When Do Endogenous Portfolios Matter for HANK?

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Portfolios in heterogeneous-agent macro

- Large part of the het. agent macro literature assumes **exogenous portfolios**
  - Agents choose consumption and savings s.t. idiosyncratic and aggregate risk
  - May save in *accounts* of differing degrees of liquidity... [liquidity vs return]
  - ...but cannot choose the mix of *assets* held in these accounts [risk vs return]
- All of the existing "HANK" literature, in particular, makes this assumption
- Emerging conclusions about the importance of heterogeneity, e.g.
  - deficit-financed fiscal transfers have large&persistent effects on activity
  - nominal asset redistribution matters for aggregate effect of monetary policy
- **Q**: what changes when agents are allowed to hedge aggregate risk?

This paper

- New method for solving for endogenous portfolios in the **sequence-space**

  Auclert-Bardóczy-Rognlie-Straub 2021; vs Bhandari-Bourany-Evans-Golosov in state-space

- Idea: study portfolio choice at date -1 when shocks realize at date 0
- With enough assets, obtain **aggregate risk-sharing** condition across agents with **different idiosyncratic states** s_t:

$$\frac{\mathbb{E}[u'(c_0(s_0))|s_{-1}]}{\mathbb{E}[u'(c_{ss}(s_0))|s_{-1}]} = \lambda_0 \quad \forall s_{-1}$$

Implications:

- Can solve for impulse responses to shocks, portfolios, and λ_0 jointly
  - Computation uses same objects as exogenous-portfolio method
  - Just add simple "correction" to sequence-space jacobian
- Can use λ_0 as stochastic discount factor to solve for s.s. risk premia

- Take a "canonical" HANK model (Auclert, Rognlie, Straub JPE/ARE)
- Let agent optimally choose asset mix, compare with exogenous portfolio
- When do endogenous portfolios matter?
    1. Sometimes **not at all**
       [monetary policy shock example: exogenous portfolios are a natural hedge]
    2. Sometimes **not,** but **provided we constrain portfolios**
       [deficit-financed shock example: hedging portfolios are implausible]
    3. Sometimes **a lot,** and **with reasonable portfolios**
       [nominal bonds example: hedging achievable with risk-free real bonds]

→ Good practice (and simple!) to check optimal portfolios for robustness

Roadmap

1. Heterogeneous-agent portfolios and risk premia
2. Canonical HANK model: exogenous vs endogenous portfolios
3. When do endogenous portfolios matter for HANK?

Heterogeneous-agent portfolios and risk premia

Setting

- Heterogeneous households i can allocate wealth a_i to $K+1$ assets
- Asset k has supply A^k; stochastic payoff $x^k(\epsilon)$, $\epsilon \equiv (\epsilon_1, \ldots, \epsilon_Z)$ (Z shocks)
- Suppose $\epsilon_z = \sigma \overline{\epsilon_z}$, with $\overline{\epsilon_z}$'s independent, $\overline{\epsilon_z} \sim \mathcal{N}(0, \overline{\sigma}_z^2)$, σ common
- Given value function W_i, prices p^k, the problem of household i is:

$$\max_{\{a_i^k\}} \quad \mathbb{E}_\epsilon \left[W_i \left(\sum_{k=0}^{K} x^k(\epsilon)\, a_i^k, \epsilon \right) \right]$$

$$\text{s.t.} \quad \sum_{k=0}^{K} p^k a_i^k = a_i$$

e.g. $W_i(a', \epsilon) = \mathbb{E}_{s'|s_i}\left[V(a', s', \epsilon)\right]$
with $s' \equiv$ idiosyncratic risk

- Classic first-order condition: e.g. $W_i'(a', \epsilon) = \mathbb{E}_{s'|s_i}\left[u'(c(a', s', \epsilon)\right]$

$$\mathbb{E}_\epsilon \left[\frac{x^k(\epsilon)}{p^k} \frac{W_i'\left(\sum_k x^k(\epsilon)\, a_i^k, \epsilon\right)}{\gamma_i} \right] = 1 \quad \forall i, k \tag{1}$$

Perturbation

- Given σ, **equilibrium** is a_i^k, p^k s.t. (1) hold and markets clear, $\int a_i^k = A^k, \ \forall k$
- Consider a perturbation of the model in σ. We look for:
  - $p^k(\sigma)$ to second order in σ around $\sigma = 0$ "second-order risk premia"
  - $\lim_{\sigma \to 0} a_i^k(\sigma)$ "zeroth-order portfolios"

  [as in Tille-van Wincoop 2010, Devereux-Sutherland 2011, Coeurdacier-Rey 2013]
- Evaluating (1) at $\sigma = 0$, we get

$$\frac{x^k(\mathbf{0})}{p^k(0)} = \frac{\gamma_i(0)}{W_i'(Ra_i, \mathbf{0})} \equiv R$$

  Rates of return on all assets equalized to a steady-state R $\left(= \frac{\sum_{k=0}^{K} x^k(\mathbf{0}) A^k}{\int a_i di}\right)$
- For first order, $\overline{\epsilon_z}$ symmetry $\Rightarrow p^k$ and γ^i are even, so $\frac{dp^k}{d\sigma}(0) = \frac{d\gamma^i}{d\sigma}(0) = 0$
- What about second order? Intuitively, we get the C-CAPM...

Second-order perturbation and complete markets

- Indeed, totally differentiating (1) twice around $\sigma = 0$, we obtain:

$$-\sum_{z=1}^{Z}\left(\frac{dx^k(0)/x^k(0)}{d\epsilon_z}-\frac{dx^0(0)/x^0(0)}{d\epsilon_z}\right)\frac{dW_i'(0)/W_i'(0)}{d\epsilon_z}\overline{\sigma}_z^2 = r^k - r^0 \qquad \forall i, k$$

where $\frac{dW_i'(0)}{d\epsilon_z}$ depends on $a_i^k(0)$, $r^k \equiv \frac{1}{2}\left(\sum_{z=1}^{Z}\frac{d^2x^k(0)/x^k(0)}{d\epsilon_z^2}\overline{\sigma}_z^2 - \frac{d^2p^k(0)/p^k(0)}{d\sigma^2}\right)$

- Assume that $K = Z$ and that a rank condition is satisfied for relative returns
- Then we have **complete markets**: for each z, there must exist a λ_z such that:

$$\boxed{\frac{dW_i'(0)/W_i'(0)}{d\epsilon_z} = \lambda_z \qquad \forall i} \tag{2}$$

→ Can use (2) to **test** for portfolio optimality and **solve** for 0th order portfolios

- We only need the **first-order solution** evaluated at **some** portfolios

Solving for complete-market allocation and portfolios

▸ Incomplete markets

- Let $\overline{W_i}(t_i,\epsilon) \equiv W_i\left(\sum_{k=0}^{K} x^k(\epsilon)\overline{a_i^k} + t_i, \epsilon\right)$ be value at portfolios $\overline{a_i^k}$. Then:

$$\frac{dW_i'(\mathbf{0})/W_i(\mathbf{0})}{d\epsilon_z} = \frac{d\overline{W_i'}(\mathbf{0})/\overline{W_i'}(\mathbf{0})}{d\epsilon_z} + R\frac{\overline{W_i''}(\mathbf{0})}{\overline{W_i'}(\mathbf{0})}\frac{dt_i}{d\epsilon_z}$$

where $dt_i/d\epsilon_z \equiv \sum_{k=0}^{K} \frac{\partial x^k(\mathbf{0})/x^k(\mathbf{0})}{\partial \epsilon_z} p^k(0)\left(a_i^k(0) - \overline{a_i^k}\right)$ is extra "transfer" to i

- Using (2), optimal complete-market transfers given λ_z are:

$$\boxed{\frac{dt_i}{d\epsilon_z} = \frac{\overline{W_i'}(\mathbf{0})}{R\overline{W_i''}(\mathbf{0})}\left(\lambda_z - \frac{d\overline{W_i'}(\mathbf{0})/\overline{W_i'}(\mathbf{0})}{d\epsilon_z}\right)} \qquad (3)$$

- Using market clearing, see that $\int (dt_i/d\epsilon_z)di = 0$, which gives λ_z:

$$\boxed{\lambda_z = \left(\int \frac{\overline{W_i'}(\mathbf{0})}{R\overline{W_i''}(\mathbf{0})}di\right)^{-1} \int \frac{\overline{W_i'}(\mathbf{0})}{R\overline{W_i''}(\mathbf{0})}\frac{d\overline{W_i'}(\mathbf{0})/\overline{W_i'}(\mathbf{0})}{d\epsilon_z}di} \qquad (4)$$

- (Can finally back out the 0th order portfolios $a_i^k(0)$ that give $dt_i/d\epsilon_z$ to i)

Second-order risk premia

- Define $R^k(\sigma) \equiv \mathbb{E}\left[x^k(\sigma\bar{\epsilon})\right]/p^k(\sigma)$ as expected return on asset k. We have:

$$\frac{R^k(\sigma)}{R} \approx r^k \sigma^2$$

so defining the random var's $\lambda(\bar{\epsilon}) \equiv \sum_z \lambda_z \bar{\epsilon}_z$ and $X^k(\bar{\epsilon}) \equiv \sum_z \frac{dx^k(\mathbf{0})/x^k(\mathbf{0})}{d\epsilon_z}\bar{\epsilon}_z$,

$$\begin{aligned} \frac{R^k(\sigma) - R^0(\sigma)}{R} &\approx \left(r^k - r^0\right)\sigma^2 \\ &\approx -\mathrm{Cov}\left(\lambda(\bar{\epsilon}), X^k(\bar{\epsilon}) - X^0(\bar{\epsilon})\right)\sigma^2 \end{aligned} \tag{5}$$

$\rightarrow$ λ is a cross-sectional sdf, gives us **second-order risk premia**

- Bottom line:

0th order portfolios $\longleftrightarrow$ 1st order impulses $\longrightarrow$ 2nd order premia

Canonical HANK model: exogenous vs endogenous portfolios

- Households face idiosyncratic risk to their efficiency level e_{it} (Markov Chain)

$$\max \; \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(u(c_{it}) - v\left(n_{it}\right)\right)$$

$$c_{it} + p_t s_{it} + b_{it} \leq \left(p_t + d_t\right) s_{it-1} + \left(1 + r_{t-1}\right) b_{it-1} + e_{it}\left(1 - \tau_t\right) w_t n_{it}$$

$$p_t s_{it} + b_{it} \geq 0$$

$s_{it} \equiv$stocks (price p_t, dividends d_t), $b_{it} \equiv$ bonds, $\tau_t \equiv$ tax rate, $w_t \equiv$ wage

- Production from labor $Y_t = N_t$, monopolistic competition, CES demand
- Flexible prices: $w_t = \frac{1}{\mu}$, dividends $d_t = (1 - \tau_t)\left(1 - \frac{1}{\mu}\right) Y_t$, mass 1 of shares
- Aggregate shock realizes at $t = 0$, perfect foresight over aggregates for $t \geq 0$
- In particular, no arbitrage for $t \geq 0 \Rightarrow p_t = \sum_{s=0}^{\infty} \left(\prod_{u=0}^{s} \frac{1}{1+r_{t+u}}\right) d_{t+s}$

The canonical HANK model, continued

- Fiscal policy sets τ_t, spends G_t and has debt B_t, with

$$B_t = (1 + r_{t-1}) B_{t-1} + G_t - \tau_t Y_t$$

  Sets plans for $G_t, T_t \equiv \tau_t Y_t$ compatible with intertemporal budget constraint

- Sticky nominal wages, implying:
  - Labor rationed, equal allocation rule $n_{it} = N_t = Y_t$
  - Phillips curve for inflation π_t (not relevant to solve for quantities)

- Monetary policy sets real rate r_t, using rule for nominal rate $i_t = r_t + \pi_{t+1}$

- Market clearing in goods, stocks, and bonds:

$$Y_t = G_t + \int c_{it} di \qquad \int s_{it} di = 1 \qquad \int b_{it} di = B_t$$

Steady state, shocks, and portfolios

- Steady-state with no aggregate risk:
  - $Y = N = 1$, $B = 0$, $G = T$, $p = \frac{1}{r}\left(1 - \frac{1}{\mu}\right)(1 - T)$
  - Given $\frac{p+d}{1+r} = p$, only total asset position $a_{it} \equiv ps_{it} + b_{it}$ defined
  - Fix r, find β such that asset market clears: $\int a_{it} di = p$
- Aggregate shock specified as follows:
  - Potential shock to fiscal policy $\{dG_t, dB_t\}_{t \geq 0}$ and monetary policy $\{dr_t\}_{t \geq 0}$
  - Before date 0, uncertainty over realization of $\epsilon = (\epsilon_G, \epsilon_B, \epsilon_r) \sim N(0, \sigma^2 \mathbf{I})$
  - At date 0, ϵ realizes, paths $\{G + \epsilon_G dG_t, B + \epsilon_B dB_t, r + \epsilon_r dr_t\}_{t \geq 0}$ become known
- Two types of portfolios at date 0:
  1. **Exogenous portfolios**: $b_{i,-1} = 0$ (100% in stocks)
  2. **Endogenous portfolios**: $(s_{i,-1}, b_{i,-1})$, optimally chosen at $t = -1$

Equilibrium after date 0 in the sequence space, given portfolios

- Fix initial dist. $\mathcal{D}$ over $\left(s_{i,-1}, b_{i,-1}, e_{i0}\right)$ and an ϵ, so $\{G_t, B_t, r_t\}_{t\geq 0}$ known
- This implies the path $T_t = (1 + r_{t-1})\, B_{t-1} + G_t - B_t$
- For $t \geq 0$, household problem is

$$\max_{c_{it}, a_{it}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(u(c_{it}) - v\left(Y_t\right)\right)$$

$$c_{it} + a_{it} \leq (1 + r_{t-1}) a_{it-1} + e_{it} \left(\frac{Y_t - T_t}{\mu}\right); \quad a_{it} \geq 0; \quad \text{all } t > 0$$

$$c_{i0} + a_{i0} \leq (p_0 + d_0)\, s_{i,-1} + (1 + r)\, b_{i,-1} + e_{it} \left(\frac{Y_0 - T_0}{\mu}\right); \quad a_{i0} \geq 0$$

- Household decisions affected only by aggregates $\{r_t\}$, $\{Y_t - T_t\}$, $p_0 + d_0$
  - $\rightarrow$ $\int a_{it} di$ is given by a "sequence-space function" $\mathcal{A}_t\left(\{r_s\}, \left\{\frac{Y_s - T_s}{\mu}\right\}; p_0 + d_0, \mathcal{D}\right)$
  - $\rightarrow$ Households indifferent between portfolios delivering $a_{it} = p_t s_{it} + b_{it}$

Equilibrium after date 0 in sequence space

- Equilibrium given $\{G_t, B_t, r_t\}$ (so T_t) and initial dist. $\mathcal{D}$ is $\{Y_t, p_t\}$ solving

$$\mathcal{A}_t\left(\{r_t\}, \left\{\frac{Y_s - T_t}{\mu}\right\}, p_0 + \left(1 - \frac{1}{\mu}\right)(Y_0 - T_0), \mathcal{D}\right) = p_t + B_t \quad \forall t \tag{6}$$

$$p_t = \sum_{s=1}^{\infty}\left(\prod_{u=0}^{s}\frac{1}{1 + r_{t+u}}\right)\left(1 - \frac{1}{\mu}\right)(Y_s - T_s) \tag{7}$$

- **Exogenous portfolios:** $\mathcal{D}$ is given
- **Endogenous portfolios**: $\mathcal{D}$ must satisfy condition (2), which reads

$$\frac{\mathbb{E}\left[u'\left(c_0\left(a,e\right)\right)|a_{-1}, e_{-1}\right]}{\mathbb{E}\left[u'\left(c_{ss}\left(a,e\right)\right)|a_{-1}, e_{-1}\right]} = \lambda_0 \quad \forall\left(a_{-1}, e_{-1}\right) \tag{8}$$

Recall the fixed point: portfolios $\rightleftharpoons$ impulse responses

- Write $\mathbf{Y} \equiv \{Y_0, Y_1, Y_2, \ldots\}'$, etc, for sequences
- Let $\mathbf{U} \equiv \{\mathbf{Y}, \mathbf{p}\}$ (unknowns), $\mathbf{Z} \equiv \{\mathbf{G}, \mathbf{B}, \mathbf{r}\}$ (exogenous), then (6)–(7) writes

$$\mathbf{H}\left(\mathbf{U}, \mathbf{Z}, \mathcal{D}\right) = 0$$

- With exogenous portfolios, for small shocks:

$$\mathbf{H}_U d\mathbf{U} + \mathbf{H}_Z d\mathbf{Z} = 0$$

$\Rightarrow$ assuming $\mathbf{H}_U$ invertible:

$$d\mathbf{U} = -\mathbf{H}_U^{-1}\mathbf{H}_Z d\mathbf{Z}$$

Traditional first-order sequence-space solution

[Auclert, Bardóczy, Rognlie, Straub 2021]

Linearization with endogenous portfolios

- With endogenous portfolios, now (heuristically)

$$\mathbf{H}_U d\mathbf{U} + \mathbf{H}_Z d\mathbf{Z} + \mathbf{H}_{\mathcal{D}} d\mathcal{D} = 0$$

  - $d\mathcal{D}$: dist change induced by the complete mkt transfers given shocks $d\mathbf{U}$, $d\mathbf{Z}$

1. Using CM transfer equation (3), we have $d\mathcal{D} = \mathbf{D}_\lambda d\lambda + \mathbf{D}_U d\mathbf{U} + \mathbf{D}_Z d\mathbf{Z}$
2. Using market clearing (4), we have $d\lambda = \lambda'_U d\mathbf{U} + \lambda'_Z d\mathbf{Z}$
3. Putting everything together, the general equilibrium solution is:

$$\left(\mathbf{H}_U + \underbrace{\mathbf{H}_{\mathcal{D}}\mathbf{D}_\lambda \lambda'_U + \mathbf{H}_{\mathcal{D}}\mathbf{D}_U}_{\mathbf{H}_U^{corr}}\right) d\mathbf{U} + \left(\mathbf{H}_Z + \underbrace{\mathbf{H}_{\mathcal{D}}\mathbf{D}_\lambda \lambda'_Z + \mathbf{H}_{\mathcal{D}}\mathbf{D}_Z}_{\mathbf{H}_Z^{corr}}\right) d\mathbf{Z} = 0$$

$$\Rightarrow \qquad d\mathbf{U} = -\left(\mathbf{H}_U + \mathbf{H}_U^{corr}\right)^{-1}\left(\mathbf{H}_Z + \mathbf{H}_Z^{corr}\right) d\mathbf{Z}$$

Just uses modified seq.-space Jacobians ($\mathbf{H}^{corr}$ simple to get in practice)

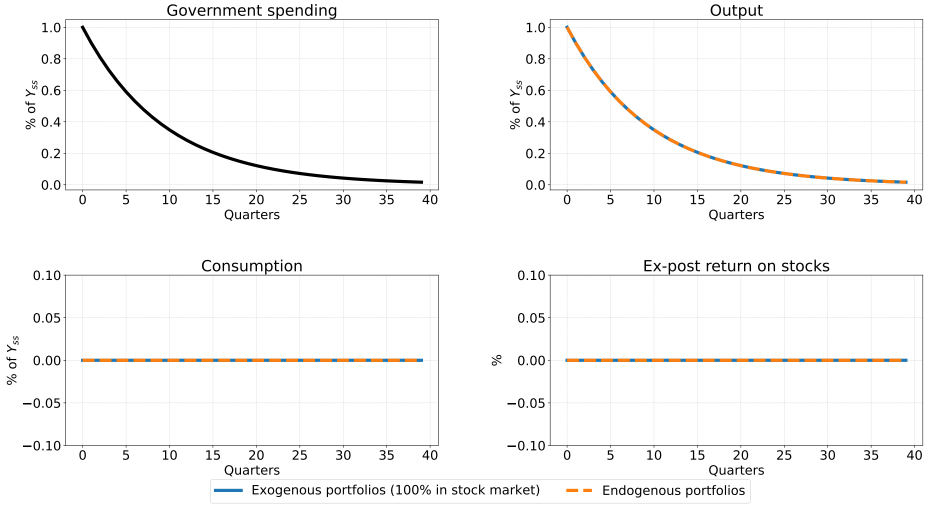
When do endogenous portfolios matter for HANK?

Illustrative calibration

- Elasticity of intertemporal substitution $EIS = 1$
- Standard calibration of income process
- $G = T = 0$
- $\mu = 1.02$, $r = 4\%$ annually $\Rightarrow p \simeq 50\% \times$annual Y
- Steady state features average quarterly income-weighted MPC of 0.18
- All three shocks are AR(1)'s with quarterly persistence $\rho = 0.9$

Example 1: balanced budget G shock

- Set $\sigma_r = \sigma_B = 0$: only shock government spending $d\mathbf{G}$, with $d\mathbf{T} = d\mathbf{G}$

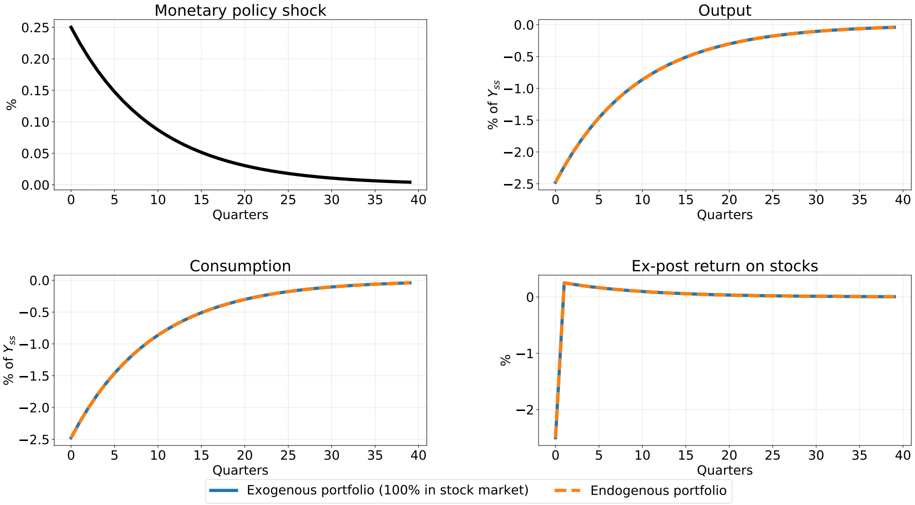


Balanced budget G shock outcome

- Balanced-budget shocks have same effect with endogenous portfolios!
- Why? $d\mathbf{G} = d\mathbf{T} = d\mathbf{Y}$, $d\mathbf{C} = d\mathbf{p} = 0$ is solution with exogenous portfolios (Balanced-budget multiplier: Haavelmo 1945, Auclert-Rognlie-Straub 2024)
- Labor and capital income unaffected for all agents $\Rightarrow dc_{i0} = 0$
- Agents are perfectly hedged against this shock, irrespective of portfolios

Example 2: monetary policy shock

- Set $\sigma_G = \sigma_B = 0$: only monetary policy shock $d\mathbf{r}$



Monetary policy shock: wrap-up

- With monetary policy shocks, 100% stock portfolios are optimal here!
- Why? With these portfolios and this shock, for all agents in equilibrium,

$$\frac{dc_{it}}{c_{it}} = -\sum_{s\geq 0} \frac{dr_{t+s}}{1+r} \quad \forall t$$

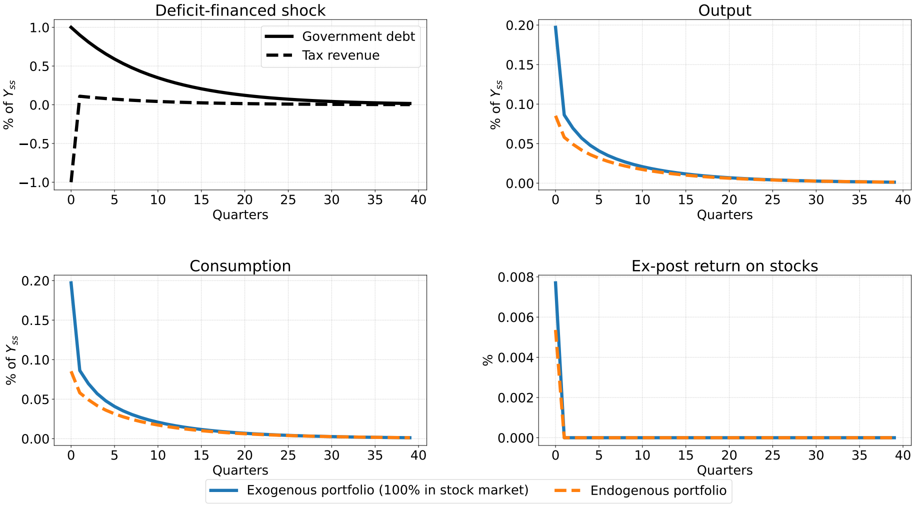
(Werning, 2015)

- Optimal risk-sharing condition (2) is satisfied

→ Endogenous portfolios **do not make a difference** when exogenous portfolios are already a natural hedge

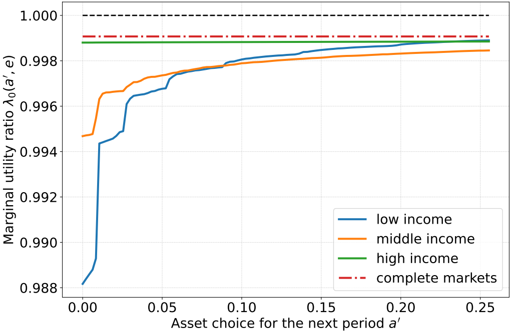
Example 3: deficit-financed transfer shock

- Set $\sigma_G = \sigma_r = 0$: only shock to debt $d\mathbf{B}$ (pure transfer)



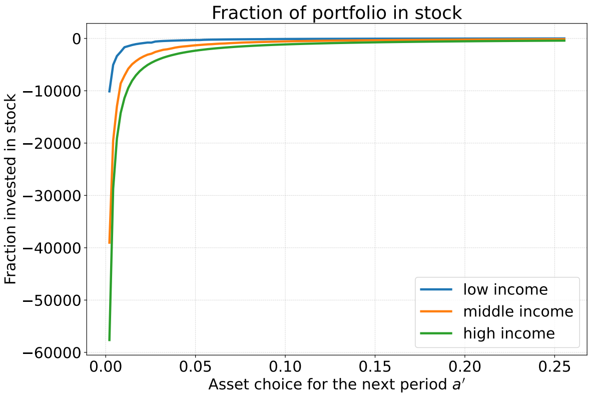
Role of endogenous portfolios

- Endogenous portfolios shrink impact transfer multiplier from 0.2 to 0.08
- Why? Study $\lambda_0(a', e) = \frac{\mathbb{E}[u'(c_0(a',e))|e]}{\mathbb{E}[u'(c_{ss}(a',e))|e]}$ at 100% stock portfolios:



- Low-(a', e) agent MU falls the most: hedge by reducing stock exposure

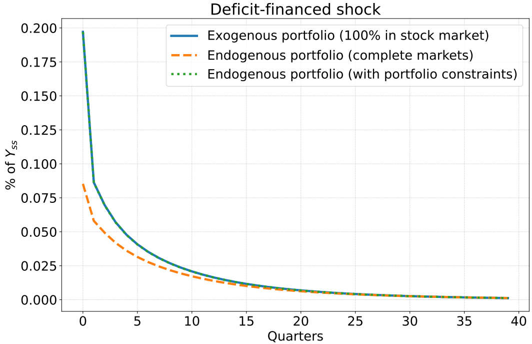
Visualizing portfolios

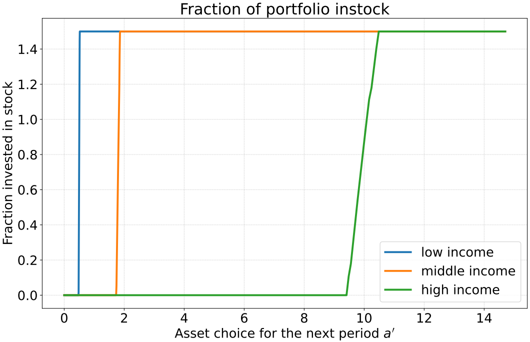


- Optimal portfolios feature implausibly high leverage for poor agents
- What if we add portfolio constraints? ▸ algorithm

Deficit-financed shock with portfolio constraints

- Consider for instance no short sales and 0.5 max leverage ratio:

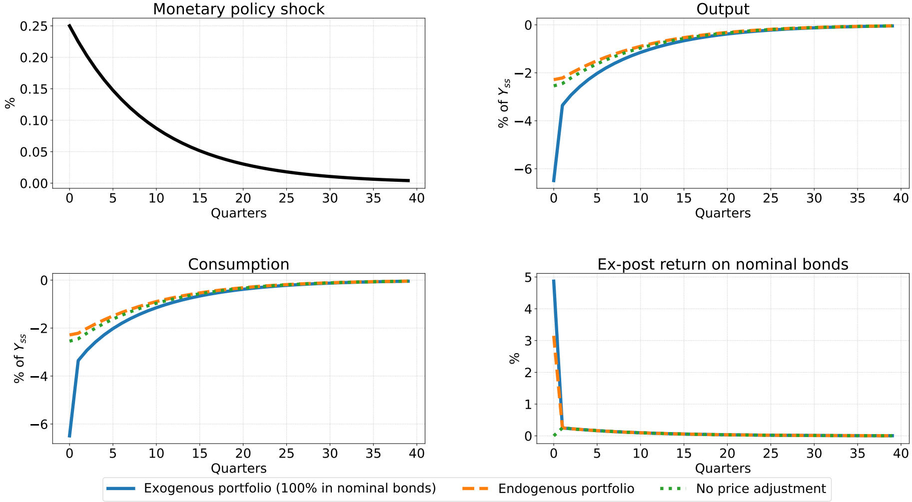




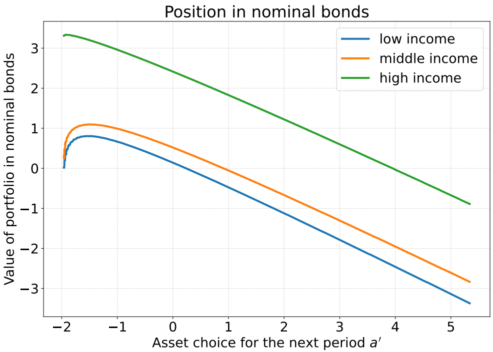
→ Endogenous portfolios **do not make a difference** when the unconstrained hedging portfolios have extreme gross positions [pf. constraints ≃ exog pf.]

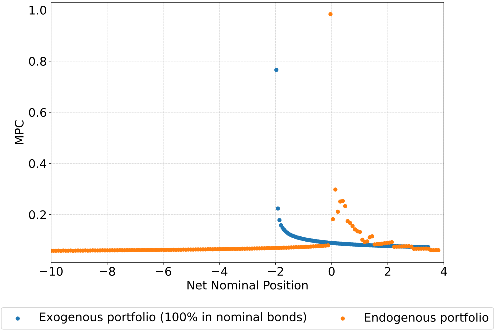
Example 4: monetary shock, nominal bonds

- Now go back to monetary policy shock, but model has nominal bonds:
  - No markups, no government, Huggett model with constraint $a' \geq -A$



3: Visualizing portfolios





→ Endogenous portfolios **can make a difference** when there exist reasonable hedging portfolios that are very different from baseline

Conclusion

- Simple modification of sequence space jacobians gives us:
  - impulse responses with endogenous portfolios
  - second-order risk premia
  - simple to add portfolio constraints, incomplete markets
- In HANK, endogenous portfolios do not always matter
- When exogenous portfolios are a bad hedge vs other assets, they do

Thank you!

Portfolio constaints

Back

- With portfolio constraints, now in the baseline case

$$\mathbf{X}'\boldsymbol{\Sigma}\lambda_i = \mathbf{r} + \boldsymbol{\Theta}'\boldsymbol{\eta}_i$$

  where $\boldsymbol{\Theta}$ collects the portfolio constraints for each asset and η_i captures shadow value of constraints for i

- Here need to solve model iteratively, imposing constraints for guesses that violate them and clearing markets with remaining degrees of freedom

- Recall our key equation from second-order perturbation:

$$\sum_{z=1}^{Z}\left(\frac{dx^k(\mathbf{0})/x^k(\mathbf{0})}{d\epsilon_z}-\frac{dx^0(\mathbf{0})/x^0(\mathbf{0})}{d\epsilon_z}\right)\frac{dW_i'(\mathbf{0})/W_i'(\mathbf{0})}{d\epsilon_z}\overline{\sigma}_z^2=r^0-r^k \qquad \forall i,k$$

- In matrix terms, this is

$$\mathbf{X}'\boldsymbol{\Sigma}\boldsymbol{\lambda}_i=\mathbf{r} \quad \forall i \tag{9}$$

  - $\mathbf{X}\equiv$ sensitivity of relative return of asset k to shock z ($Z\times K$)
  - $\boldsymbol{\lambda}_i\equiv$ sensitivity of value function of agent i to shock z ($Z\times 1$)
  - $\boldsymbol{\Sigma}\equiv$ shock variances ($Z\times Z$)
  - $\mathbf{r}\equiv$ asset-specific relative risk premia ($K\times 1$ vector)

- We also know that the underlying portfolios $\omega_i a_i$ satisfy

$$\mathbf{t}_i=\mathbf{X}\omega_i a_i$$

Incomplete markets

Back

- With incomplete markets, we project complete market transfers on the column space of $\mathbf{X}$:
$$\mathbf{t}_i = \mathbf{X}(\mathbf{X}'\mathbf{\Sigma}\mathbf{X})^{-1}\mathbf{X}'\mathbf{\Sigma}\mathbf{t}_i^{CM}$$
- The risk premia r^k as the same as in the complete markets allocation
- Projection applies to Jacobians, but have to solve the impulse responses to all shocks jointly
- Note also that $\mathbf{X}$ is endogenous, so there is a fixed point