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The Mirage of Falling R-stars

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The Mirage of Falling R-stars

Aleš Bulíř and Jan Vlček*

Abstract

Was the recent decline in real interest rates driven by a diminishing natural real interest rate, or have we observed a long sequence of shocks that have pushed market rates below the equilibrium level? In this paper we show on a sample of 12 open economies that once we account for equilibrium real exchange rate appreciation/depreciation, the natural real interest rate in the 2000s and 2010s is no longer found to be declining to near or below zero. The explicit inclusion of equilibrium real exchange rate appreciation in the identification of the natural rate is the main deviation from the Laubach-Williams approach. On top of that, we use a full-blown semi-structural model with a monetary policy rule and expectations. Bayesian estimation is used to obtain parameter values for individual countries.

Abstrakt

Byl nedávný pokles reálných úrokových sazeb způsoben klesající přirozenou reálnou úrokovou sazbou, nebo jsme pozorovali dlouhou sekvenci šoků, které stlačily tržní sazby pod rovnovážnou úroveň? V tomto článku na vzorku 12 otevřených ekonomik ukážeme, že jakmile zohledníme zhodnocení/znehodnocení rovnovážného reálného kurzu, přirozená reálná úroková míra prvních dvou desetiletích 21. století již neklesá blízko k nule nebo pod nulu. Explicitní zahrnutí apreciacie rovnovážného reálného kurzu do identifikace přirozené úrokové sazby je hlavní rozdílem oproti Laubachovu-Williamsovu přístupu. Kromě toho používáme plnohodnotný semistrukturální model s pravidlem měnové politiky a očekáváními. K parametrizaci modelu pro jednotlivé země se používá bayesovský odhad.

JEL Codes: E43, E52, E58

Keywords: Equilibrium real appreciation, natural rate of interest (r-star), Penn effect, zero lower bound.

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Nontechnical Summary

In recent years, many economies have seen a significant decline in real interest rates. This has sparked a debate about whether this trend is due to a fundamental decrease in the natural real interest rate, r^* , or is a result of persistent economic shocks. The natural real interest rate is critical for guiding monetary policy, as it represents the equilibrium rate at which inflation neither accelerates nor decelerates. Understanding the true behavior of the natural real interest rate would help central banks and policymakers navigate economic conditions more effectively.

The paper argues that by accounting for equilibrium real exchange rate appreciation or depreciation, the decline in r^* observed during the 2000s and 2010s should be reinterpreted. Specifically, once equilibrium real exchange rate developments are considered, r^* does not appear to be falling to zero or negative levels as previously argued. Instead, it remains relatively stable, generally hovering around 2–3 percent for most advanced and emerging economies, with a slight decline in the early 2020s. Given the sizable negative difference between observed and natural real interest rates, this implies that monetary policy in most sample countries was continuously loose during the 2013–2022 period.

The evidence in the paper is based on an examination of 12 small open economies, both advanced and emerging, over the past two decades. To identify the natural rate of interest, we deviate from the commonly used methodology of Laubach-Williams in several aspects.

First, we adjust the natural rate of interest for appreciation of the equilibrium real exchange rate (the “Penn effect”) and we use an open-economy IS schedule, which is more relevant for small open economies experiencing β convergence. In countries with equilibrium appreciation, r^* should be lower, as nonresidents collect a part of the “convergence gain” through the return on the exchange rate rather than through higher GDP growth and the corresponding profits. In contrast, equilibrium depreciation lowers yields expressed in the foreign currency and nonresidents demand a higher return on country assets, thus pushing r^* up. This is the major deviation from the Laubach and Williams (2003) methodology, where the natural real interest rate is linked only to the growth rate of potential output.

Second, we use a full-blown semi-structural model closed by a monetary policy rule, in contrast to the HLW framework, which has no expectations in the model and no policy rule. Third, although we impose a canonical semi-structural model on all the sample countries, we use Bayesian estimation to obtain parameter values for individual countries. The equations are modified to capture alternative monetary regimes and idiosyncratic features of advanced, emerging market, and low-income countries.

1. Introduction

Was the recent decline in real interest rates driven by a decline in r^* , the natural real interest rate? Or have we observed a long sequence of shocks—from monetary and fiscal policy, demographics, and globalization—that have pushed the market and policy interest rates below the equilibrium level? The answer to this question matters for the conduct of monetary and fiscal policies, past and future. In this paper we argue that once we account for equilibrium real exchange rate appreciation/depreciation, the natural real interest rate in the 2000s and 2010s is no longer found to be declining to near or below zero in most countries. If anything, during the first two decades of the 21st century the natural rate of interest for many advanced and emerging market economies appears to have been stable at around 2–3 percent, with a modest decline in the early 2020s.

The calculation of r^* matters a lot for policymaking. Fed Chairman Powell (2018) likened monetary policymakers to sailors navigating by the “stars” when plotting a course for the economy, referring to concepts such as the natural real interest rate, r^* . Blanchard (2022) wrote: “The mandate of central banks is to set the actual safe real interest rate, r , as close as they can to the neutral interest rate, r^* , and in so doing, keep output close to potential output.”¹ Underestimate the natural rate for your country and your country’s monetary conditions will be too loose and likely lead to overheating and higher-than-desired inflation. Overestimate r^* and monetary conditions will be too tight and likely lead to higher unemployment and undershooting of the central bank’s inflation target.

What is r^* and how can we measure it? In new Keynesian models the natural real interest, r^* , is equivalent to the growth rate of output in the closed-economy Euler equation. Over time, r^* should converge to the long-run average of observed real interest rates, either as the theoretical return absent policies and shocks, as the actual return without cyclical factors, or simply as the real interest rate path at very low frequencies, perhaps driven by the integration of financial markets across regions (Miranda-Agrippino and Rey 2022). Laubach and Williams (2003) and Holston, Laubach, and Williams (HLW, 2017) brought together these two strands of literature by extracting trends in real interest rates and in output growth and tying them together using the closed-economy IS schedule. Their methodology has been replicated for many countries, although most of them are classified as small open economies. The HLW closed-economy approach is problematic for open economies that have received massive capital flows which have (1) influenced the structure of their economies and increased real growth and (2) led to real appreciation of their currencies.

While real interest rates have declined relative to the 1990s and stayed low in the post-pandemic period, there is little agreement on why r^* should be declining or close to zero. The secular decline has been attributed to four developments (Goodhart and Pradhan 2020): (1) changes in the natural rate of interest; (2) effects of the “savings glut”; (3) easy monetary policy and unconventional monetary policy in the aftermath of the Global Financial Crisis (GFC); and (4) lower and more stable inflation prior to the COVID-19 shock. Goodhart and Pradhan were skeptical that these conditions could prevail in the long run. In contrast, IMF (2023) and Blanchard (2022) have argued that once the current inflationary episode has passed, interest rates are likely to revert toward pre-pandemic levels in advanced countries. Reis (2022) has reasoned that the long-standing focus on

¹ In the literature, the terms natural rate and neutral rate have often been used interchangeably. Our preference is for the former, where the natural real interest rate refers to the real rate of interest that would prevail in a long-run equilibrium where there are no price rigidities or other frictions (Obstfeld 2023).

the decline in short-term bond rates, or “safe rates”, is misplaced and has demonstrated that measures of the return on private aggregate capital have remained roughly constant, or have even increased slightly, since the Global Financial Crisis.

Our contribution has two dimensions. First, we adjust the natural rate of interest for appreciation of the equilibrium real exchange rate (the “Penn effect”) and we use an open-economy IS schedule, which is more relevant for small open economies with β convergence. Most economies are not in a steady state and experience some convergence or divergence either in per capita income or in equilibrium real exchange rates vis-à-vis their advanced-economy peers. These convergence gains have been approximated by trend (equilibrium) real appreciation (Samuelson 1994). In countries with equilibrium appreciation, r^* should be lower, as nonresidents collect a part of the “convergence gain” through the return on the exchange rate (rather than through higher GDP growth). In contrast, equilibrium depreciation should lower yields in the foreign currency and nonresidents would demand a higher return on country assets, thus pushing r^* up.

Second, we use a full-blown semi-structural model closed by a monetary policy rule, in contrast to the HLW framework, which has no expectations in the model and no policy rule. Third, although we impose a canonical semi-structural model on all sample countries, we use Bayesian estimation to obtain parameter values for individual countries. The equations are modified to capture alternative monetary regimes and features of advanced, emerging market, and low-income countries.

The framework allows us to answer two policy-relevant questions in the small open-economy framework. First, has the natural rate of interest been broadly stable before, during, and after the Great Recession or has it drifted downward? Second, conditional on the natural rate estimates, has monetary policy been too loose in response to the Great Recession and the accompanying low-inflation environment?

Our results suggest that the natural rate of interest has been stable and higher than when estimated using the HLW framework, with the difference being mostly attributable to real equilibrium exchange rate developments. The average r^* difference of about 150 basis points between the HLW framework and the open-economy framework suggests that monetary policy informed by HLW estimates of natural rate of interest may have been too loose during the pre- and post-pandemic period. Only in a couple of countries do we find r^* dipping below the zero lower bound, and only at the very end of the sample, at the height of the pandemic.

We begin by outlining our extensions to the HLW framework, Bayesian estimation, and our sample. The next section compares alternative estimates of the natural real rate of interest, comparing our estimates with those obtained using the HLW framework. The penultimate section suggests some policy implications and the final section concludes.

2. The Analytical Framework

2.1 The HLW Framework and Its Applications

The unobserved component model proposed by HLW has been the empirical workhorse of the r^* literature. The framework starts with the neoclassical growth model and its Euler equation: the

natural rate of interest varies over time in response to shifts in preferences and the growth rate of output. To model r^* as a time-varying process, HLW combine a new Keynesian Phillips curve with an IS schedule, arriving at

$$r_t^* = g_t^* + z_t$$

where g_t^* is the trend growth rate of the natural rate of output and z_t captures all other determinants of r^* , including, for example, the real exchange rate. Empirically, the closed-economy real output gap is modeled as an autoregressive process that is also affected by movements in the short-term real interest rate, r_t , and the natural real interest rate, r^* . External developments are captured by the rest-of-the-world output gap.

The HLW framework did not explicitly capture the salient features of small open economies, specifically the role of the real exchange rate in output and inflation, and it was soon extended.² Berger and Kempa (2014) introduced the real effective exchange rate, q_t , expressing it as a deviation from its equilibrium level.³ The exchange rate enters the Phillips and IS schedules and thus affects inflation directly through its first difference and indirectly through the output gap (Ball 1999). They close the model by tying the real interest rate gap, \hat{r}_t , to the real exchange rate gap, \hat{q}_t : $\hat{r}_t = \gamma \hat{q}_{t-1} + \kappa_{t-1}$, where the error term, κ_t , captures time-varying risk premia and other distortions in international capital markets. However, by leaving the equilibrium HLW relationship as $r_t^* = g_t^* + z_t$, the authors create an inconsistency between the exchange rate channel in the IS schedule in the underlying structural model and the HLW relationship.

The modifications of the HLW framework did not question the assumption that r^* depends solely on economic growth. The implication is that fast-growing, emerging market countries exhibit higher r^* by construction and the HLW framework then struggles to explain the decline in observed real rates in emerging market countries. Arena and others (2020) reported that around one-half of the overall estimated decline in r^* since the GFC must be attributed to the behavior of the unexplained component, z_t , rather than to a change in g^* . In our view the unexplained component is mostly equilibrium real appreciation/depreciation.

We account explicitly for the impact of equilibrium movements in the exchange rate. The ‘‘Penn effect’’ established a link between equilibrium real growth and equilibrium appreciation: a country ‘‘converging’’ in per capita GDP terms experiences real appreciation (Samuelson 1994). The gain from the ‘‘convergence process’’ in West Germany in the 1960s and 1970s, in the Asian Tigers in the 1970s and 1980s, and in the European transition countries in the 1990s and 2000s was split between a faster rate of growth of real GDP and real exchange rate appreciation. De Broeck and Sløk (2006) argued that about two-fifths of the real convergence was realized through equilibrium real appreciation. Ignoring equilibrium appreciation in the closed-economy HLW framework will overestimate the natural rate of interest in fast-growing countries that face equilibrium appreciation and underestimate r^* in countries with equilibrium depreciation.

² HLW-inspired extensions for small open economies include ones for Canada (Berger and Kempa 2014), the Czech Republic (Hlédik and Vlček 2018), Denmark (Pedersen 2015), and Slovakia (Kupkovič 2020).

³ The real exchange rate, q_t , is the nominal rate, s_t , adjusted for the ratio of the foreign price level, p_t^F , to the domestic price level, in logs: $q_t = s_t + p_t^F - p_t$. A negative value of Δq_t implies appreciation of the real exchange rate.

2.2 The HLW Framework and Its Applications

How does the Penn effect relate to r_t^* ? On the one hand, trend GDP growth accounts for capital yields from production. On the other hand, real appreciation accounts for investment yields realized in foreign currency. Empirically, real exchange rate appreciation has been closely linked to FDI inflows and associated productivity gains (Bulíř and Šmídková 2005; Bini Smaghi 2007; Babecký, Bulíř, and Šmídková 2009). First, following Hlédik and Vlček (2018) we extend HLW for equilibrium real appreciation, q_t^* . Second, we assume no stochastic term in the natural rate equation with persistence:

$$r_t^* = \rho r_{t-1}^* + (1 - \rho)(2c_1(c_2 g_t^* + (1 - c_2)q_t^*)), \quad (1)$$

where the contributions of the potential growth and equilibrium exchange rate components add up to unity. We estimate parameters c_1 and c_2 below and cannot reject the null hypothesis of them being equal to roughly 1 and $\frac{1}{2}$, respectively, in our sample countries (as in Arena and others 2020, De Broeck and Sløk 2006).

The addition of the real exchange rate variable—in both trends and gaps—necessitates extending the underlying modeling framework also for uncovered interest rate parity and the policy reaction function. The full model therefore resembles the semi-structural quarterly projection model (QPM) used in many central banks—see Berg and others (2006) for a canonical version. The model used here consists of four key equations as follows:

The Phillips curve

$$\pi_t = a_1 \pi_{t-1} + (1 - a_1) \pi_{t+1} + a_2 RMC_t + \varepsilon_t^\pi, \quad (2)$$

where π is quarter-on-quarter inflation, RMC are real marginal costs, and ε^π is a cost push shock. The RMC variables are expressed as deviations from the long-term trends, denoted by “ \wedge ”. Specifically, they are defined as a weighted sum of the output gap, \hat{y} , and the real exchange rate gap, \hat{q} :

$$RMC_t = a_3 \hat{y}_t + (1 - a_3) \hat{q}_t. \quad (3)$$

Aggregate demand

$$\hat{y}_t = b_1 \hat{y}_{t-1} - b_2 MCI_t + b_3 \hat{y}_t^F + \varepsilon_t^{\hat{y}}, \quad (4)$$

where MCI is the monetary conditions index, \hat{y}^F is the foreign output gap, and $\varepsilon^{\hat{y}}$ is a demand shock. The monetary conditions index combines the real interest rate gap, \hat{r} , and the real exchange rate gap, \hat{q} , with the weight of b_4 of the real interest rate gap and $(1 - b_4)$ of the real exchange rate gap:

$$MCI_t = b_4 \hat{r}_t + (1 - b_4)(-\hat{q}_t) \quad (5)$$

Uncovered interest rate parity

$$s_t = h_2(s_{t-1} + \Delta s^{TAR}) + (1 - h_2)((1 - e_1)s_{t+1}^e + e_1(s_{t-1} + 2(\pi^{TAR} - \pi^{TAR,US} + \Delta q^*)) + (-i_t + i_t^{EA} + prem_t)/4) + \varepsilon_t^s, \quad (6)$$

where s is the nominal exchange rate, Δs^{TAR} is the exchange rate depreciation target, π^{TAR} is the domestic inflation target, i is the domestic policy interest rate, i^{EA} is the ECB policy rate, $prem$ is the risk premium, and ε^S is a UIP shock. The specification allows us to model a range of alternative exchange rate regimes with and without foreign exchange interventions that either smooth the exchange rate trajectory (e_1) or target a specific exchange rate trajectory (h_2).

Policy reaction function

$$i_t = h_1(4(s_{t+1} - s_t) + i_t^{EA} + prem_t) + (1 - h_1)(g_1 i_{t-1} + (1 - g_1)(r_t^* + \Delta_4 \pi_{t+3}) + g_2(\Delta_4 \pi_{t+3} - \pi^{TAR}) + g_3 \hat{y}_t) + \varepsilon_t^i, \quad (7)$$

where ε^i is a monetary policy shock. Again, this extended specification allows us to model rate-setting environments that are either focused on inflation stability ($h_1 = 0$) or combine inflation and exchange rate stability objectives. In Appendix A we provide a complete overview of our parametrization of the model under different monetary policy regimes.

2.2.1 Estimation

We strive to minimize country-specific adjustments to the model and its coefficients, limiting our ad hoc changes to modifications to the policy rule and to the UIP to reflect the exchange rate and monetary policies. To this end, Bayesian estimation gives us a disciplined rule-based calibration and estimation framework. We start by pre-filtering the foreign data, namely, real GDP and the real rate, using the Hodrick-Prescott filter.⁴ The rest of the trends and gaps, including the inflation objective/target, are identified in the full model by the Kalman filter jointly with the model parameters, including their standard deviations, which are identified using Bayesian estimation.⁵ The prior distributions of the individual parameters are set at either $\frac{1}{4}$ or $\frac{1}{2}$ of the initial parameter calibration or the empirical standard deviation of the series (Appendix A). Robustness checks are provided in Appendix B. The country-specific results are summarized in Appendix C.

2.2.2 The Sample and Data

Our sample contains 12 countries in or near Europe that trade principally with the euro area countries. Their currencies are either floating or pegged to the euro, covering a range of monetary policy regimes, from floats and inflation targeting through “stabilized regimes” to a currency board (Table 1). The sample is also diverse in terms of economic development—the richest country, Switzerland, has a GDP per capita more than eight times higher than the poorest, Morocco. The rest-of-the-world trading partner is therefore the euro area, and the relevant exchange rate is the domestic currency against the euro.

We use seasonally adjusted quarterly data for our sample countries from 2002Q1 to 2022Q4 from Eurostat and national statistical offices. The series for Türkiye, Serbia, North Macedonia, Bosnia and Herzegovina, and Morocco are slightly shorter, based on availability. Inflation is the annualized

⁴ Prefiltering the foreign variables ensures that the foreign gaps are the same across countries when the Kalman filter is applied.

⁵ The identification of the inflation objective begins with the official inflation target, if available. However, the final estimate of the inflation objective is obtained using the Kalman filter, as in Ireland (2007). Intuitively, should inflation stay above or below the target for an extended period, the official target may cease to be credible.

first difference of the log of seasonally adjusted core CPI,⁶ output is quarterly real GDP in natural logs multiplied by 100, and the exchange rate is the country's real exchange rate in domestic currency terms (a decline denotes appreciation of the domestic currency against the euro). Our measure of the (ex post) average real interest rate is based either on the policy rate or on an interbank rate that has closely mirrored the policy rate (see Appendix C for the interest rate definitions for our sample countries). All data are either from Eurostat or from central bank websites.

Table 1: The Sample Countries and Their Characteristics

Country	ISO code	GDP per capita, PPP US\$	Average real interest rate	Exchange rate and monetary framework	Sample period
Switzerland	CHE	70,764	-0.3	Crawl-like arr.	2002Q1–2022Q4
Norway	NOR	64,443	0.3	Float/IT	2002Q1–2022Q4
Sweden	SWE	54,240	-0.2	Float/IT	2002Q1–2022Q4
United Kingdom	GBR	45,989	-0.7	Float/IT	2002Q1–2022Q4
Czech Republic	CZE	40,917	-0.9	Float/IT	2002Q1–2022Q4
Poland	POL	34,587	1.4	Float/IT	2002Q1–2022Q4
Hungary	HUN	33,863	1.8	Float/IT	2002Q1–2022Q4
Türkiye	TUR	31,753	-2.6	Float/IT	2010Q1–2022Q4
Serbia	SRB	19,695	3.3	Stabilized ER/IT	2007Q1–2022Q4
North Macedonia	MKD	16,372	6.4	Soft peg	2006Q1–2022Q4
Bosnia and Herzegovina	BIH	14,813	1.7	Currency board	2015Q1–2022Q4
Morocco	MAR	8,353	1.1	Peg with a band	2008Q1–2022Q4

Note: 2021 GDP per capita is measured in purchasing power parity, 2017 international dollars; the real interest rate is measured as the mean of the policy interest rates *minus* the one-period ahead rate of inflation during 2008–2022; IT denotes inflation targeting. The classification of the monetary regime is based on the IMF's 2021 AREAER Database (IMF 2022).

Source: IMF, World Economic Outlook Database, October 2022; IMF, Annual Report on Exchange Arrangements and Exchange Restrictions (AREAER), 2021; Eurostat; authors' calculations.

The decline in global real interest rates—policy rates as well as short- and long-term market rates—is a well-established fact (Gamber 2020; Goodhart and Pradhan 2020; and Rogoff, Rossi, and Schmelzing 2022). After peaking at about 4 percent in the early 1990s, real rates have continued their downward trend over the following three decades. After the GFC, real policy and short-term interbank rates dropped close to or below zero in almost all industrial countries.

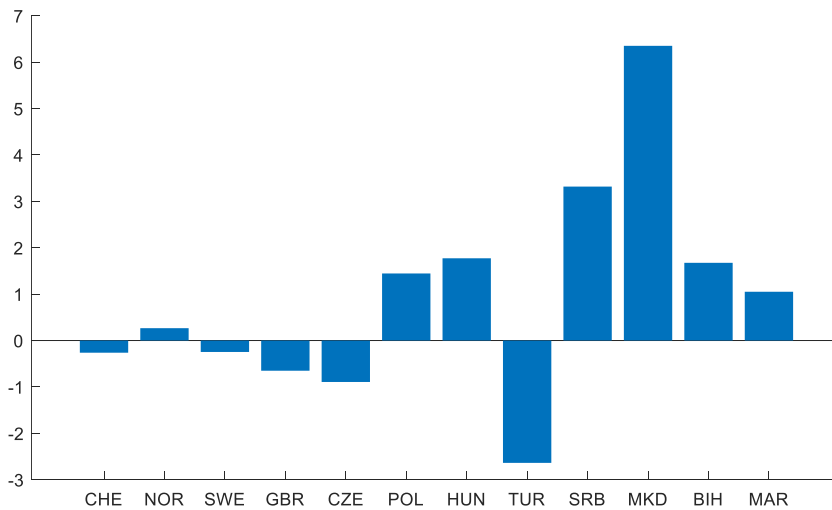
The sample central banks chose to keep their low post-GFC real interest rates during the pandemic and early post-pandemic period (Figure 1). Excluding Bosnia and Herzegovina with its domestic political instability and correspondingly high sovereign risk premium, only in Serbia and North Macedonia in 2008–2022 did real policy rates stay close to 3 percent, a level that used to be considered appropriate for emerging market countries. In the subsample of EU countries—all with

⁶ The definition of the core CPI is country specific. The estimates using headline inflation were not materially different from those using core inflation.

central banks practicing inflation targeting—the average real policy rate was minus 50 basis points. Such low real rates mostly reflect nominal policy rates being brought close to the zero lower bound rather than above-target inflation. Only at the beginning and end of the sample period was the rate of inflation higher than targeted.

Moreover, market participants—such as the respondents to the Consensus Forecasts surveys—have expected real interbank and bond rates to stay low well past the pandemic. In each of the four sample countries for which Consensus Forecasts nominal rate projections are available—Norway, Sweden, Switzerland, and the UK—the 2022 average of five-year ahead expectations of real interest rates was well below 1 percent. Five-year ahead projections of interest rates and inflation should be free of cyclical effects, so the real interest rate series are a good proxy for market expectations of the natural rate. It is reasonable to assume that market expectations of r^* would be comparable for the rest of our sample.

Figure 1: Real Policy Interest Rate, 2008–2022 (in percent)



Note: The average ex-post real policy interest rate is calculated as the mean of the nominal policy rate minus one-period ahead inflation. The sample for Türkiye starts in 2010 and that for Bosnia and Herzegovina in 2015.

Source: Eurostat, national central banks; authors’ calculations.

3. Estimates of the Natural Rate of Interest

In this section we present our estimates of the natural real interest rate. These fall into two groups: (1) time-invariant estimates (assuming that both trend real output growth and trend real appreciation/depreciation were constant during the sample period); (2) time-varying estimates (assuming that both trends were stochastic). We then compare our results with those of HLW.⁷ To avoid polluting our estimates with the COVID-19 shock, our Bayesian estimation of the model parameters employs data up to the first quarter of 2020.

⁷ All Matlab codes are presented in an online appendix available at: https://ales-bulir.wbs.cz/code/bulir_vlcek_code.zip

3.1 Time-Invariant Estimates of r^*

The time-invariant r^* was positive in all countries and well above 1 percent (Table 2). These estimates are based on a restrictive assumption that both the trend (potential) rate of real output growth, g^* , and the trend (equilibrium) real exchange rate appreciation, q^* , were constant at the sample average during 2002–2022. The advantage of this assumption is that we can easily compute the sample contribution of potential growth and equilibrium real exchange rate appreciation to the r^* estimate. We will relax the restrictive assumption of constant trend rates of growth in the next section.

Our starting hypothesis that r^* ought to be lower, other things being equal, in countries with equilibrium appreciation—that is, “converging countries” experiencing the Penn effect—is broadly confirmed. But let us mention the outliers first. Switzerland, the richest country in the sample, experienced persistent real appreciation, as the Swiss franc acted as a “safe haven” currency. The (presumably disequilibrium) appreciation pushed r^* down by 80 basis points (see Table 2).⁸ Poland and Türkiye, the two fast-growing countries, bucked the Penn effect—their currencies depreciated in real terms during the sample period. As a result, their period-average r^* estimates are high at 5.4 percent and 9.5 percent, respectively.

Table 2: Country Estimates of Time-Invariant r^* and Its Components, 2002Q1–2022Q4
(in percent)

Modified r^* equation: $\bar{r}^* = 2c_1(c_2\bar{g}^* + (1 - c_2)\bar{q}^*)$

Estimation period: 2002Q1–2022Q4

Country	r^*	g^*	q^*	c_1	c_2
Switzerland	0.6	1.7	−1.0	0.8	0.5
Norway	2.8	1.6	0.8	1.1	0.6
Sweden	2.8	2.1	0.8	1.0	0.4
United Kingdom	1.9	1.4	0.6	1.0	0.4
Czech Republic	1.5	2.8	−1.1	0.9	0.5
Poland	5.4	3.8	0.7	1.1	0.6
Hungary	2.5	2.4	0.1	1.0	0.5
Türkiye	9.5	5.7	5.0	0.9	0.5
Serbia	2.0	2.3	−0.6	1.0	0.5
North Macedonia	3.4	2.3	−0.1	1.3	0.6
Bosnia and Herzegovina	2.2	3.0	−0.7	1.1	0.5
Morocco	2.3	3.0	−0.4	1.0	0.5

Note: The estimate of the average (\bar{r}^*) is evaluated at the sample average for potential growth (g^*) and appreciation (q^*). A negative value of g^* implies real appreciation, as the exchange rate is expressed in units of the domestic currency for one euro, with parameters c_1 and c_2 obtained from the Bayesian estimates. We multiply the second term by 2 to check that we obtain a c_1 of 1 as in HLW and to test whether the contribution of equilibrium real appreciation is indeed about $\frac{1}{2}$ (we use the β -distribution to test the hypothesis that $c_2 = 0.5$). See Appendix I and II for the Bayesian priors and estimation results.

Source: Authors’ calculations.

⁸ The calculation of the contribution of trend appreciation to r^* is as follows: the equilibrium appreciation of the Swiss franc, q^* , is estimated at −1.0 percent per annum, c_1 is estimated at 0.8, and c_2 is estimated at 0.5. The contribution of q^* to r^* is therefore $2c_1(1 - c_2)(\bar{q}^*)$, or $2 * 0.8 * (1 - 0.5) * -1 = -0.8$ percent, or 80 basis points.

For the rest of the sample, we cannot reject our hypothesis of lower r^* in fast-growing, converging countries as compared to slower-growing, advanced countries. The average r^* for the Czech Republic, Poland, Hungary, Türkiye, Serbia, North Macedonia, Bosnia and Herzegovina, and Morocco is 3.6 percent. However, if we exclude both Türkiye and Poland, it declines further to 2.3 percent. In contrast, the average advanced-economy r^* in Norway, Sweden, and the United Kingdom is about 2.5 percent, largely on account of real depreciation of their currencies.

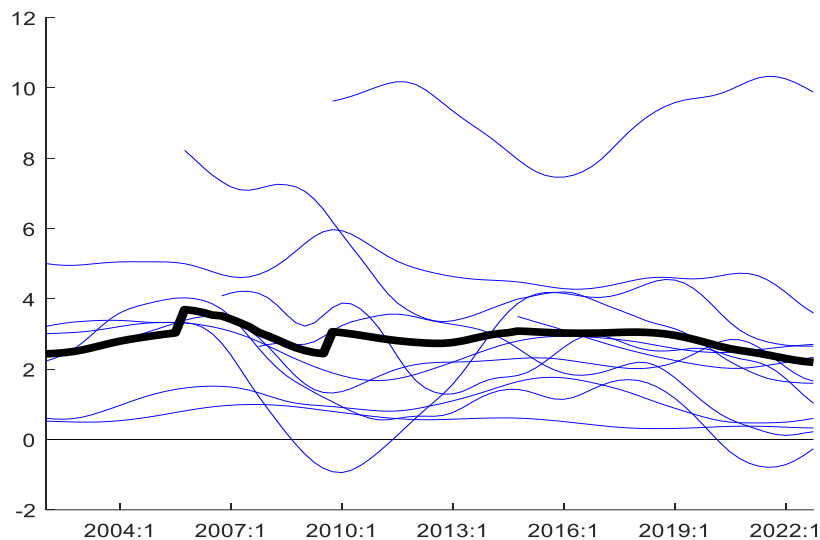
Two findings stand out. First, we do not observe the sharp decline in r^* reported in past literature. Second, after excluding the three outliers, Switzerland, Poland, and Türkiye, the advanced country estimates of r^* seem to be marginally higher than those of the faster-growing, converging countries, with the difference mostly attributable to the real appreciation-to-depreciation differential.

What is the reason for such markedly different estimates of r^* than those obtained in previous studies? On the one hand, equilibrium real appreciation lowered r^* in some of the converging countries, as nonresidents realized higher yields in euro terms. In the Czech Republic, where FDI inflows gradually brought the trade balance into surplus (Bulíř and Šmídková 2005), equilibrium appreciation of more than 1 percent per annum pushed r^* down by a full 100 basis points. On the other hand, equilibrium depreciation would lower yields in euro terms, and nonresidents would correspondingly request higher returns on country assets. We observed this effect, for example, in the UK and Türkiye: long-term depreciation of the pound and the lira pushed the estimate of r^* 54 basis points and 540 basis points higher, respectively.

3.2 Time-Varying Estimates of r^*

Both the time-varying, individual-country estimates of r^* (Figure 2, thin blue lines) and the sample average (the thick black line) are rather stable. We bring together the market real interest rate, r , and the time-invariant and time-varying estimates of r^* in Figure 3. See Appendix B for individual-country charts showing the various interest rate variables.

After fluctuating between 2 percent and 3 percent prior to the COVID-19 pandemic, our estimates of average r^* decline to close to 2 percent in the early 2020s. The end-of-sample decline in average r^* can be attributed to the pandemic-induced growth shock; that shock was partly offset by initially lower inflation during 2020–2021, which depreciated the real exchange rate in our sample countries. Recall that consumer price inflation peaked in 2022/2023, hence the negative contribution to r^* from the real appreciation of the national currencies is not yet pronounced in the sample data.

Figure 2: Time-Varying Estimates of r^* , 2002Q1–2022Q4 (in percent)

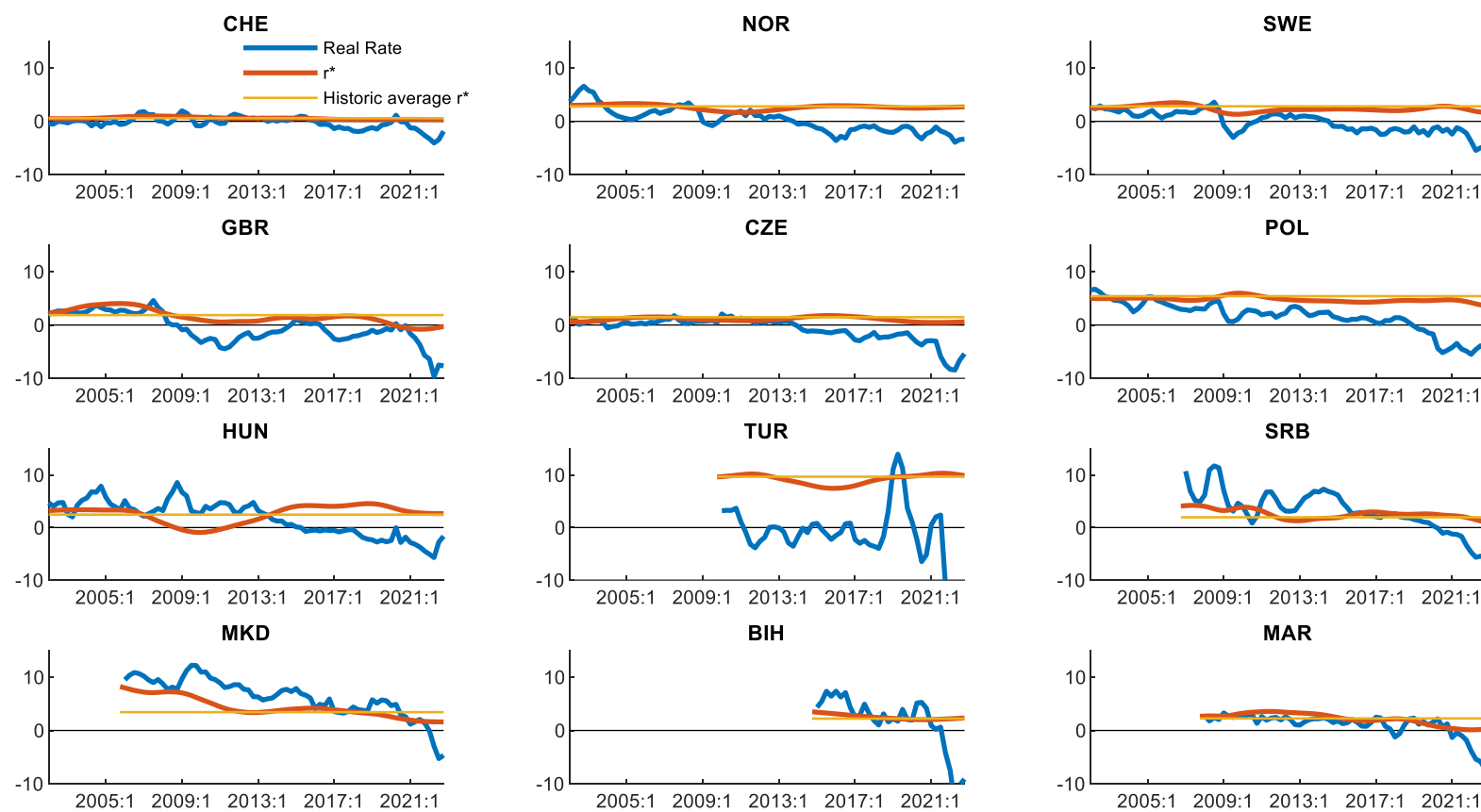
Note: The thick black line is the unweighted mean of r^* in our 12 sample countries; the thin blue lines are individual-country estimates.

Source: Authors' calculations.

We highlight three key findings. First, the time-varying estimates of r^* are close to or fluctuating around the time-invariant estimates of r^* (Figure 3). This result gives us some assurance that the impact of trend appreciation has been largely stable across time and sample countries. Second, we see only limited evidence of a secular decline in r^* . Only in the UK, the Czech Republic, Serbia, and Morocco did r^* trend visibly to or below zero toward the end of the sample. The Swiss estimates of r^* were close to zero for most of the sample period. This result has intriguing policy implications—the frequently voiced concerns about economies getting close to the effective lower bound do not seem validated by our estimates. Natural real rates may have been a lot higher than policymakers thought.

Third, in every advanced country the observed real rate has been well below our estimate of the natural rate during the post-GFC period, sometimes to the tune of 70–100 basis points. This was not true for some of the emerging countries: in Serbia, North Macedonia, Bosnia and Herzegovina, and Morocco the observed real rate was very close to or even above r^* until 2015 or even 2020. The observed rates declined well below r^* afterward, however.

Figure 3: Time-Varying Estimates of r^* , 2002Q1–2022Q4 (in percent)



Note: The blue line is the ex-post real policy rate, the red line is the country-specific estimate of r^* , and the yellow line is the estimate of r^* when both g^* and q^* are set equal to the sample average, that is, the time-invariant estimate of r^* .

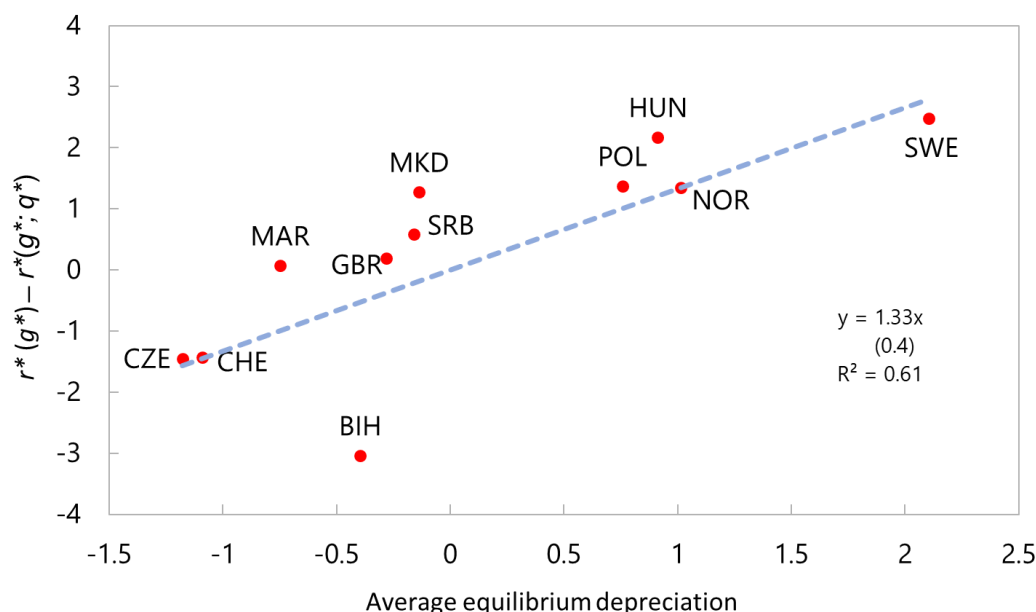
Source: Authors' calculations.

3.3 Robustness Check: Comparison with HLW

The HLW model is a natural benchmark against which to compare our extended framework. We applied the HLW methodology to our sample countries and estimated the natural rate, asking two questions. First, does equilibrium appreciation lower the open-economy estimate of the natural interest rate? Second, does the extended, open-economy framework imply a different “average” real natural interest rate as compared to the closed-economy (HLW) framework?

Regarding the former, we hypothesized that our estimate of r^* should be below the HLW estimate in countries that experience equilibrium real appreciation and above the HLW estimate in countries with equilibrium depreciation.⁹ We find clear evidence of this effect for our sample countries during 2013–2019: every percentage point of real equilibrium appreciation (q^*) is associated with the natural rate being about 1.3 percentage points lower (Figure 4).¹⁰

Figure 4: Exchange Rate Appreciation and r^* Adjustment, 2013–2019 (in percentage points)



Note: The horizontal axis measures the mean equilibrium exchange rate, with negative numbers indicating appreciation and positive numbers depreciation. The vertical axis measures the difference between the HLW (closed-economy) estimate of r^* , denoted as $r^*(g^*)$ and this paper’s (open-economy) estimate of r^* , denoted as $r^*(g^*; q^*)$. The point estimate for the intercept was not significantly different from zero, hence the linear regression shown on the chart is estimated without the intercept; the standard error of the point estimate is in parentheses. The sample excludes Türkiye, although its inclusion does not change the slope and increases the coefficient of determination only marginally.

Source: Authors’ calculations.

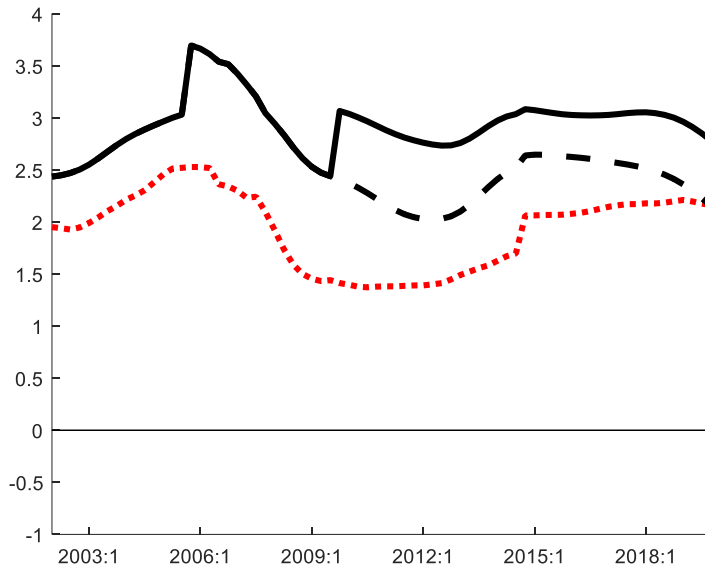
Regarding the latter, the magnitude of the difference between the HLW and our estimates of r^* surprised us. We find that the closed-economy HLW estimates of r^* are some 150 basis points

⁹ The HLW estimation procedure in R was replicated using their code transformed to Matlab. Our version of the HLW Matlab codes is a part of the package available online.

¹⁰ Extending the sample to 2002–2019 does not change the results materially—the slope coefficient is somewhat steeper at 1.6. Of course, the sample becomes unbalanced, as the series for Türkiye, Serbia, North Macedonia, Bosnia and Herzegovina, and Morocco are available from different dates.

lower on average than the open-economy estimates of r^* (Figure 5).¹¹ In our view, the key difference between the HLW and our estimates of r^* stems from the different weights each framework imposes on the actual data. The lower HLW estimates of r^* suggest that their methodology puts greater emphasis on actual real interest rates. Indeed, to a casual observer, the HLW estimates may look a lot like a moving average of the observed real interest rates, r . In contrast, our extended model puts more weight on the inflationary pressures determining r^* , the estimates of which can thus deviate significantly from the actual real rates.

Figure 5: Estimates of r^* Using HLW and Our Methodology, 2002Q1–2019Q4 (in percent)



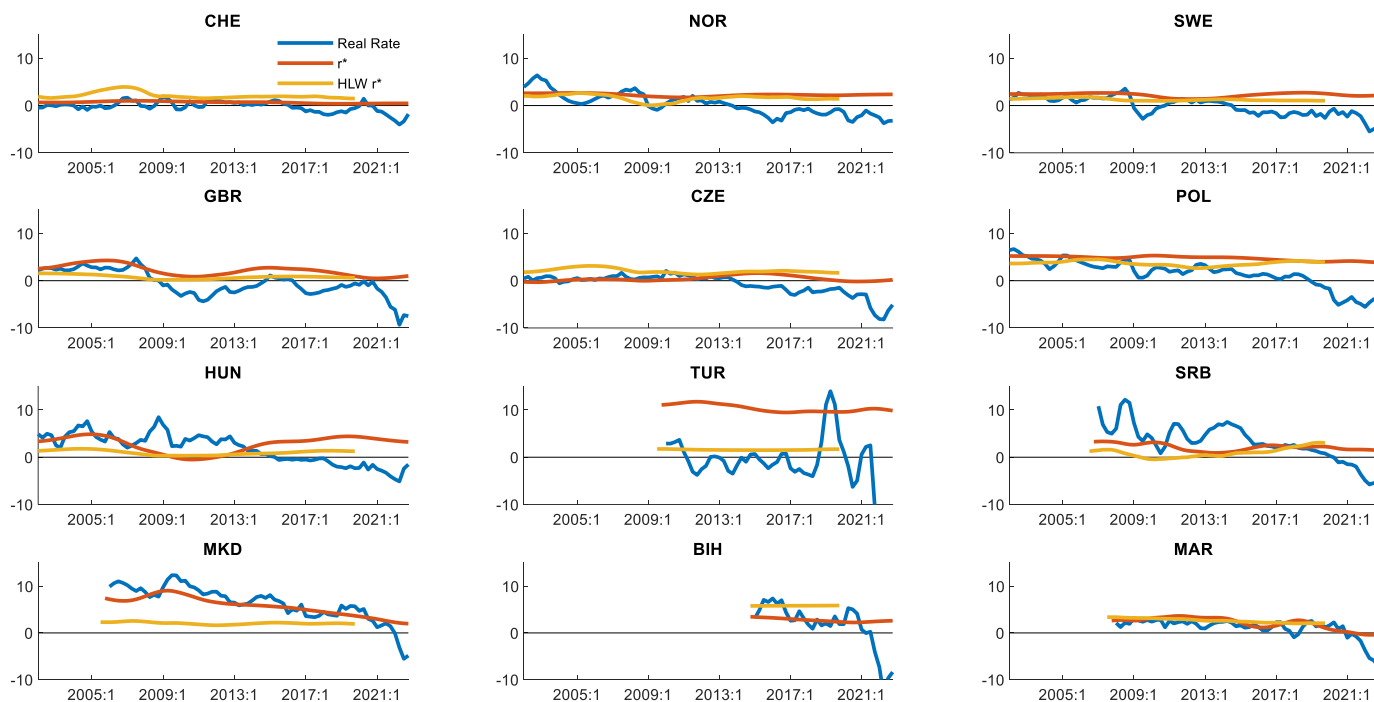
Note: The solid black line replicates the unweighted mean of r^* for our extended, open-economy model from Figure 3 and the dashed black line excludes Türkiye. The dotted red line is the unweighted mean of r^* using the closed-economy HLW methodology, including Türkiye.

Source: Authors' calculations.

Comparing the individual country estimates of r^* , only in Switzerland, the Czech Republic, and Bosnia and Herzegovina do we find the HLW estimate to be consistently and visibly above the open-economy estimate (Figure 6). These three countries recorded, of course, fast trend real appreciation of their domestic currencies, pushing the natural rate downward. For the remainder of the sample, either the two estimates overlap (Morocco and the post-GFC UK) or the HLW estimates are lower than the open-economy ones. Interestingly, neither approach suggests a secular decline in r^* after 2000.

¹¹ Figure 5 shows the results for 2002–2019 only, as the HLW-based simulations become unstable during the COVID period. We shortened the simulation period for our methodology commensurately.

Figure 6: Country-Specific Real Time-Varying Estimate of r^* Using LHW methodology, 2001Q1–2019Q4 (in percent)



Note: The blue line is the ex-post real policy rate, the red line is our country-specific estimate of r^* as defined in Equation 1, and the yellow line is the estimate of r^* based on the HLW methodology.

Source: Authors' calculations.

3.4 r and r^* : Too Tight or Too Loose?

Looking back, did the sample central banks keep r broadly equal to r^* ? Figure 6 suggests that the answer depends on the chosen r^* estimation technique and on the chosen time period. On average, we found monetary policy to have been loose ($r^* > r$) in all countries except Serbia and North Macedonia, where the results point to tight monetary policy (Table 3). We quantify the average gap between r and r^* at about minus 120 basis points during the full sample period (2002–2022), doubling to minus 240 basis points during 2013–2022.¹²

Table 3: Loose or Tight Monetary Policy: r minus r^* (in percent)

Country	Our methodology		HLW methodology	
	2002–22	2013–22	2002–19	2013–19
Switzerland	-0.8	-1.2	-2.2	-2.3
Norway	-2.4	-4.3	-0.9	-2.9
Sweden	-2.5	-3.8	-1.0	-2.2
United Kingdom	-2.2	-2.7	-0.8	-1.9
Czech Republic	-1.9	-3.5	-2.1	-3.2
Poland	-3.2	-5.0	-1.2	-2.5
Hungary	-0.8	-4.7	1.6	-1.3
Türkiye	-12.1	-12.8	-1.3	-1.2
Serbia	0.6	-0.5	3.6	2.1
North Macedonia	1.9	0.9	5.3	3.4
Bosnia and Herzegovina	-0.8	-0.8	-4.2	-4.2
Morocco	-1.1	-1.2	-0.8	-0.9
Unweighted average (excluding Türkiye)	-1.2	-2.4	-0.2	-1.4

Note: Each cell contains the sample period mean of $r - r^*$.

Source: Authors' calculations.

Why have a break in 2013? While it is somewhat arbitrary, we noticed two well-known policy events in 2013 that had a profound impact on interest rates in our sample economies, namely, the “taper tantrum” episode and the ECB’s decision to bring its main refinancing rate close to zero in November 2023. These events made it easier and cheaper for financial market participants to obtain liquidity. Consequently, during 2013–2022 only North Macedonia’s r^* was higher than the observed real rate under our methodology and also Serbia’s under the HLW methodology. Looking only at the EU countries and Switzerland, Norway, and the UK, the average gap between r and r^* was a staggering minus 360 basis points during 2013–2022. The same-period gap in the three countries that joined European Union in mid-2000 (the Czech Republic, Poland, and Hungary) was even higher at 440 basis points.

4. Policy Implications

Monetary policy is about guiding real interest rates toward their natural level. Misjudging the natural real interest rate is likely to have long-term consequences for output, unemployment, and inflation, especially if the r^* estimation error is large and long-lasting.

First, our estimates suggest that the sample central banks may have underestimated the value of r^* , on average by more than 100 basis points during 2002–2022 and by twice as much after the taper

¹² The corresponding averages for the HLW methodology are *minus* 20 and *minus* 140 basis points, smaller than the open-economy estimates but pointing in the same direction for all countries.

tantrum period. Guided by a low estimate of r^* , they kept the policy rate close to the zero lower bound for too long and, hence, $r < r^*$. These estimates are consistent with the narrative that the 2021–2023 period of high inflation had, at least partly, monetary roots (Bordo, Taylor, and Cochrane 2023).

Second, and related to the first point, one can ask whether central banks in small open economies can really set their policy rates independently of the dominant central banks such as the US Federal Reserve System and the European Central Bank (the “dilemma-rather-than-trilemma” argument of Miranda-Agrippino and Rey 2022). Or perhaps these banks can set rates independently but they just stopped trying after the Global Financial Crisis. The hypothesis of a near-zero r^* was repeated often enough by fellow policymakers and was all too convenient for small open economy central bankers to resist its lure.

Finally, our estimates do not support the claim that all the advanced and emerging market economies have been operating close to the effective (nominal) zero lower bound. The estimates suggest that the true natural interest rate— r^* plus expected inflation (or the inflation target)—may have been higher by 100 basis points on average and three times as much for some countries. Only in the UK and the Czech Republic do we observe the open-economy estimates of the natural real interest rate dipping below zero, and even then only during the COVID period.

5. Conclusions

This paper challenges the conventional narrative of declining and persistently low real natural rates of interest. We modify the Holston, Laubach, and Williams (2017) framework for the effects of equilibrium real exchange rate appreciation or depreciation. Our contribution is to include the real exchange rate (1) as a gap in the IS schedule, as done in previous extensions, but (2) also in the natural rate estimation as the estimated real exchange rate trend (equilibrium). The model parameters are estimated using Bayesian techniques and the latent variables using the Kalman filter.

Three important results emerge. First, we cannot reject the hypothesis that r^* should be lower in countries with currencies appreciating in real terms. Second, the open-economy extension of the original framework suggests that in our sample of 12 countries located in and around Europe the natural real rate of interest has been stable and more than 100 basis points higher than in the original, closed-economy HLW framework. Third, the sizable negative difference between real observed interest rates and real natural interest rates, $r - r^*$, suggests that monetary policy may have been too loose during the 2013–2022 period in most of the sample countries.

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Appendix A: The Model and Its Bayesian Estimation

In this appendix we describe the model structure in detail and discuss our Bayesian estimation. The model is taken to the country data by combining calibration and Bayesian estimation techniques.

First, based on the AREAER database (IMF 2022), we assign each country the relevant monetary policy and exchange rate regime, choosing parameters h_1 and h_2 in the UIP and the monetary policy reaction function (Table A2). Second, we calibrate the remaining parameters in the equations for the UIP, the monetary policy reaction function, and the natural rate equation function identically for all countries, conditional on the monetary regime, and compute the standard deviations of shocks in these equations from the data. We choose this strategy (1) because it is tricky to estimate a policy rule from the data (Carare and Tchaidze 2005) and (2) to ensure that the shape and calibration of the policy rule is not binding for the identification of the natural rate. Third, we use Bayesian estimation to obtain all the remaining parameters and standard deviations of shocks in the Phillips curve and IS curve. Fourth, we use the Kalman filter to filter all the data and derive the unobservable variables, namely, the trends and gaps in real interest rates, the exchange rate, and output.

A.1 Data

The observed data encompass four variables: the short-term policy interest rate, or an interbank rate mirroring the policy rate (Table A1), real GDP, the exchange rate against the euro, and core CPI.

Table A1: Interest Rate Definitions

Country	Name of the series	Type	Source
Switzerland	SNB 3-Month LIBOR Target Rate	Interbank	BIS
Norway	Deposit Rate	Policy	BIS
Sweden	Repo Rate	Policy	BIS
United Kingdom	Official Bank Rate	Policy	BIS
Czech Republic	3M PRIBOR	Interbank	CNB ¹
Poland	Reference Rate	Policy	BIS
Hungary	Hungary: Average Base Rate	Policy	BIS
Türkiye	1-Week Repo Rate	Policy	BIS
Serbia	1-Week Repo Rate	Policy	BIS
North Macedonia	Overnight Lending Rate	Policy	FSI
Bosnia and Herzegovina	Overnight Lending Rate	Policy	FSI
Morocco	Monetary Policy Rate	Policy	BAM ¹

¹ CNB stands for the Czech National Bank and BAM for the Bank Al-Maghrib.

Source: Authors.

A.2 The Model, Its Initial Calibration, and Bayesian Priors

We list the model equations and prior distributions of all the parameters in Table A2. If a parameter is not estimated, only the mean of its calibration is reported, and the type and prior distribution cells are empty. Priors for estimated and non-estimated parameters are identical across all sample countries. The exception are the priors for the standard deviations of supply and demand shocks, which are set according to the variability observed in the country data. For example, $std(\hat{\pi}^{obs})$ denotes the standard deviation of detrended CPI core inflation for each country and $std(\Delta\hat{y}^{obs})$ denotes that of the detrended growth rate of real GDP. We use the Hodrick-Prescott filter to detrend the variables. Country-specific setting of standard deviations is needed, as the differences in the

actual data variability are large and the standard deviations help us determine the signal-to-noise ratios.

Table A2 reports the mean and the standard deviation of the prior for each parameter estimated. The standard deviation of the prior is reported as multiple of its mean. In other words, this product indicates whether we have imposed a “tight” or a “loose” prior. For example, parameter a_3 has a prior with a mean of $\frac{1}{2}$ and a standard deviation of 0.5, which we report as $\frac{1}{2}$ 0.5. This is a “loose” prior relative to the others.

The parameter denoting inflation persistence in the Phillips curve, a_1 , is calibrated to 0.5 for all countries. The calibration reflects (1) the standard micro foundations of the underlying model and assumed Calvo pricing with full backward indexation, and (2) the fact that the parameter cannot be identified along with a_2 due to their observational equivalence.

Table A2: Model Equations

Equation	Prior distribution		
	Type	Mean	Standard deviation
Phillips curve			
$\pi_t = a_1\pi_{t-1} + (1 - a_1)\pi_{t+1} + a_2RMC_t + \varepsilon_t^\pi$ $RMC_t = a_3\hat{y}_t + (1 - a_3)\hat{q}_t$			
a_1	-	0.5	-
a_2	Inv. γ	0.2	0.2
a_3	β	0.5	$\frac{1}{2}$ 0.5
standard deviation of ε^π	Inv. γ	$\frac{1}{4}\text{std}(\hat{\pi}^{obs})$	$\frac{1}{2}$ or $\frac{1}{4}$ std($\hat{\pi}^{obs}$) Standard deviation during COVID period (2021Q1–2022Q4) is set six times higher than before COVID
IS curve			
$\hat{y}_t = b_1\hat{y}_{t-1} - b_2MCI + b_3\hat{y}_t^F + \varepsilon_t^y$ $MCI_t = b_4\hat{r}_t + (1 - b_4)(-\hat{q}_t)$			
b_1	β	0.7	$\frac{1}{4}$ 0.7
b_2	γ	0.1	$\frac{1}{4}$ 0.1
b_3	γ	0.15	$\frac{1}{4}$ 0.15
b_4	β	0.6	$\frac{1}{4}$ 0.6
standard deviation of ε^y	Inv. γ	$\frac{1}{2}$ std($\Delta\hat{y}^{obs}$)	$\frac{1}{2}$ std($\Delta\hat{y}^{obs}$)
Natural rate			
$r^* = \rho r_{t-1}^* + 2(1 - \rho)c_1(c_2g_t^* + (1 - c_2)q_t^*)$			
ρ	-	0.85	-
c_1	γ	1	$\frac{1}{15}$ 1
c_2	β	0.5	$\frac{1}{15}$ 0.5
UIP			
$s_t = h_2(s_{t-1} + \Delta s^{TAR})$ $+ (1 - h_2)\left((1 - e_1)s_{t+1}^e + e_1(s_{t-1} + 2(\pi^{TAR} - \pi^{TAR,US} + \Delta q^*)) + \frac{-i_t + i_t^{EA} + prem_t}{4}\right) + \varepsilon_t^s$			
h_2	-	1 for float or similar ER regimes (CHE, NOR, SWE, GBR, CZE, POL, HUN, TUR, SRB) and 0 for peg (BIH, MKD, MAR)	-
e_1	-	0.6	-
standard deviation of ε^s	-	$\frac{1}{2}$ std($\Delta\hat{s}^{obs}$)	-
Monetary policy reaction function			
$i_t = h_1((s_{t+1} - s_t)4 + i_t^{EA} + prem_t) + (1 - h_1)(g_1i_{t-1} + (1 - g_1)(r_t^* + \Delta_4\pi_{t+3}) + g_2(\Delta_4\pi_{t+3} - \pi^{TAR}) + g_3\hat{y}_t) + \varepsilon_t^i$			
h_1	-	0 for float or similar ER regimes (CHE, NOR, SWE, GBR, CZE, POL, HUN, TUR, SRB) and 1 for peg (BIH, MKD, MAR)	-
g_1	-	0.75	-
g_2	-	1.25	-
g_3	-	0.25	-
standard deviation of ε^i	-	4 std(\hat{i}^{obs}) In aftermath of GFC, std is set two times higher than during 2010Q1–2022Q4	-

Source: Authors' calculations.

A.3 The Bayesian Estimation

To avoid polluting our estimates with the COVID-19 shock, our Bayesian estimation of the model parameters employs data up to the first quarter of 2020. The posterior distribution of the estimated parameters is constructed using Metropolis-Hasting simulation with 100,000 draws. Country-specific results, including all country-specific priors and posteriors, are presented Appendix B.

From the Bayesian estimation we obtain the relative weights of potential growth and equilibrium appreciation on r^* , c_1 , and c_2 as in Equation 1. The parameter estimates come out as roughly equal to $\frac{1}{2}$ for all countries, adding up to the HLW proportional relationship between r^* and g^* . These shares are also broadly consistent with Arena and others (2020), who were looking for additional factors to explain the large z_t residual during the post-GFC period.

A.4 The Kalman Filter

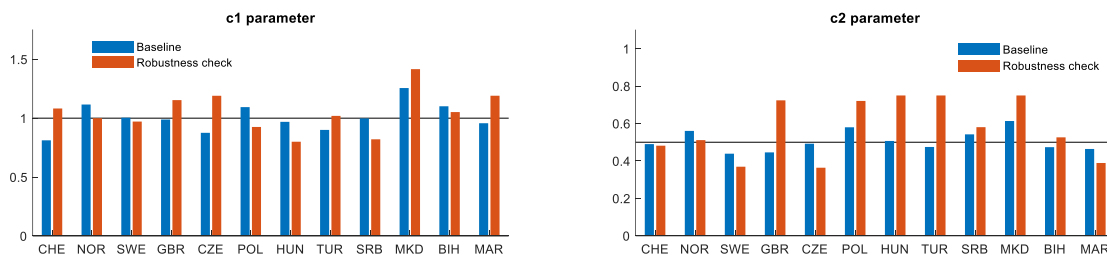
In the final step, we use the Kalman smoother to filter the historical data and obtain the latent variables. All the available data, including those for the COVID-19 period of 2020–2022, are used. The estimates presented are the smoothed states.

Appendix B: Sensitivity Analysis

Estimated parameters c_1 and c_2 drive the identification of the natural rate and, for the purposes of Bayesian estimation, we started with the standard deviation of the prior being relatively small. That is, we treat these priors as the “tight” ones. This both reflects the theoretical foundations of the Penn effect and provides sufficient information for the estimation procedure. This appendix evaluates the implications of less tight priors of these parameters for their posterior estimates and for the natural rates identified. In summary, we find that our results are only marginally affected by an alternative choice of key priors.

Making the priors for c_1 and c_2 less tight has only a limited impact on the estimated parameters (Figure B1). To make the “tight” priors looser, we increased the standard deviations to five times the original calibration. The table shows that the differences between the two sets of estimated parameters are not one-sided—the estimates with looser priors are larger for some countries and smaller for others. On average, the c_1 parameters are different by less than 20 percent and the c_2 ones by about 30 percent.

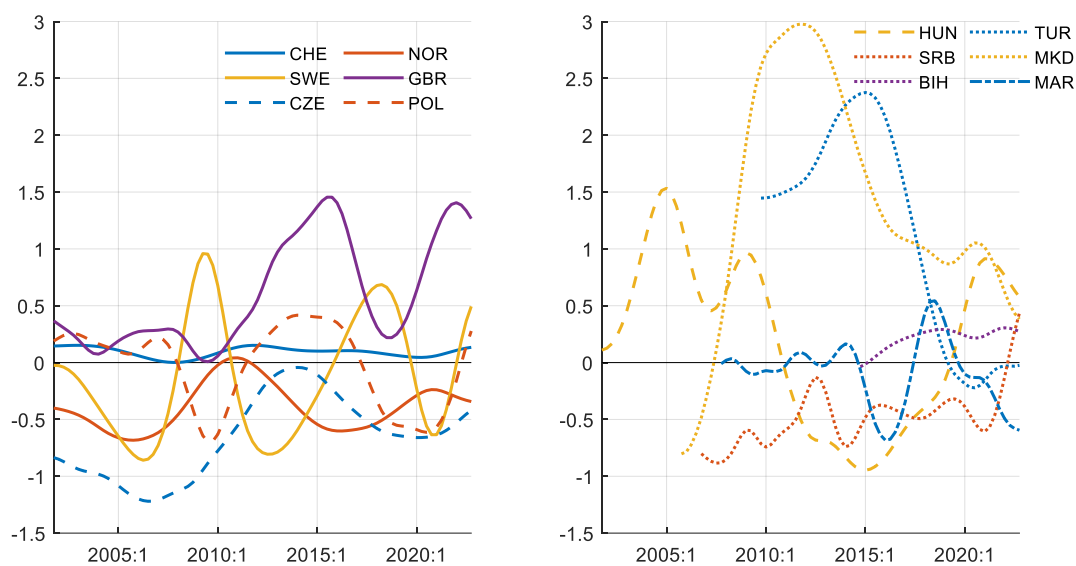
Figure B1: Sensitivity Analysis of c_1 and c_2



Note: The original parameters (blue bars) are estimated with tight priors as in Table A2. The relaxed-prior estimates (red bars) assume that the standard deviations of the prior are five times as large.

Source: Authors’ calculations.

Predictably, relaxing the priors makes the estimates of r^* somewhat more volatile, but the differences are not large (Figure B2). We highlight two observations. First, the differences are concentrated mostly within ± 100 basis points for all countries except North Macedonia and Türkiye. In the UK, the Czech Republic, and Hungary the differences are also occasionally outside this narrow band. Second, for one-half of the sample the relaxed priors result in a more volatile estimate of r^* , while for the rest we notice a directional shift. Specifically, the relaxed priors push the r^* estimate down in the Czech Republic (by about 75 basis points), Norway (by less than 50 basis points), and Serbia (by about 50 basis points). The post-GFC estimates of r^* are about 100 basis points higher in the UK and even higher in North Macedonia and Türkiye.

Figure B2: Differences Between the r^* Estimates with “Tight” and “Loose” Priors

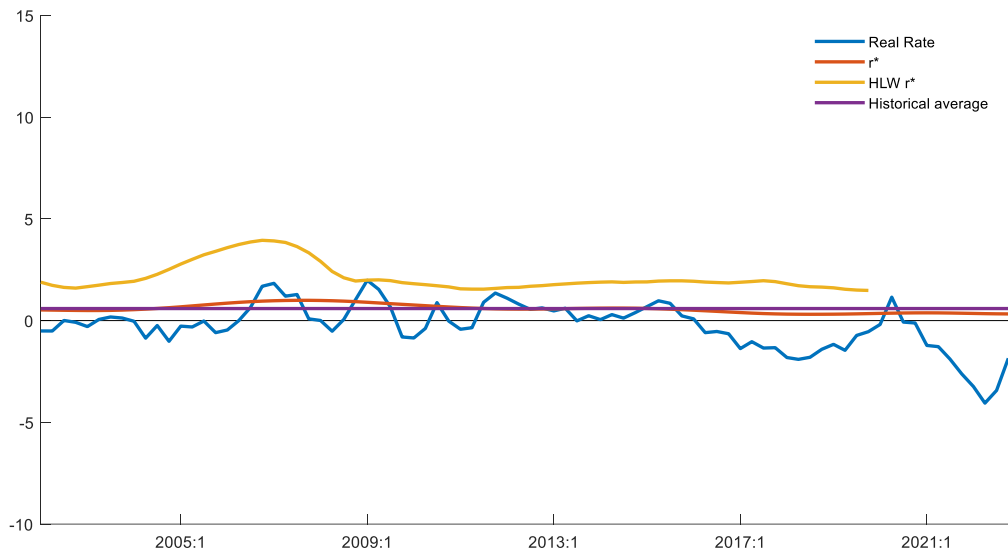
Note: We use the new c_1 and c_2 parameters to recalculate the natural rate for our sample countries under the relaxed priors and compare them with the baseline estimates. The lines depict the difference between the original estimate of r^* (under the tight priors) and the alternative estimate of r^* (under the relaxed priors).

Source: Authors' calculations.

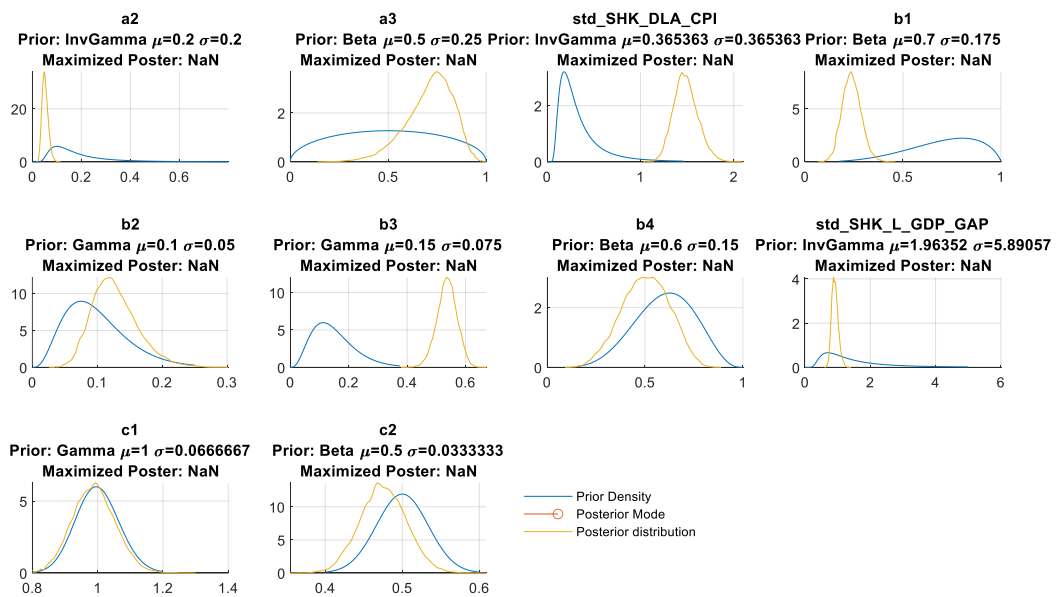
Appendix C: Country-Specific Results

This appendix summarizes both the HLW and our extended-model estimates of the natural rate of interest for all 12 sample countries and the prior and posterior modes from each Bayesian estimation. The historical average for r^* corresponds to the time-invariant estimate of the natural rate.

Figure C1: Switzerland: Alternative Estimates of r^* and Bayesian Estimation



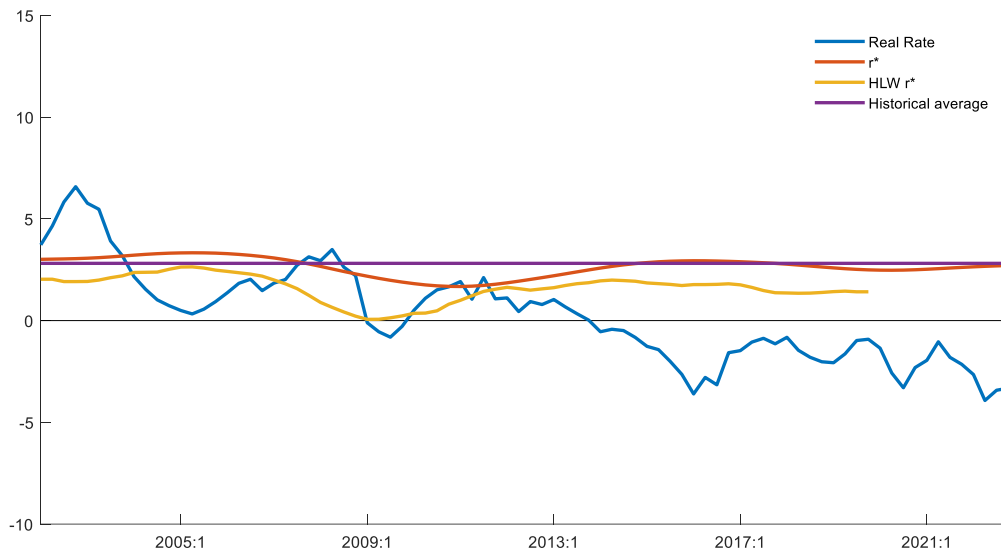
Prior Distributions and Posterior Modes - No System Prior



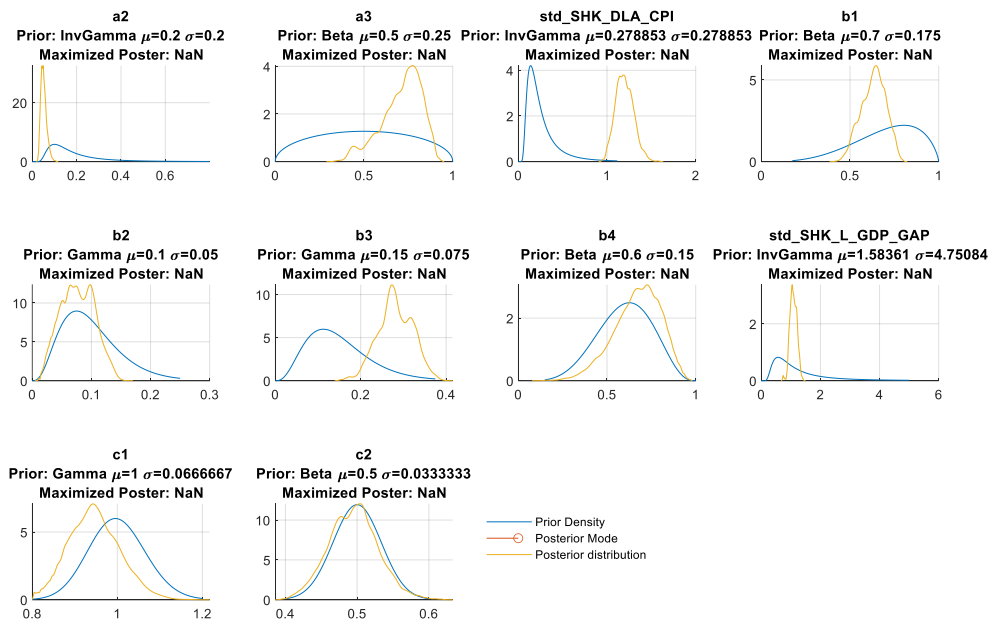
Note: The blue line is the ex-post real policy rate, the red line is our country-specific estimate of r^* , and the yellow line is the estimate of r^* based on the HLW methodology.

Source: Authors' calculations.

Figure C2: Norway: Alternative Estimates of r^* and Bayesian Estimation



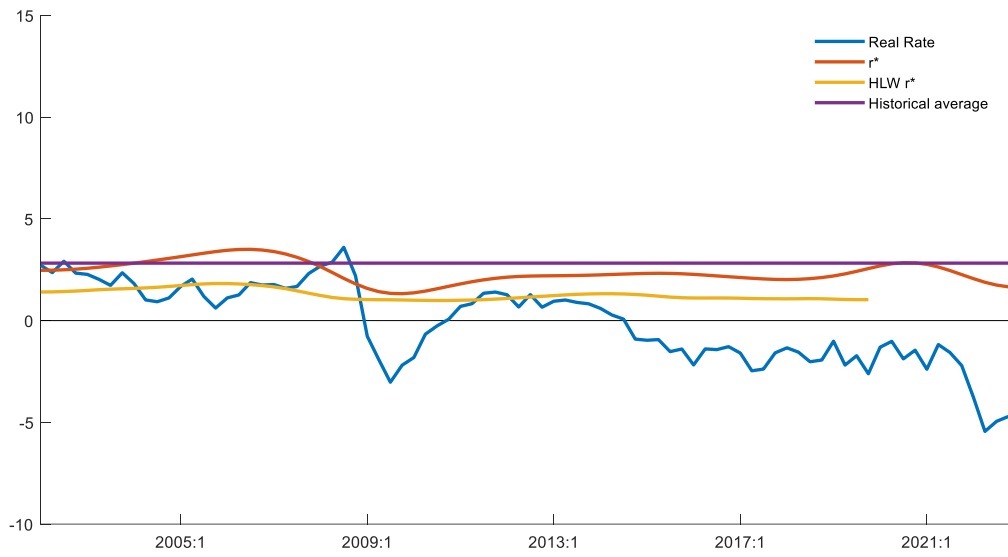
Prior Distributions and Posterior Modes - No System Prior



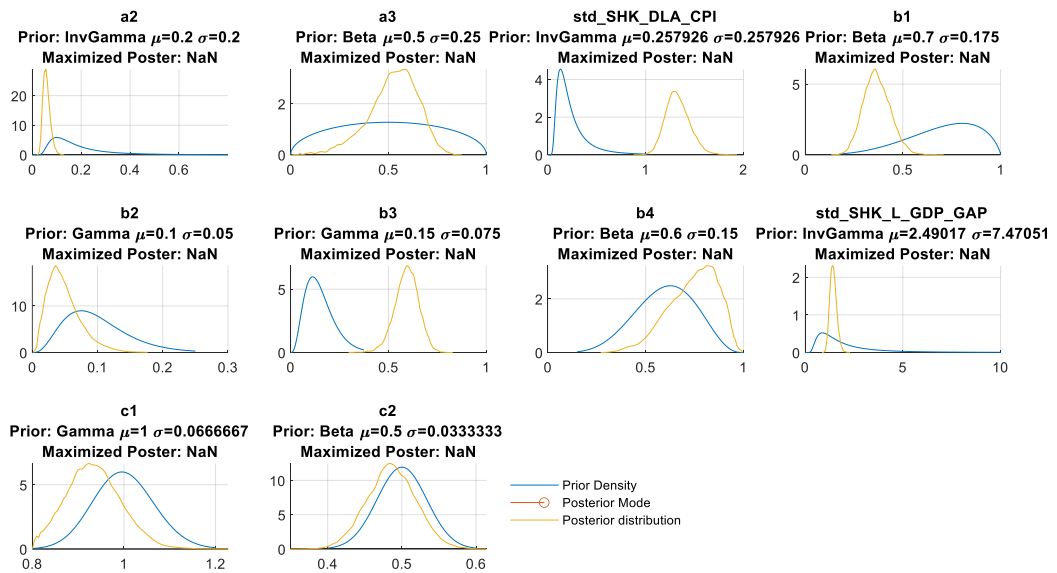
Note: The blue line is the ex-post real policy rate, the red line is our country-specific estimate of r^* , and the yellow line is the estimate of r^* based on the HLW methodology.

Source: Authors' calculations.

Figure C3: Sweden: Alternative Estimates of r^* and Bayesian Estimation



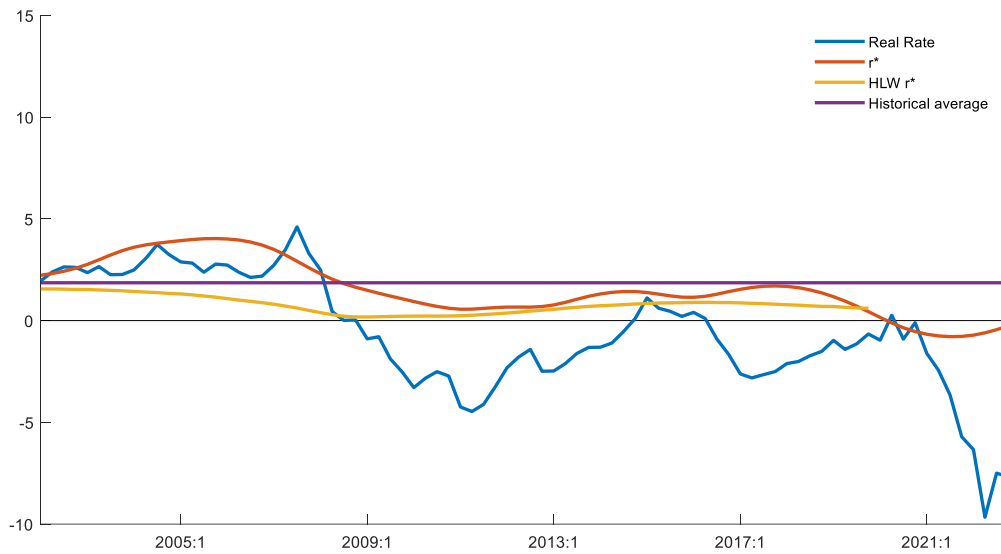
Prior Distributions and Posterior Modes - No System Prior



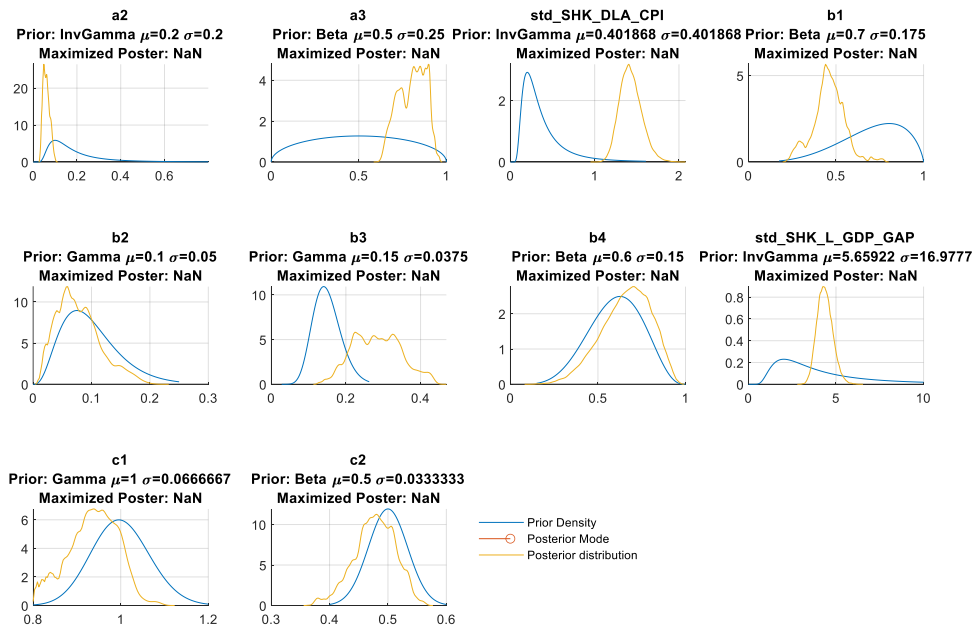
Note: The blue line is the ex-post real policy rate, the red line is our country-specific estimate of r^* , and the yellow line is the estimate of r^* based on the HLW methodology.

Source: Authors' calculations.

Figure C4: United Kingdom: Alternative Estimates of r^* and Bayesian Estimation



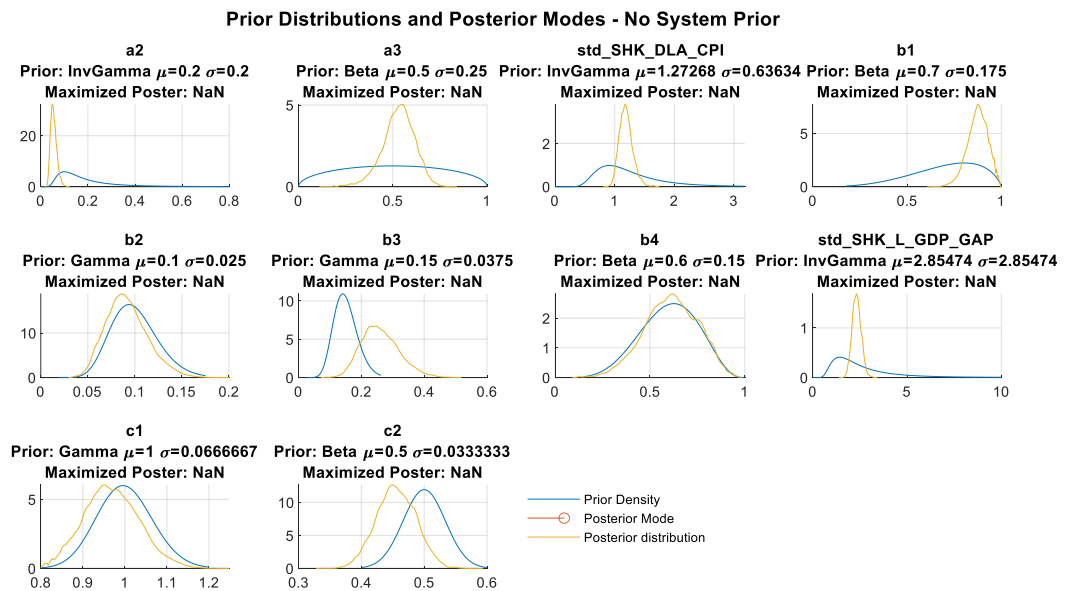
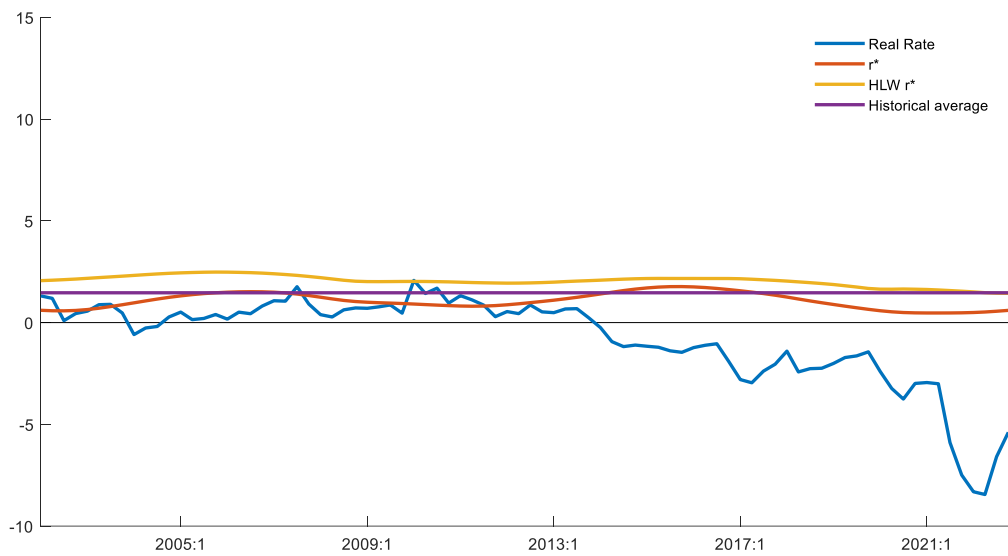
Prior Distributions and Posterior Modes - No System Prior



Note: The blue line is the ex-post real policy rate, the red line is our country-specific estimate of r^* , and the yellow line is the estimate of r^* based on the HLW methodology.

Source: Authors' calculations.

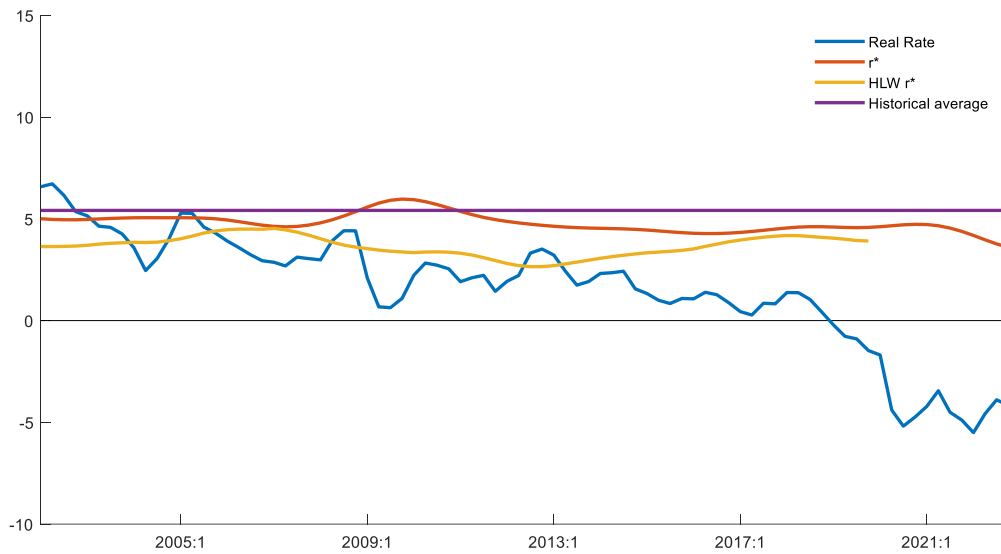
Figure C5: Czech Republic: Alternative Estimates of r^* and Bayesian Estimation



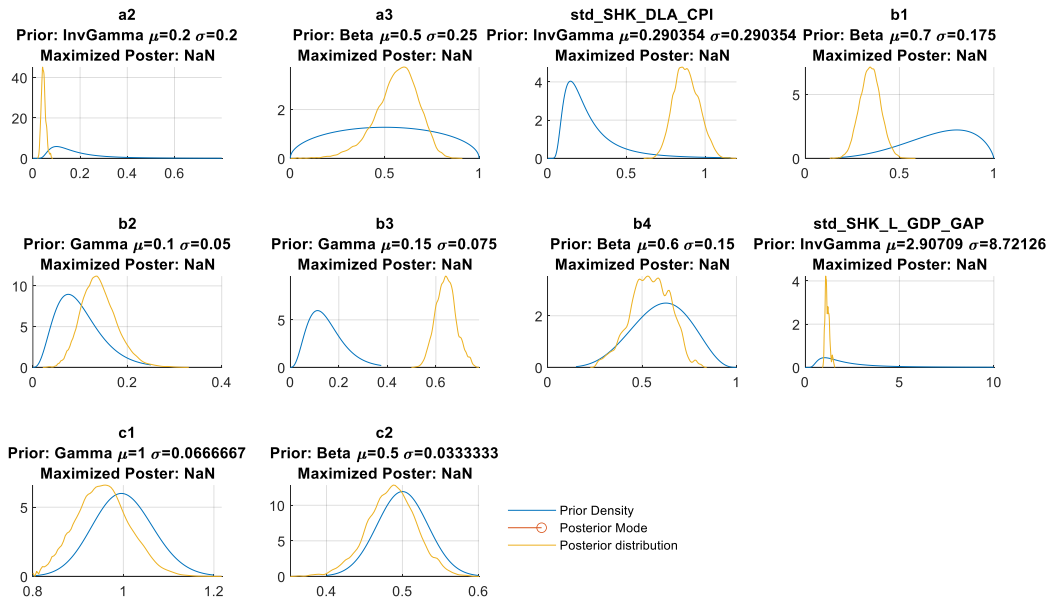
Note: The blue line is the ex-post real policy rate, the red line is our country-specific estimate of r^* , and the yellow line is the estimate of r^* based on the HLW methodology.

Source: Authors' calculations.

Figure C6: Poland: Alternative Estimates of r^* and Bayesian Estimation



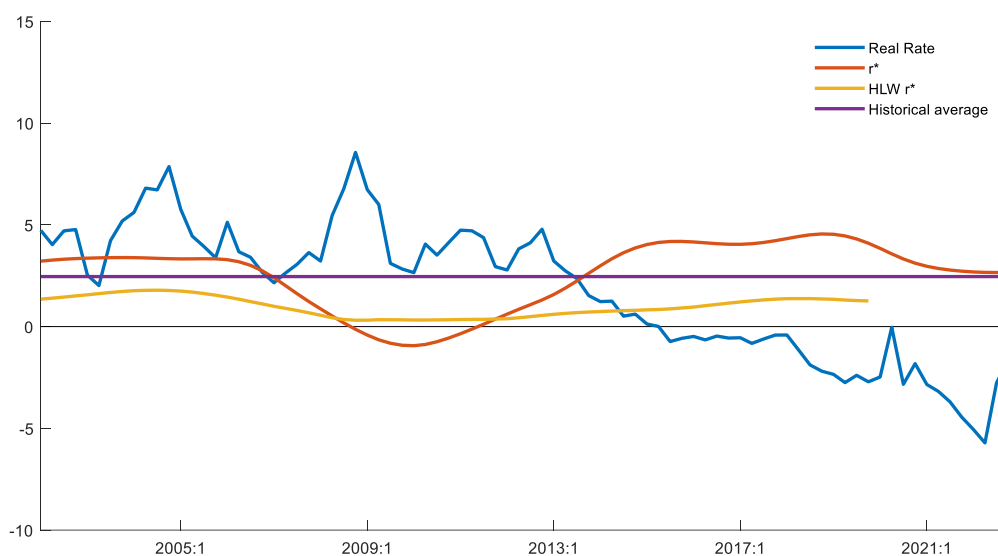
Prior Distributions and Posterior Modes - No System Prior



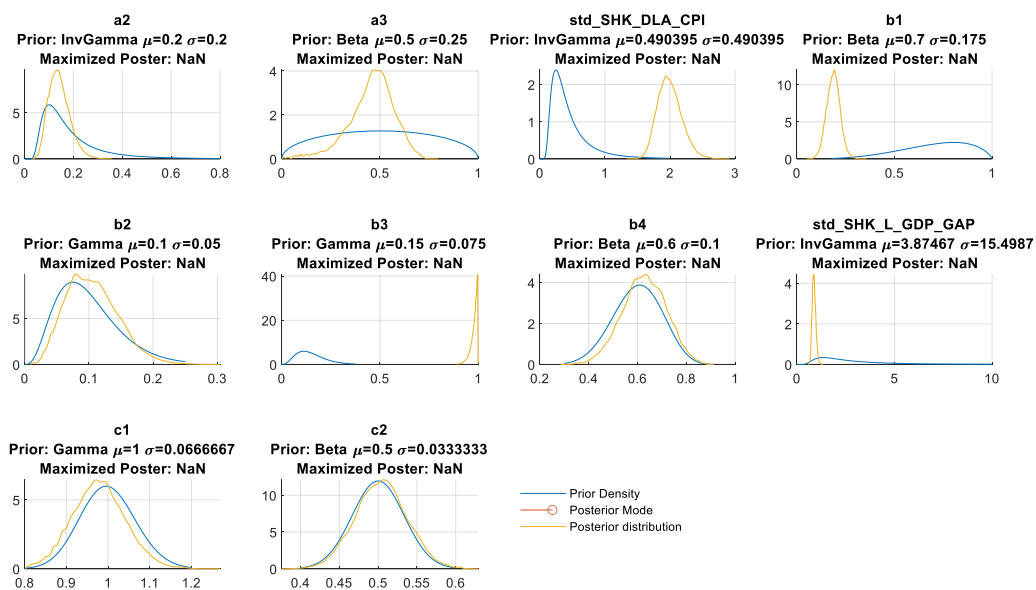
Note: The blue line is the ex-post real policy rate, the red line is our country-specific estimate of r^* , and the yellow line is the estimate of r^* based on the HLW methodology.

Source: Authors' calculations.

Figure C7: Hungary: Alternative Estimates of r^* and Bayesian Estimation



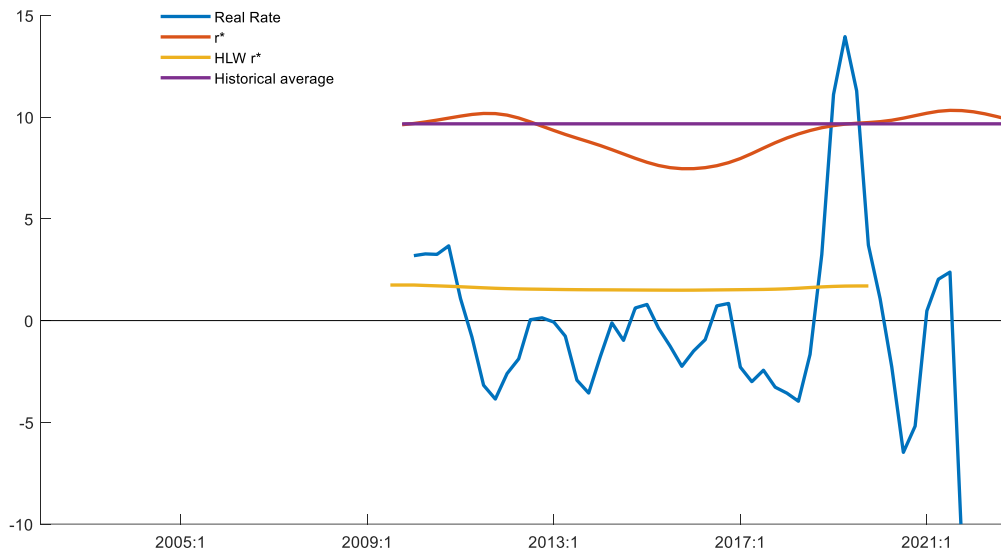
Prior Distributions and Posterior Modes - No System Prior



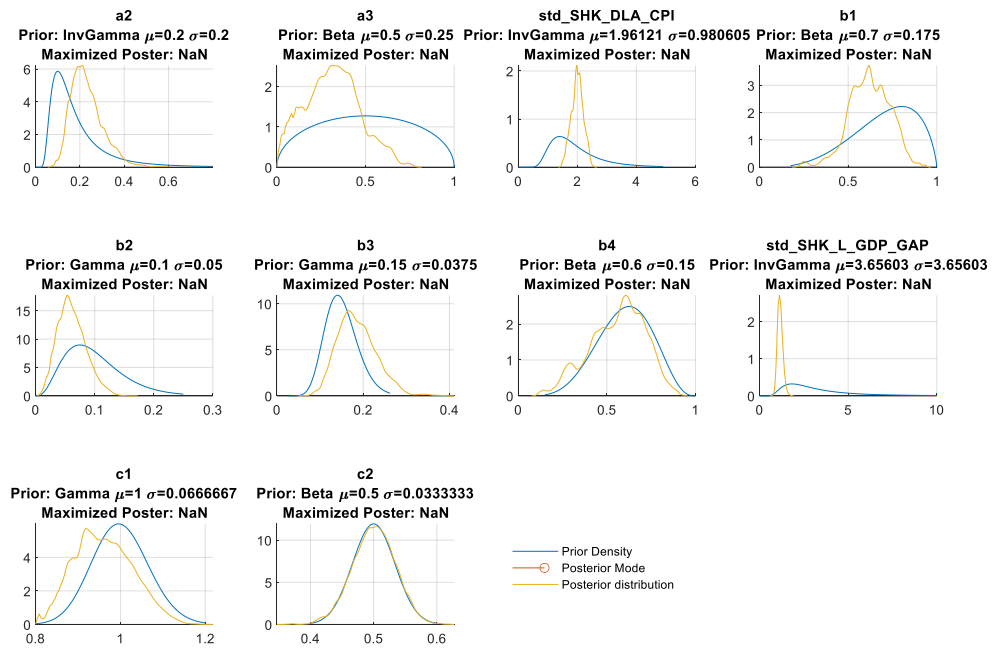
Note: The blue line is the ex-post real policy rate, the red line is our country-specific estimate of r^* , and the yellow line is the estimate of r^* based on the HLW methodology.

Source: Authors' calculations.

Figure C8: Türkiye: Alternative Estimates of r^* and Bayesian Estimation



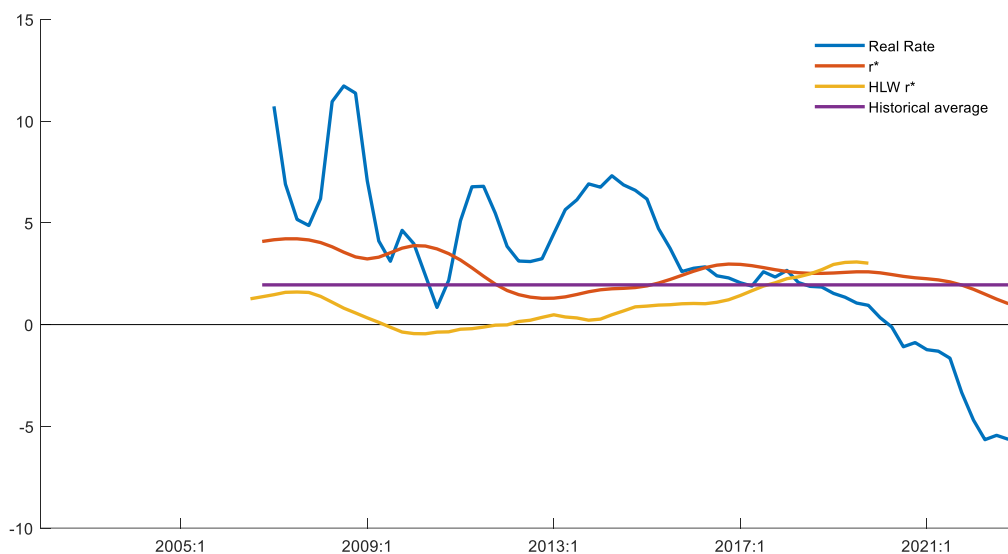
Prior Distributions and Posterior Modes - No System Prior



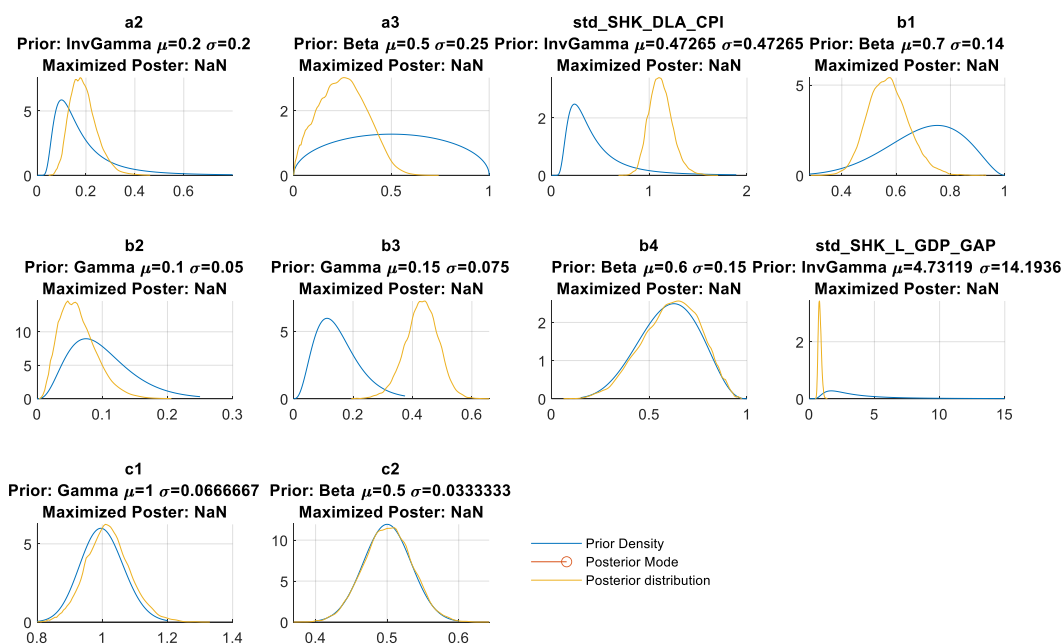
Note: The blue line is the ex-post real policy rate, the red line is our country-specific estimate of r^* , and the yellow line is the estimate of r^* based on the HLW methodology.

Source: Authors' calculations.

Figure C9: Serbia: Alternative Estimates of r^* and Bayesian Estimation



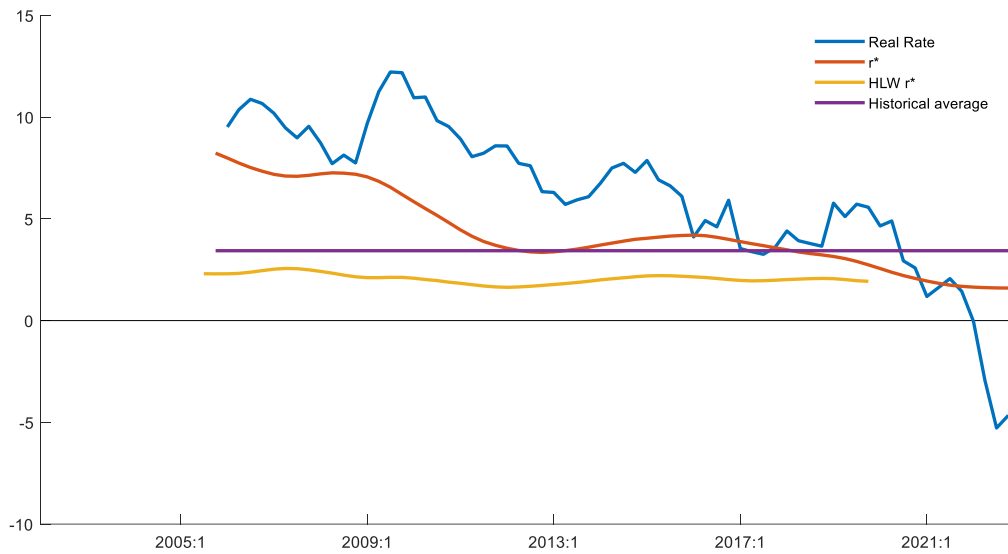
Prior Distributions and Posterior Modes - No System Prior



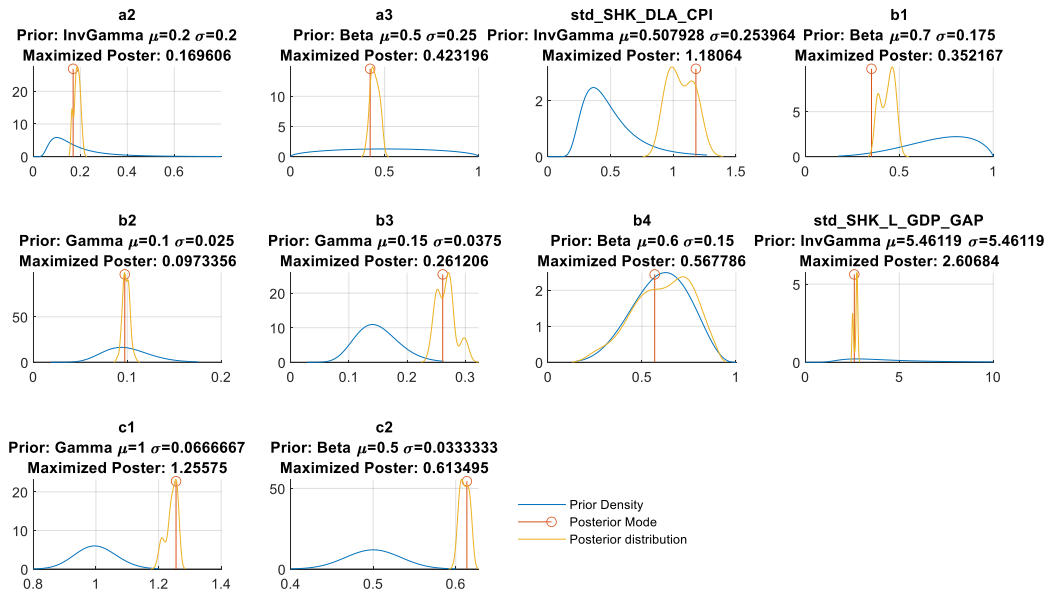
Note: The blue line is the ex-post real policy rate, the red line is our country-specific estimate of r^* , and the yellow line is the estimate of r^* based on the HLW methodology.

Source: Authors' calculations.

Figure C10: North Macedonia: Alternative Estimates of r^* and Bayesian Estimation



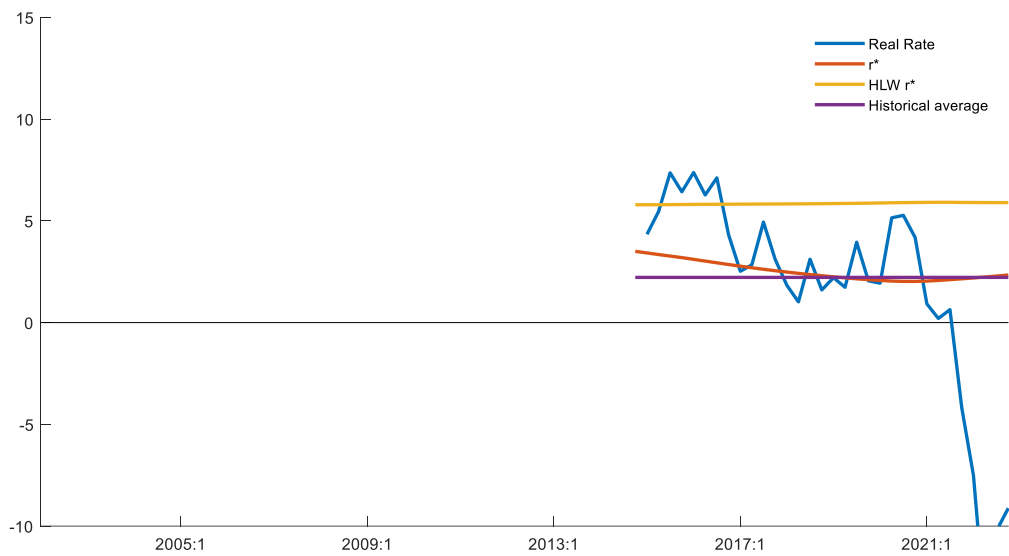
Prior Distributions and Posterior Modes - No System Prior



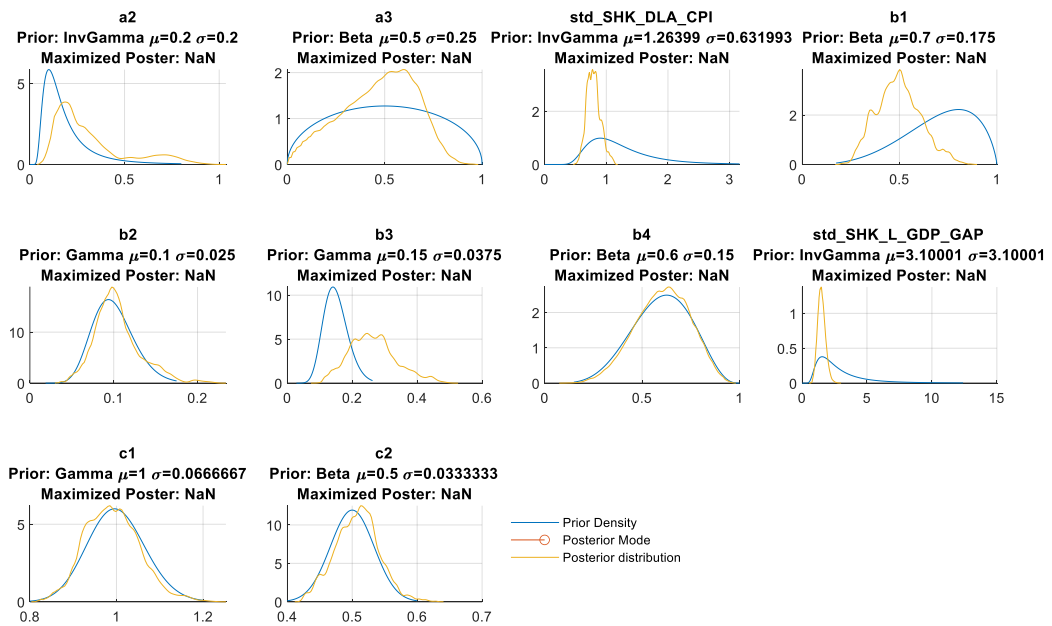
Note: The blue line is the ex-post real policy rate, the red line is our country-specific estimate of r^* , and the yellow line is the estimate of r^* based on the HLW methodology.

Source: Authors' calculations.

Figure C11: Bosnia and Herzegovina: Alternative Estimates of r^* and Bayesian Estimation



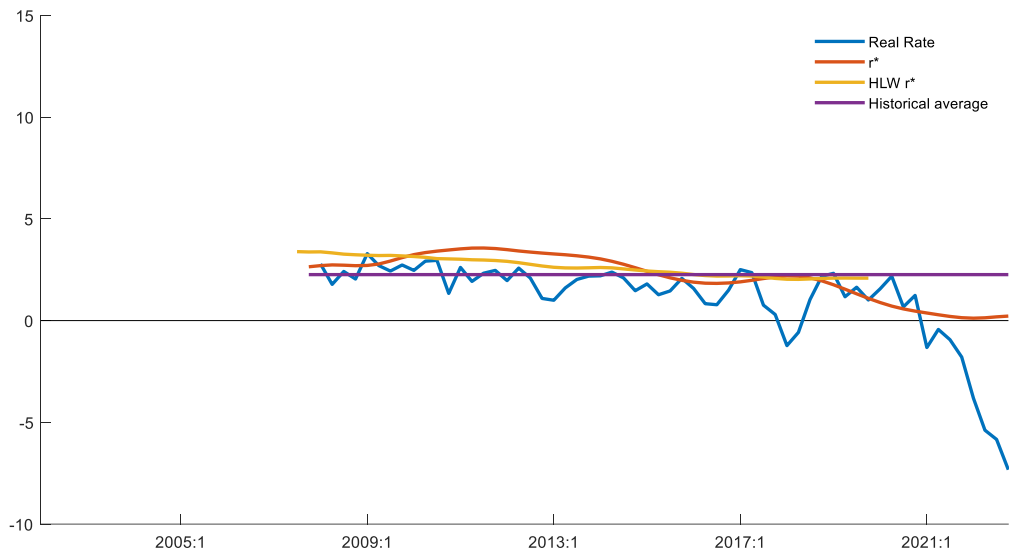
Prior Distributions and Posterior Modes - No System Prior



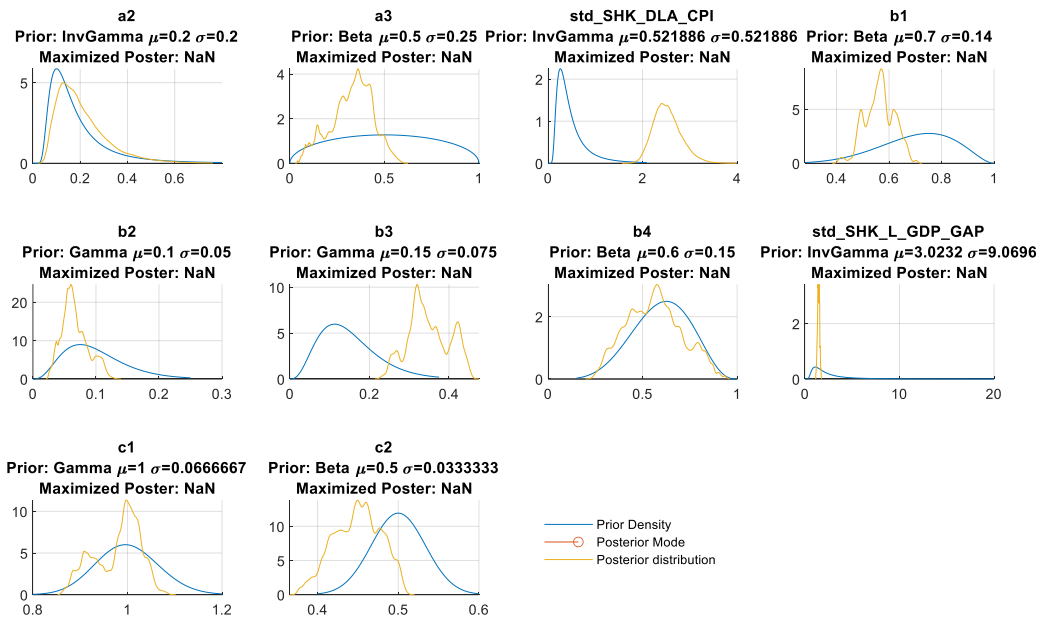
Note: The blue line is the ex-post real policy rate, the red line is our country-specific estimate of r^* , and the yellow line is the estimate of r^* based on the HLW methodology.

Source: Authors' calculations.

Figure C12: Morocco: Alternative Estimates of r^* and Bayesian Estimation



Prior Distributions and Posterior Modes - No System Prior



Note: The blue line is the ex-post real policy rate, the red line is our country-specific estimate of r^* , and the yellow line is the estimate of r^* based on the HLW methodology.

Source: Authors' calculations.

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