Non-Keynesian stabilizers and inflation dynamics

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Introduction: Motivation

Surge in inflation in Europe and the US since 2020 raises some economic issues:

- Should we fight wage-inflation or price-inflation when both increase at a different pace?
- Inflation sometimes handled by "unconventional fiscal policy" (D'Acunto et al., 2018; Dao et al., 2023).
- Should we take advantage from fiscal-monetary interactions?

What we do

- Study HANK model with price and wage Phillips curves (no capital).
- Derive and simulate optimal monetary-fiscal policies with *many* fiscal instruments,
- Consider both supply and demand shocks.

$\rightarrow$ Ramsey allocation (with commitment) after an MIT shock.

Why do we care?

Three questions:

1. When should we deviate from price stability?
2. How do fiscal and monetary policies interact?
3. When do allocations differ between HA and RA economies?

How do we do it?

- Lagrangian approach + refined truncation (LeGrand and Ragot, 2022a, 2022b, 2023).
- Bring closer macro and public finance in the spirit of Heathcote and Tsujiyama (2021).

Selected literature (1/2)

RA economies

- Sticky price and sticky wages in Blanchard (1986), Blanchard and Galí (2007a and 2007b).
- Optimal policies in Erceg et al. (2000), Chugh (2006), Galí (2015, chap. 6), Lorenzoni and Werning (2023).

HA economies

- Monetary policy in HA: Bilbiie (2008), Kaplan, et al. (2018), Nuño and Moll (2018), Auclert (2019).
- Optimal policy HA: Bhandari et al. (2021), Nuño and Thomas (2022), Dyrda and Pedroni (2022), McKay and Wolf (2022), Davila and Schaab (2022).

Selected literature (2/2)

Tools for normative HA:

- Computing transitions: Dyrda and Pedroni (2022).
- Continuous-time: Nuño and Thomas (2022).
- Primal Approach + time-varying perturbations: Bhandari et al. (2022).
- Lagrangian approach: Açikgöz et al. (2022).

What we find

- Equivalence result à la Correia et al. 2008-2013.
  - With a sufficiently rich fiscal system: restore price and/or wage stability.
  - Should be thought as an analytical benchmark.
  - Fiscal system features employee/employer social contributions + income tax. Consistent with the public finance literature (Saez et al. 2012 and Lehmann et al. 2013)
- Deviations from price stability: (mainly for supply shocks)
  - inflation affects the real wage (Keynes, 1936; Mitchell 1986).
  - substitute for missing labor tax.
- RA and HA allocations differ for demand shock because of debt dynamics.

The model

1. Households face uninsurable productivity risk, hold public debt to self-insure (no capital).
2. Firms produce with labor, face sticky prices à la Rotemberg (Bhandari et al., 2022; Le Grand et al., 2022; Acharya, et al. 2022)
3. Unions bargain on behalf of households, choose the same wage-labor for all households, face sticky wages à la Rotemberg as in Auclert et al. (2023b); Alves and Violante (2023).
4. Government finances public spending with a rich fiscal structure.

The model: Different wages

Nominal wages (e.g., W):

+ capital tax τ_t^K

Real wages (e.g., $w = W/P$)

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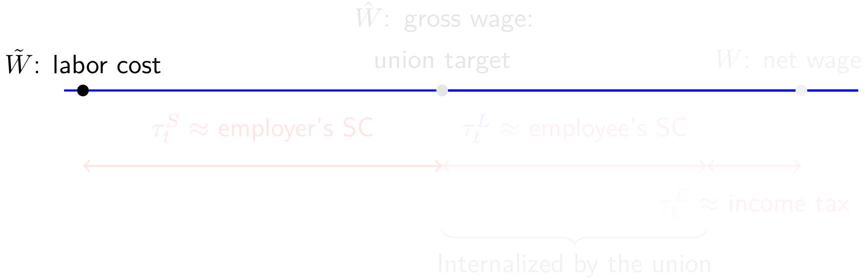

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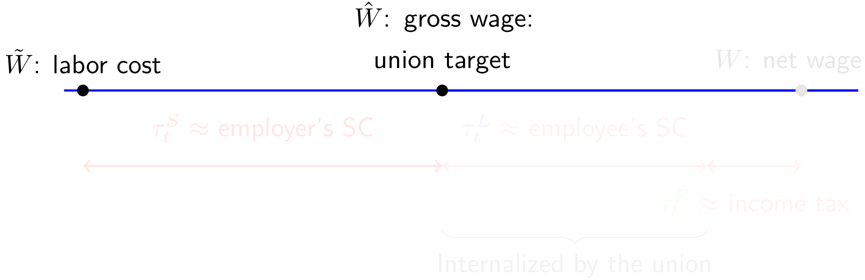

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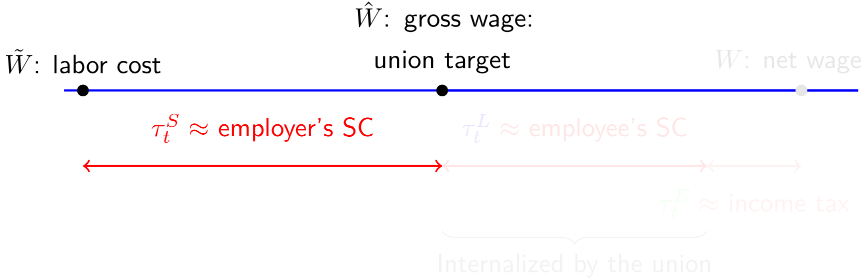

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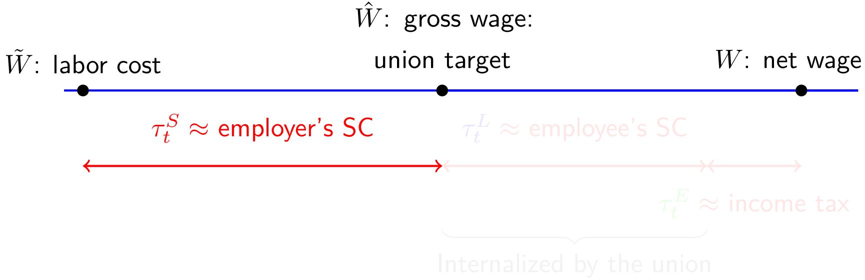

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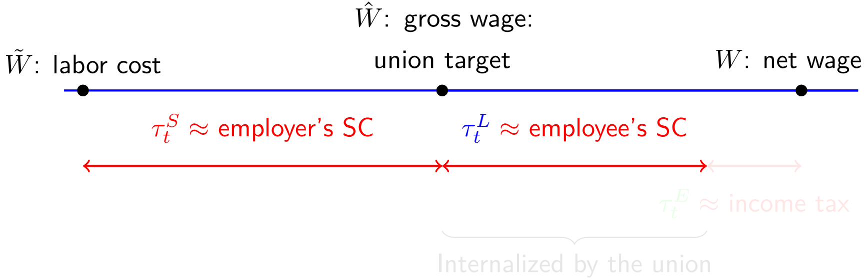

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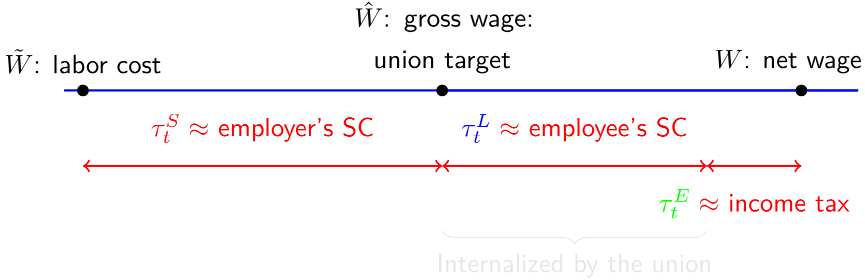

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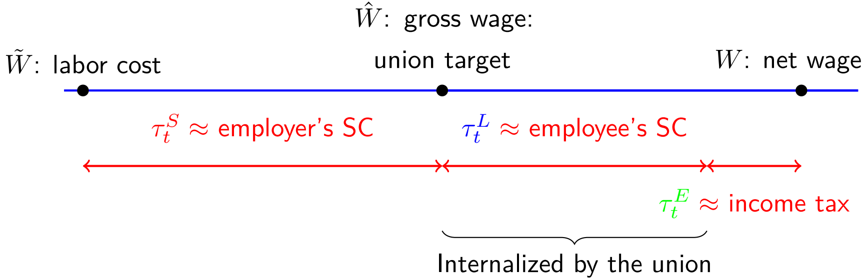

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Real wages (e.g., $w = W/P$)

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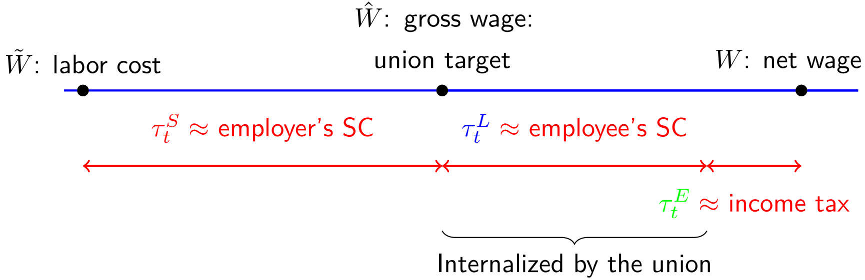

$+$ capital tax τ_t^K

Real wages (e.g., $w = W/P$)

The model: The taxation scheme

- Three non-substitute labor taxes: employer/employee social contribution + income tax.
- Consistent with observed tax schemes in major OECD countries.
- Consistent with the empirical public finance literature on tax incidence.
  - Saez et al. (2012) on Greek data: duality of employer vs employee social contributions
  - Lehmann et al. (2013) on French data: rigid pre-income tax wage target for bargaining

The model: Households' program

Households' program is:

$$\max_{\{c_{i,t},a_{i,t}\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(u(c_{i,t}) - v(L_t)\right),$$

$$\text{s.t. } c_{i,t} + a_{i,t} = (1+r_t)a_{i,t-1} + w_t y_{i,t} L_t,$$

with $c_{i,t} > 0, \; a_{i,t} \geq 0$, and with post-tax rates:

$$w_t = (1-\tau_t^L)(1-\tau_t^E)\hat{w}_t,$$

$$\hat{w}_t = (1-\tau_t^S)\tilde{w}_t,$$

$$\tilde{r}_t = \frac{1+i_{t-1}}{1+\pi_t^P},$$

$$r_t = (1-\tau_t^E)(1-\tau_t^K)\tilde{r}_t.$$

Sticky wages

Differentiated labor, unions have market power, bargain for workers, same hours for all workers. Rotemberg adjustment cost for workers, based on Erceg et al. (2000) (e.g., as in Auclert et al., 2023b; Alves and Violante, 2023).

Wage Phillips curve:

$$\pi_t^W(\pi_t^W+1) = \frac{\varepsilon_W}{\psi_W}\left(v'(L_t) - \frac{\varepsilon_W - 1}{\varepsilon_W}(1-\tau_t^L)\hat{w}_t \int_i y_{i,t} u'(c_{i,t})\ell(di)\right)L_t + \beta\mathbb{E}_t\left[\pi_{t+1}^W(\pi_{t+1}^W+1)\right].$$

Sticky prices

Production: Firms produce with labor (CRS) with productivity Z.
Rotemberg pricing for firms.

Price Phillips curve:

$$\pi_t^P(1+\pi_t^P) = \frac{\varepsilon_P - 1}{\psi_P Z_t}\Big(\tilde{w}_t - Z_t\Big) + \beta \mathbb{E}_t\Big[\pi_{t+1}^P(1+\pi_{t+1}^P)\frac{Y_{t+1}}{Y_t}\Big]$$

Government

Real governmental budget constraint: financing of a public spending stream (G_t) with many taxes, and public debt.

$$G_t + \frac{1+i_t}{1+\pi_t^P} B_{t-1} \leq B_t + \tau_t^E (1-\tau_t^L)\hat{w}_t L_t$$
$$+ \tau_t^K \tilde{r}_t \int_i a_{i,t-1} \ell(di) + \tau_t^L \hat{w}_t L_t + \tau_t^S \tilde{w}_t L_t + \Omega_t.$$

which can also be written as:

$$G_t + (1+r_t) \int_i a_{i,t-1} \ell(di) + w_t L_t \leq \left(1 - \frac{\psi_P}{2}(\pi_t^P)^2\right) Z_t L_t + \int_i a_{i,t} \ell(di).$$

The Ramsey problem

- The planner wants chooses the monetary-fiscal policy mix $(\tau_t^S, \tau_t^E, \tau_t^L, \tau_t^K, B_t, i_t)$, which
- affect prices and allocations $(\pi_t^P, \pi_t^W, w_t, r_t, L_t, (c_{i,t}, a_{i,t}))_{t\geq 0}$,
- to maximize aggregate welfare,
- given possible demand shocks G_t or supply shocks Z_t.

Solution method

Steps:

1. Writing the Ramsey problem (see below)
2. Introducing Lagrange multipliers Multipliers
3. Factorization of the Lagrangian $\mathcal{L}$ to simply derive the FOC of the planner. Lagrangian
4. Relation with public finance: Saez and Santcheva (2016) Public
5. Truncation method to simulate the model (finite state space representation). Truncation

The Ramsey problem: Aggregate welfare

Aggregate welfare.

$$\mathcal{W}_0 = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\int_i \omega(y_t^i) u(c_t^i) \ell(di) - v(L_t) - \frac{\psi_W}{2} (\pi_t^W)^2 \right]$$

- $\omega(y_t^i)$: planner's weights (see also LeGrand, Martin-Baillon, and Ragot 2022; Davila and Schaab, 2022; McKay and Wolf 2023) associated to productivity level y_t^i.
- Chosen to solve the *inverse optimum taxation problem*, as in Heathcote and Tsujiyama (2021),...
- ...while preserving Pareto-optimality.

The Ramsey problem: Formulation

$\max$ over $\left\{\tau_t^S, \tau_t^E, \tau_t^L, \pi_t^P, \pi_t^W, w_t, r_t, L_t, (c_{i,t}, a_{i,t})_{t\geq 0}\right\}$ of $\mathcal{W}_0$ s.t.:

$$G_t + (1+r_t)\int_i a_{i,t-1}\ell(di) + w_t L_t \leq \left(1 - \frac{\psi_P}{2}(\pi_t^P)^2\right) Z_t L_t + \int_i a_{i,t}\ell(di),$$

for all i: $c_{i,t} + a_{i,t} = (1+r_t)a_{i,t-1} + w_t y_{i,t} L_t,$

$$u'(c_{i,t}) = \beta\mathbb{E}_t\left[(1+r_{t+1})u'(c_{i,t+1})\right] + \nu_{i,t},$$

$$\pi_t^W(\pi_t^W + 1) = \frac{\varepsilon_W}{\psi_W}\left(v'(L_t) - \frac{\varepsilon_W - 1}{\varepsilon_W}\frac{w_t}{1-\tau_t^E}\int_i y_{i,t}u'(c_{i,t})\ell(di)\right)L_t + \beta\mathbb{E}_t[\cdots],$$

$$\pi_t^P(1+\pi_t^P) = \frac{\varepsilon_P - 1}{\psi_P}\left(\frac{1}{Z_t}\frac{w_t}{(1-\tau_t^L)(1-\tau_t^S)(1-\tau_t^E)} - 1\right) + \beta\mathbb{E}_t[\cdots],$$

$$(1+\pi_t^W)\frac{w_{t-1}}{(1-\tau_{t-1}^E)(1-\tau_{t-1}^L)} = \frac{w_t}{(1-\tau_t^E)(1-\tau_t^L)}(1+\pi_t^P).$$

The Ramsey problem: Formulation

$$\max \text{ over } \left\{\tau_t^S, \tau_t^E, \tau_t^L, \pi_t^P, \pi_t^W, w_t, r_t, L_t, (c_{i,t}, a_{i,t})_{t\geq 0}\right\} \text{ of } \mathcal{W}_0 \text{ s.t.:}$$

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The Ramsey problem: Formulation

max over $\left\{\tau_t^S, \tau_t^E, \tau_t^L, \pi_t^P, \pi_t^W, w_t, r_t, L_t, (c_{i,t}, a_{i,t})_{t\geq 0}\right\}$ of $\mathcal{W}_0$ s.t.:

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Equivalence result

Proposition (An equivalence result)

When all instruments are available, the government implements an allocation with zero inflation for prices and wages in all periods.

- τ_t^S neutralizes the gap between mpl and wage $\rightarrow$ turns off price Phillips curve.
- τ_t^E neutralizes the gap between mrs and wage $\rightarrow$ turns off wage Phillips curve.
- τ_t^L and τ_t^K neutralize any residual distributional effect of inflation (see LeGrand et al., 2022, Bhandari, et al. 2022; Acharya et al., 2023).

$\Rightarrow$ inflation rates can optimally be set to 0.

Same spirit as Correia and Teles (2008) and Correia et al. (2013).

What about missing instruments? **Demand** shocks

Instruments	**RA**	**HA**
1. $\tau^L + \tau^S + \tau^E$	$\pi^P = \pi^W = 0$ (First-best)	$\pi^P = 0$ and $\pi^W = 0$
2. $\tau^L + \tau^S$	$\pi^P = \pi^W = 0$ (First-best)	$\pi^P = 0$ and $\pi^W \neq 0$
3. τ^S	$\pi^P = \pi^W = 0$	$\pi^P \neq 0$ and $\pi^W \neq 0$
4. τ^L	$\pi^P = \pi^W = 0$	$\pi^P \neq 0$ and $\pi^W \neq 0$

What about missing instruments? **Supply** shocks

Instruments	**RA**	**HA**
1. $\tau^L + \tau^S + \tau^E$	$\pi^P = \pi^W = 0$ (First-best)	$\pi^P = 0$ and $\pi^W = 0$
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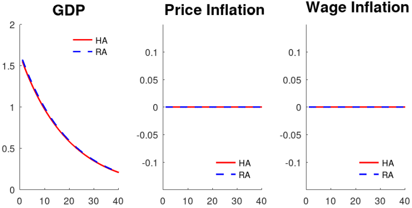
4. τ^L	$\pi^P \neq 0$ and $\pi^W \neq 0$	$\pi^P \neq 0$ and $\pi^W \neq 0$

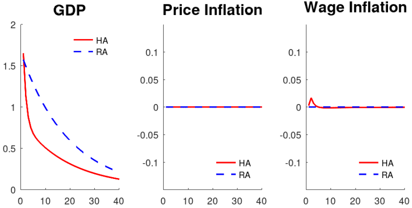
	Preference and technology		
β	Discount factor	0.99	Quarterly calibration
σ	Curvature utility	2	
χ	Scaling param. labor supply	0.01	$L = 1/3$
φ	Frisch elasticity labor supply	0.5	Chetty, et al. 2011
	Shock process		
ρ_y	Autocorrelation idio. income	0.993	Krueger et al. 2018
σ_y	Standard dev. idio. income	6%	Gini$= 0.78$
ρ_z	Autocorrelation TFP shock	0.95	
	Tax system		
τ^L	Labor tax	16%	$G/Y = 15$
τ^S,τ^E,τ^K	Other tax	0%	
B/Y	Public debt over yearly GDP	128%	$MPC = 0.3$
G/Y	Public spending over yearly GDP	15%	Targeted
	Monetary parameters		
ε_p	Elasticity of sub. between goods	6	Schmitt-Grohe and Uribe, 2005
ψ_p	Price adjustment cost	100	Price PC 5%
ε_w	Elasticity of sub. labor inputs	21	Schmitt-Grohe and Uribe, 2005
ψ_w	Wage adjustment cost	2100	Wage PC 1%

Demand shocks

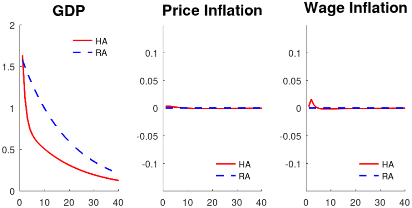
Economy 1: Optimal τ^E, τ^S, τ^L



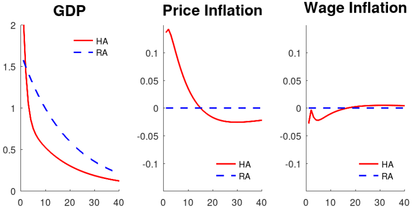
Economy 2: Optimal τ^S, τ^L



Economy 3: Optimal τ^L

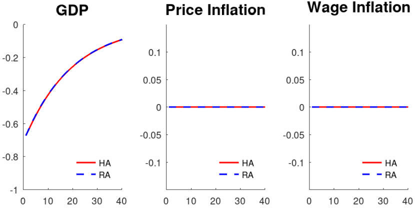


Economy 4: Optimal τ^S

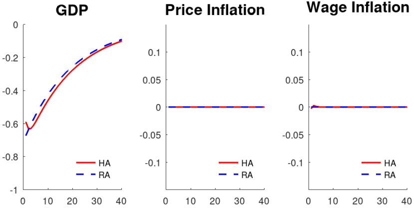


Supply shocks

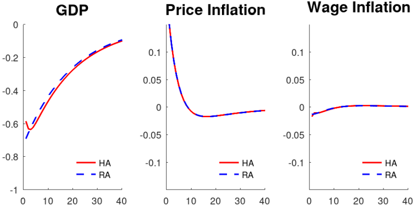
Economy 1: Optimal τ^E, τ^S, τ^L



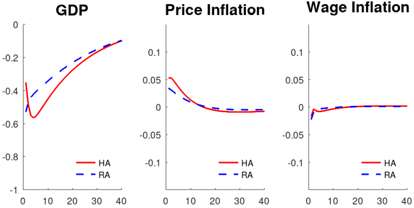
Economy 2: Optimal τ^S, τ^L



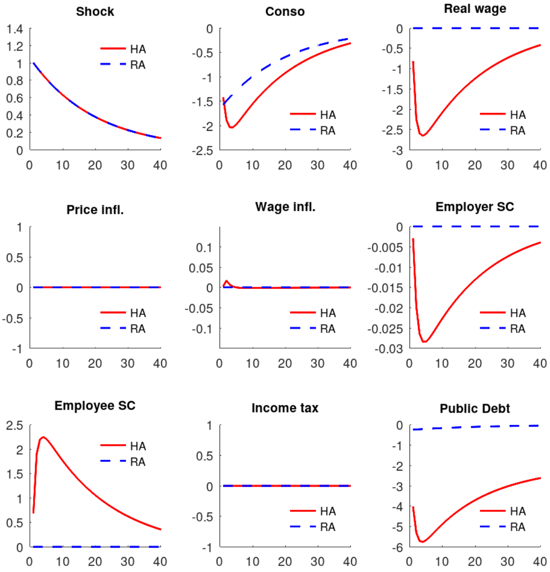
Economy 3: Optimal τ^L



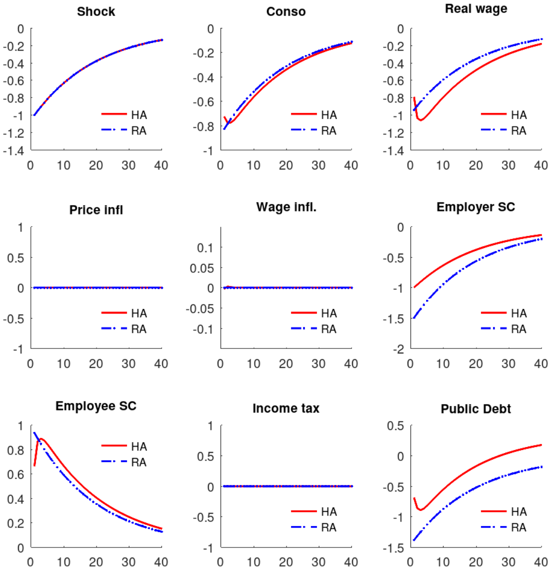
Economy 4: Optimal τ^S



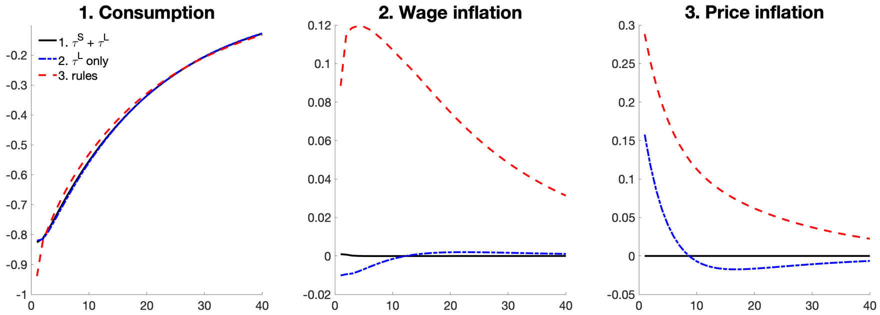
The case τ^E constant (demand shock)



The case τ^E constant (supply shock)



The strange world of rules... ($\phi^{\pi} = 1.5$ and $\rho^{B} = 0.08$)



Conclusion

We have studied 16 economies (supply-demand, RA-HA, 4 fiscal systems)

- HA differ from RA for demand shocks, due to debt management.
- Demand shock: Deviation from price stability when labor tax (τ_t^L) is missing.
- Supply shocks: Deviation from price stability when wage subsidy (τ_t^S) is missing.
  - Germany: *kurzarbeit*, France *activité partielle* were time-varying wage subsidies

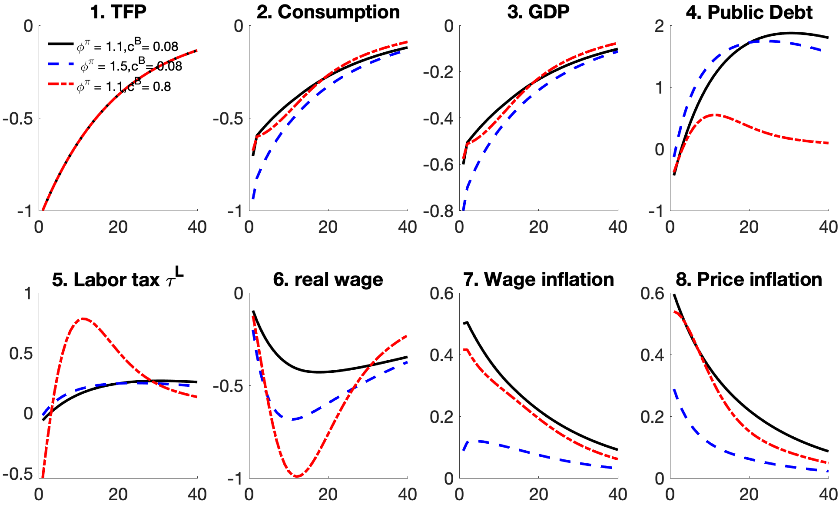
  $\rightarrow$ Non-Keynesian stabilizers.
- Opens the road to thinking about automatic (non-Keynesian) stabilizers, connected to the public finance literature.

Thank you !

1 - Appendix

The sensitivity of the economy to rules

back



The Ramsey problem Back

(Post-tax formulation, Chamley, 1986):

max over $(\tau_t^S, \tau_t^E, \tau_t^L, \pi_t^P, \pi_t^W, w_t, r_t, L_t, (c_{i,t}, a_{i,t})_{t\geq 0}$ of
$\mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \int_i \omega(y_t^i) \left(u(c_t^i) - v(L_t) \right) \ell(di) - \frac{\psi_W}{2} (\pi_t^W)^2 \right]$ with:

μ_t
$$G_t + (1+r_t) \int_i a_{i,t-1} \ell(di) + w_t L_t \leq \left(1 - \frac{\psi_P}{2} (\pi_t^P)^2 \right) Z_t L_t + \int_i a_{i,t} \ell(di),$$

for all $i \in \mathcal{I}$: $c_{i,t} + a_{i,t} = (1+r_t) a_{i,t-1} + w_t y_{i,t} L_t,$

λ_t^i
$$u'(c_{i,t}) = \beta \mathbb{E}_t \left[(1+r_{t+1}) u'(c_{i,t+1}) \right] + \nu_{i,t},$$

γ_t^W
$$\pi_t^W (\pi_t^W + 1) = \frac{\varepsilon_W}{\psi_W} \left(v'(L_t) - \frac{\varepsilon_W - 1}{\varepsilon_W} \frac{w_t}{1-\tau_t^E} \int_i y_{i,t} u'(c_{i,t}) \ell(di) \right) L_t + \beta \mathbb{E}_t [\cdots],$$

γ_t^P
$$\pi_t^P (1 + \pi_t^P) = \frac{\varepsilon_P - 1}{\psi_P} \left(\frac{1}{Z_t} \frac{w_t}{(1-\tau_t^L)(1-\tau_t^S)(1-\tau_t^E)} - 1 \right) + \beta \mathbb{E}_t [\cdots],$$

Λ_t
$$(1+\pi_t^W) \frac{w_{t-1}}{(1-\tau_{t-1}^E)(1-\tau_{t-1}^L)} = \frac{w_t}{(1-\tau_t^E)(1-\tau_t^L)} (1+\pi_t^P)$$

Lagrangian Back

$$
\begin{aligned}
\mathcal{L} = {} & \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \int_i \omega_t^i (u(c_{i,t}) - v(L_t))\ell(di) - \frac{\psi_W}{2}(\pi_t^W)^2 \\
& - \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \int_i (\lambda_{i,c,t} - (1+r_t)\lambda_{i,c,t-1})\, u'(c_{i,t})\ell(di) \\
& - \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t (\gamma_{W,t} - \gamma_{W,t-1})\pi_t^W (1+\pi_t^W) \\
& + \frac{\varepsilon_W}{\psi_W}\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \gamma_{W,t} \left(v'(L_t) - \frac{\varepsilon_W - 1}{\varepsilon_W} w_t \int_i y_{i,t} u'(c_{i,t})\ell(di) \right) L_t \\
& - \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t (\gamma_{P,t} - \gamma_{P,t-1})\pi_t^P (1+\pi_t^P) Z_t L_t + \frac{\varepsilon_P - 1}{\psi_P}\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \gamma_{P,t} \left(\frac{w_t}{(1-\tau_t^L)} - Z_t \right) L_t \\
& + \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \mu_t \left((1 - \frac{\psi_P}{2}(\pi_t^P)^2) Z_t L_t + \int_i a_{i,t}\ell(di) - G_t - (1+r_t)\int_i a_{i,t-1}\ell(di) - w_t L_t \right) \\
& + \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \Lambda_t \left((1+\pi_t^W)\frac{w_{t-1}}{1-\tau_{t-1}^L} - \frac{w_t}{1-\tau_t^L}(1+\pi_t^P) \right)
\end{aligned}
$$

Factorization of the Lagrangian

Back

$$
\begin{aligned}
\mathcal{L} = {} & \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \int_i \omega_t^i (u(c_{i,t}) - v(L_t))\ell(di) - \frac{\psi_W}{2}(\pi_t^W)^2 \\
& - \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \int_i \left(\lambda_{i,c,t} - (1 + r_t)\lambda_{i,c,t-1}\right) u'(c_{i,t})\ell(di) \\
& - \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t (\gamma_{W,t} - \gamma_{W,t-1})\pi_t^W (1 + \pi_t^W) \\
& + \frac{\varepsilon_W}{\psi_W} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \gamma_{W,t} \left(v'(L_t) - \frac{\varepsilon_W - 1}{\varepsilon_W} w_t \int_i y_{i,t} u'(c_{i,t})\ell(di) \right) L_t \\
& - \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t (\gamma_{P,t} - \gamma_{P,t-1})\pi_t^P (1 + \pi_t^P) Z_t L_t + \frac{\varepsilon_P - 1}{\psi_P} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \gamma_{P,t} \left(\frac{w_t}{(1 - \tau_t^L)} - Z_t \right) L_t \\
& + \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \mu_t \left((1 - \frac{\psi_P}{2}(\pi_t^P)^2) Z_t L_t + \int_i a_{i,t}\ell(di) - G_t - (1 + r_t) \int_i a_{i,t-1}\ell(di) - w_t L_t \right) \\
& + \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \Lambda_t \left((1 + \pi_t^W) \frac{w_{t-1}}{1 - \tau_{t-1}^L} - \frac{w_t}{1 - \tau_t^L}(1 + \pi_t^P) \right)
\end{aligned}
$$

Relationship with public finance Back

Define $\psi_{i,t} := \frac{\partial \mathcal{L}}{\partial c_{i,t}}$. It is the Social value of liquidity (equivalent to GSMWW, of Saez and Stantcheva, 2016).

$$\psi_{i,t} := \underbrace{\omega_t^i u'(c_{i,t})}_{\text{direct effet}} - \underbrace{(\lambda_{i,t} - (1+r_t)\lambda_{i,t-1})\, u''(c_{i,t})}_{\text{effect on savings}}$$
$$\underbrace{- \frac{\varepsilon_W - 1}{\psi_W} \gamma_{W,t} w_t L_t y_{i,t} u''(c_{i,t})}_{\text{effect on wage inflation}}.$$

Define $\hat{\psi}_{i,t} := \psi_{i,t} - \mu_t$, then

$$\hat{\psi}_{i,t} = \beta \mathbb{E}_t \left[(1 + r_{t+1}) \hat{\psi}_{i,t+1} \right]$$

$\rightarrow$ the FOCs of the planner can be expressed as a function of $\hat{\psi}_{i,t}$ and of the available instruments.

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Simulating the Model: The truncation back

Problem: Equilibrium = joint distribution over (a, λ_c). Hard to solve.

Main ideas:

- Construct a finite-dimensional state-space representation.
- Go back in the sequential representation (space of idiosyncratic histories).
- Truncate consistently the histories for the N previous periods, to produce a recursive representation: as if there were a Representative A gent for each history.
- "Exact" truncation: Same prices and same aggregate quantities (L, K, C) as in the true model.
- Can prove convergence to the true allocation when the truncation length increases (LeGrand and Ragot, 2022a).
- Problem of the curse of dimensionality: Number of histories k^N, now solved by a **refined truncation**.

Simulating the Model: The truncation

$$\{y_{-\infty}, ..., y_{-N}, y_{-N+1}, .., y_{-1}, y_0\} = y^i$$

back

Simulating the Model: The truncation

$$\{y_{-\infty}, ..., y_{-N}, \underbrace{y_{-N+1}, .., y_{-1}, y_0}_{h}\} = y^i$$

back

Simulating the Model: The truncation

$$\{\underbrace{y_{-\infty}, ..., y_{-N}}_{\sim \xi_h}, \underbrace{y_{-N+1}, .., y_{-1}, y_0}_{h}\} = y^i$$

back

Simulating the Model: The truncation

$$\{\underbrace{y_{-\infty}, ..., y_{-N}}_{\sim\xi_h}, \underbrace{y_{-N+1}, .., y_{-1}, y_0}_{h}\} = y^i$$

$$\begin{aligned}
\pi_{\hat{h}h} &= 1_{h \succeq \hat{h}} \pi_{y_0^{\hat{h}} y_0^h} \\
S_h &= \sum_{\hat{h} \in \mathcal{H}} S_{\hat{h}} \pi_{\hat{h}h} \\
\tilde{a}_{t,h} &= \sum_{\hat{h} \in \mathcal{H}} \frac{S_{t-1,\hat{h}}}{S_{t,h}} \pi_{\hat{h}h} a_{t-1,\hat{h}}. \\
a_{t,h} + c_{t,h} &= w_t y_0^h l_{t,h} + (1 + r_t)\tilde{a}_{t,h} + T_t, \\
\xi_h u'(c_{t,h}) &= \beta \mathbb{E}_t \left[(1 + r_{t+1}) \sum_{\tilde{h} \succeq h} \pi_{h\tilde{h}} \xi_{\tilde{h}} u'(c_{t+1,\tilde{h}}) \right] + \nu_{t,h}.
\end{aligned}$$

back

Simulating the model with rules: The inflation "spiral"

Back Taylor Rule $\phi^{\pi} = 1.1$ and $c^B = 0.08$.