. E. AND A. O. NAKAMURA (2009): "Accounting for Housing in a CPI." Federal Reserve Bank of Philadelphia Working Paper 09-4.

FRAIT, J. AND L. KOMÁREK (2007): "Monetary Policy and Asset Prices: What Role for Central Banks in New EU Member States?" *Prague Economic Papers 16 (1)*, pp. 3–23.

GOODHART, C. (2001): "What Weight Should be Given to Asset Prices in the Measurement of Inflation?" *The Economic Journal 111 (472)*, pp. 335–356.

HAMPL, M. AND T. HAVRÁNEK (2017): "Should Inflation Measures Used by Central Banks Incorporate House Prices? The Czech National Bank's Approach." CNB Research and Policy Note 1/2017.

HLÉDIK, T. AND J. VLČEK (2018): "Quantifying the Natural Rate of Interest in a Small Open Economy – The Czech Case." CNB Working Paper 7/2018.

KONÜS, A. A. (1939): "The Problem of the True Index of the Cost of Living." *Econometrica 7 (1)*, pp. 10–29.

POLLACK, R. A. (1975): "The Intertemporal Cost of Living Index." *Annals of Economic and Social Measurement 4 (1)*, pp. 179–198.

REIS, R. (2005): "A Dynamic Measure of Inflation." NBER Working Paper 11746.

RŮŽIČKA, L. AND R. ŠNOBL (2017): "An Experimental Price Index Including Prices of Older Properties." In: Czech National Bank Inflation Report III/2017.

SHIBUYA, H. (1992): "Dynamic Equilibrium Price Index: Asset Price and Inflation." *Monetary and Economic Studies 10 (1)*, pp. 95–109.

SHIRATSUKA, S. (1999): "Asset Price Fluctuation and Price Indices." *Monetary and Economic Studies 17 (3)*, pp. 103–128.

VERBRUGGE, R. (2008): "The Puzzling Divergence of Rents and User Costs." *Review of Income and Wealth 54 (4)*, pp. 671–699

VICKERS, J. (2000): "Monetary Policy and Asset Prices." *The Manchester School 68 (5)*, pp. 1–22.

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